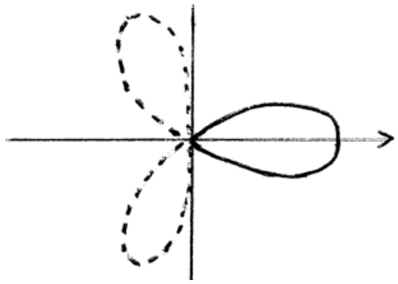
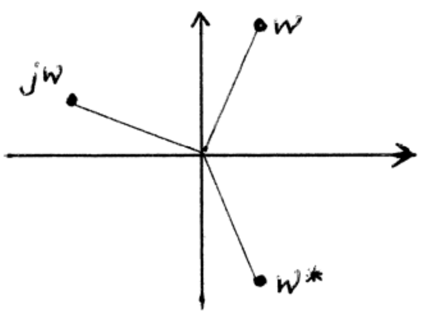
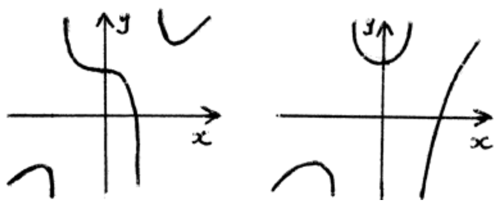
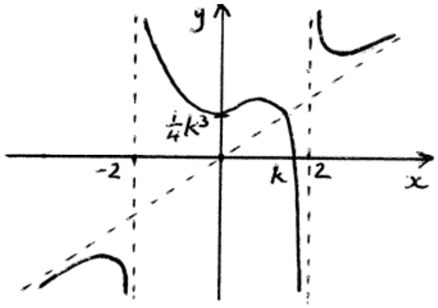
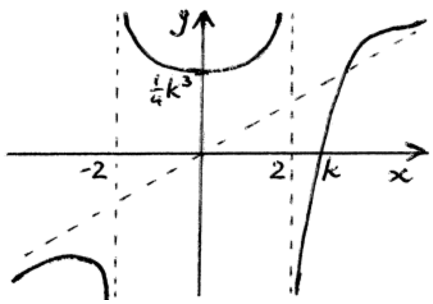
1(a)(i)		B1 B1 B1 **3**	For one loop in correct quadrant(s) For two more loops Continuous and broken lines *Dependent on previous B1B1*
(ii)	Area is $\int \frac{1}{2}r^2\,d\theta = \int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{1}{2}a^2\cos^2 3\theta\,d\theta$ $= \int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{1}{4}a^2(1+\cos 6\theta)\,d\theta$ $= \left[\frac{1}{4}a^2(\theta + \frac{1}{6}\sin 6\theta) \right]_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}$ $= \frac{1}{12}\pi a^2$	M1 A1 M1 A1 B1 **5**	For $\int \cos^2 3\theta\,d\theta$ For a correct integral expression including limits *(may be implied by later work)* For $\int \cos^2 3\theta\,d\theta = \frac{1}{2}\theta + \frac{1}{12}\sin 6\theta$ Accept $0.262a^2$
(b)	$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx = \left[\frac{1}{2}\arcsin\left(\frac{2x}{\sqrt{3}}\right) \right]_0^{\frac{3}{4}}$ $= \frac{1}{2}\arcsin\left(\frac{3}{2\sqrt{3}}\right)$ $= \frac{1}{6}\pi$	M1 A1A1 M1 A1 **5**	For arcsin For $\frac{1}{2}$ and $\frac{2x}{\sqrt{3}}$ *Dependent on previous M1*
(c)	Putting $\sqrt{3}x = \tan\theta$ Integral is $\int_0^{\frac{1}{3}\pi} \frac{1}{\sec^3\theta}\left(\frac{\sec^2\theta}{\sqrt{3}}\right)d\theta$ $= \int_0^{\frac{1}{3}\pi} \frac{\cos\theta}{\sqrt{3}}\,d\theta = \left[\frac{\sin\theta}{\sqrt{3}}\right]_0^{\frac{1}{3}\pi}$ $= \frac{1}{2}$	M1 A1A1 M1 A1 **5**	For any tan substitution For $\frac{1}{(\sec^2\theta)^{\frac{3}{2}}}$ and $\frac{\sec^2\theta}{\sqrt{3}}$ Including limits of θ
	OR Putting $2x = \sqrt{3}\sin\theta$ — M1 A1 Integral is $\int_0^{\frac{1}{3}\pi} \frac{1}{2}\,d\theta$ — A1 M1 $= \frac{1}{6}\pi$ — A1		For any sine substitution For $\int \frac{1}{2}\,d\theta$ For changing to limits of θ *Dependent on previous M1*

