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1. (a) Use algebra to find the set of values of x for which

$$x + 2 > \frac{12}{x + 3} \tag{6}$$

(b) Hence, or otherwise, find the set of values of x for which

$$x + 2 > \frac{12}{|x + 3|} \tag{1}$$



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2.

$$z = -2 + (2\sqrt{3})i$$

(a) Find the modulus and the argument of z .

(3)

Using de Moivre's theorem,

(b) find z^6 , simplifying your answer,

(2)

(c) find the values of w such that $w^4 = z^3$, giving your answers in the form $a + ib$ where $a, b \in \mathbb{R}$.

(4)

Lined area for student answers.



4. (a) Show that

$$r^2(r+1)^2 - (r-1)^2 r^2 \equiv 4r^3 \quad (3)$$

Given that $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$

(b) use the identity in (a) and the method of differences to show that

$$(1^3 + 2^3 + 3^3 + \dots + n^3) = (1 + 2 + 3 + \dots + n)^2 \quad (4)$$



6.

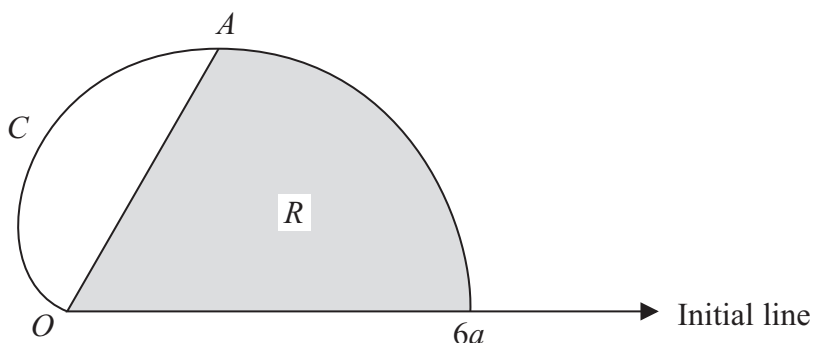


Figure 1

The curve C , shown in Figure 1, has polar equation

$$r = 3a(1 + \cos\theta), \quad 0 \leq \theta < \pi$$

The tangent to C at the point A is parallel to the initial line.

- (a) Find the polar coordinates of A . (6)

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the initial line and the line OA .

- (b) Use calculus to find the area of the shaded region R , giving your answer in the form $a^2(p\pi + q\sqrt{3})$, where p and q are rational numbers. (5)



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Question 6 continued

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Question 7 continued

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8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 7x \frac{dy}{dx} + 16y = 2 \ln x, \quad x > 0 \quad (I)$$

into the differential equation

$$\frac{d^2y}{du^2} - 8 \frac{dy}{du} + 16y = 2u \quad (II)$$

(6)

(b) Find the general solution of the differential equation (II), expressing y as a function of u .

(7)

(c) Hence obtain the general solution of the differential equation (I).

(1)



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Question 8 continued

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