2 (i)	$\lvert w \rvert = \frac{1}{2}$, $\arg w = 3\theta$ $\lvert w^* \rvert = \frac{1}{2}$, $\arg w^* = -3\theta$ $\lvert jw \rvert = \frac{1}{2}$, $\arg jw = 3\theta + \frac{1}{2}\pi$  	B1 B1 ft B1B1 ft B2 **6**	w^* and jw in correct positions relative to their w in first quadrant Give B1 for at least two points in correct quadrants
(ii)	$(1+w)(1+w^*) = 1 + \frac{1}{2}e^{3j\theta} + \frac{1}{2}e^{-3j\theta} + (\frac{1}{2}e^{3j\theta})(\frac{1}{2}e^{-3j\theta})$ $= 1 + \frac{1}{2}(\cos 3\theta + j\sin 3\theta) + \frac{1}{2}(\cos 3\theta - j\sin 3\theta) + \frac{1}{4}$ $= \frac{5}{4} + \cos 3\theta$	M1 A1 M1 A1 (ag) **4**	for $w^* = \frac{1}{2}e^{-3j\theta}$ for $1 + \frac{1}{4}$ correctly obtained for $w = \frac{1}{2}(\cos 3\theta + j\sin 3\theta)$ for $\cos 3\theta$ correctly obtained
(iii)	$C + jS = e^{2j\theta} - \frac{1}{2}e^{5j\theta} + \frac{1}{4}e^{8j\theta} - \ldots$ $= \dfrac{e^{2j\theta}}{1 + \frac{1}{2}e^{3j\theta}}$ $= \dfrac{e^{2j\theta}(1 + \frac{1}{2}e^{-3j\theta})}{(1 + \frac{1}{2}e^{3j\theta})(1 + \frac{1}{2}e^{-3j\theta})}$ $= \dfrac{e^{2j\theta}(1 + \frac{1}{2}e^{-3j\theta})}{\frac{5}{4} + \cos 3\theta}$ $= \dfrac{e^{2j\theta} + \frac{1}{2}e^{-j\theta}}{\frac{5}{4} + \cos 3\theta}$ $\left(= \dfrac{4e^{2j\theta} + 2e^{-j\theta}}{5 + 4\cos 3\theta} \right)$ $C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$ $S = \dfrac{4\sin 2\theta - 2\sin\theta}{5 + 4\cos 3\theta}$	M1 M1 A1 M1 A1 M1 A1 (ag) A1 **8**	Obtaining a geometric series Summing an infinite geometric series Using complex conjugate of denom Equating real or imaginary parts Correctly obtained

3 (i)	$(1-\lambda)[\,(-3-\lambda)(-4-\lambda)-12\,]$ $-2[\,-2(-4-\lambda)-12\,]+3[\,-4-2(-3-\lambda)\,]=0$ $(1-\lambda)(\lambda^2+7\lambda)-2(2\lambda-4)+3(2\lambda+2)=0$ $\lambda^3+6\lambda^2-9\lambda-14=0$	M1 A1 A1 (ag) **3**	Evaluating $\det(\mathbf{M}-\lambda\mathbf{I})$ *Allow one omission and two sign errors* $\det(\mathbf{M}-\lambda\mathbf{I})$ correct Correctly obtained (=0 is required)
(ii)	When $\lambda=-1$, $-1+6+9-14=0$ $(\lambda+1)(\lambda^2+5\lambda-14)=0$ $(\lambda+1)(\lambda-2)(\lambda+7)=0$ Other eigenvalues are $2,\ -7$	B1 M1 A1 **3**	or showing that $(\lambda+1)$ is a factor, and deducing that -1 is a root for $(\lambda+1)\times$quadratic factor
(iii)	$x+2y+3z=-x$ $-2x-3y+6z=-y$ $2x+2y-4z=-z$ $z=0,\ x+y=0$ An eigenvector is $\begin{pmatrix}1\\-1\\0\end{pmatrix}$	M1 M1 A1 **3**	At least two equations Solving to obtain an eigenvector
	OR $\begin{pmatrix}2\\2\\3\end{pmatrix}\times\begin{pmatrix}-2\\-2\\6\end{pmatrix}=\begin{pmatrix}18\\-18\\0\end{pmatrix}$ M1 M1 A1		Appropriate vector product Evaluation of vector product
(iv)	$\mathbf{M}\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}6\\0\\2\end{pmatrix}=2\begin{pmatrix}3\\0\\1\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}0\\3\\-2\end{pmatrix}=\begin{pmatrix}0\\-21\\14\end{pmatrix}=-7\begin{pmatrix}0\\3\\-2\end{pmatrix}$	M1 A1A1 **3**	Any method for verifying or finding an eigenvector
(v)	$\mathbf{P}=\begin{pmatrix}1&3&0\\-1&0&3\\0&1&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}-1&0&0\\0&2&0\\0&0&-7\end{pmatrix}^3$ $=\begin{pmatrix}-1&0&0\\0&8&0\\0&0&-343\end{pmatrix}$	B1 ft M1 A1 ft **3**	 seen or implied (ft) *(condone eigenvalues in wrong order)* Order must be consistent with **P** (when B1 has been awarded)
(vi)	By CHT, $\mathbf{M}^3+6\mathbf{M}^2-9\mathbf{M}-14\mathbf{I}=\mathbf{0}$ $\mathbf{M}^2+6\mathbf{M}-9\mathbf{I}-14\mathbf{M}^{-1}=\mathbf{0}$ $\mathbf{M}^{-1}=\frac{1}{14}\mathbf{M}^2+\frac{3}{7}\mathbf{M}-\frac{9}{14}\mathbf{I}$	B1 M1 A1 **3**	Condone omission of **I** Condone dividing by **M**

4 (a)	$\frac{1}{2}(e^x - e^{-x}) + 2(e^x + e^{-x}) = 8$ $5e^{2x} - 16e^x + 3 = 0$ $(5e^x - 1)(e^x - 3) = 0$ $e^x = \frac{1}{5},\ 3$ $x = -\ln 5,\ \ \ln 3$	M1 M1 M1 A1A1 A1 ft **6**	Exponential form Quadratic in e^x Solving to obtain a value of e^x Exact logarithmic form from 2 positive values of e^x *Dependent on M3*
	OR $\sqrt{c^2 - 1} = 8 - 4c$ $15c^2 - 64c + 65 = 0$ M1 M1 $c = \frac{5}{3},\ \frac{13}{5}$ A1A1 $x = \pm\ln 3,\ \pm\ln 5$ M1 $x = \ln 3,\ -\ln 5$ A1		Obtaining quadratic in c (or s) $(15s^2 + 16s - 48 = 0)$ Solving to obtain a value of c (or s) or $s = \frac{4}{3},\ -\frac{12}{5}$ Logarithmic form (including $\pm$ if c) cao
(b)	$\int_0^2 \frac{1}{2}e^x(e^x - e^{-x})\,dx$ $= \left[\frac{1}{4}e^{2x} - \frac{1}{2}x\right]_0^2$ $= (\frac{1}{4}e^4 - 1) - (\frac{1}{4})$ $= \frac{1}{4}(e^4 - 5)$	M1 M1 A1 A1 **4**	Exponential form Integrating to obtain a multiple of e^{2x}
(c)(i)	$\dfrac{\frac{2}{3}}{\sqrt{1 + (\frac{2}{3}x)^2}}$ $\left(= \dfrac{2}{\sqrt{9 + 4x^2}}\right)$	B2 **2**	Give B1 for any non-zero multiple of this
(ii)	$\left[x\,\text{arsinh}(\frac{2}{3}x)\right]_0^2 - \int_0^2 \dfrac{2x}{\sqrt{9+4x^2}}\,dx$ $= \left[x\,\text{arsinh}(\frac{2}{3}x) - \frac{1}{2}\sqrt{9 + 4x^2}\right]_0^2$ $= \left(2\,\text{arsinh}(\frac{4}{3}) - \frac{5}{2}\right) - (-\frac{3}{2})$ $= 2\ln\left(\frac{4}{3} + \sqrt{1 + \frac{16}{9}}\right) - 1$ $= 2\ln 3 - 1$	M1 A1 ft B1 M1 M1 A1 (ag) **6**	Integration by parts applied to $\text{arsinh}(\frac{2}{3}x) \times 1$ for $\int \dfrac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\sqrt{9+4x^2}$ Using both limits (provided both give non-zero values) Logarithmic form for arsinh (intermediate step required)

5 (i)	$x = 2$, $x = -2$ $y = x + \dfrac{4x - k^3}{x^2 - 4}$ Asymptote is $y = x$	B1 M1 A1 **3**	 Dividing out or B2 for $y = x$ stated
(ii)	 $k < 2$ $k > 2$	B1 B1 B1 B1 **4**	$k < 2$ for LH and RH sections for central section, with positive intercepts on both axes $k > 2$ for LH and central sections for RH section, crossing x-axis
(iii)	$\dfrac{dy}{dx} = \dfrac{(x^2 - 4)(3x^2) - (x^3 - k^3)(2x)}{(x^2 - 4)^2}$ $= \dfrac{x(2k^3 + x^3 - 12x)}{(x^2 - 4)^2}$ $\dfrac{dy}{dx} = 0$ when $x = 0$ When $x \approx 0$, $2k^3 + x^3 - 12x > 0$ $\dfrac{dy}{dx} < 0$ when $x < 0$, $\dfrac{dy}{dx} > 0$ when $x > 0$ Hence there is a minimum when $x = 0$	M1 A1 A1 (ag) M1 A1 (ag) **5**	Using quotient rule (or equivalent) Any correct form Correctly shown or evaluating $\dfrac{d^2y}{dx^2}$ when $x = 0$ or $\dfrac{d^2y}{dx^2} = \frac{1}{8}k^3 > 0$ when $x = 0$
(iv)	Curve crosses $y = x$ when $x^3 - k^3 = x(x^2 - 4)$ $x = \frac{1}{4}k^3$ So curve crosses this asymptote	M1 A1 (ag) **2**	

(v)	$k < 2$ 	B2		Asymptotes shown Intercepts $\frac{1}{4}k^3$ and k indicated Minimum on positive y-axis Maximum shown Give B1 for minimum and maximum on central section
	$k > 2$ 	B2	**4**	Asymptotes shown Intercepts $\frac{1}{4}k^3$ and k indicated Minimum on positive y-axis RH section crosses $y = x$ and approaches it from above Give B1 for RH section approaching both asymptotes correctly