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2. Using algebra, find the set of values of x for which

$$3x - 5 < \frac{2}{x}$$

(5)



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3. (a) Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y \tan x = e^{4x} \cos^2 x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

giving your answer in the form $y = f(x)$.

(6)

(b) Find the particular solution for which $y = 1$ at $x = 0$

(2)



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4.

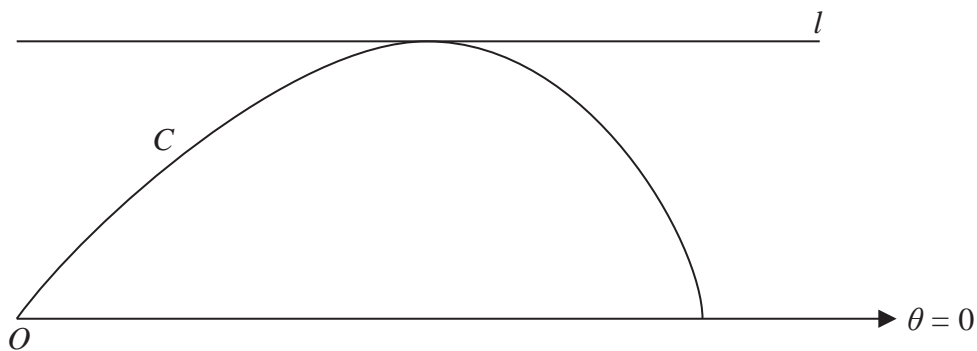


Figure 1

Figure 1 shows the curve C with polar equation

$$r = 2 \cos 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{4}$$

The line l is parallel to the initial line and is a tangent to C .

Find an equation of l , giving your answer in the form $r = f(\theta)$.

(9)



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7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta \tag{5}$$

(b) Hence find the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{2} = 0$$

giving your answers to 3 decimal places where necessary. (5)

(c) Use the identity given in (a) to find

$$\int_0^{\frac{\pi}{4}} (4 \sin^5 \theta - 5 \sin^3 \theta) d\theta$$

expressing your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers. (4)



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8. (a) Show that the substitution $x = e^z$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 2y = 3 \ln x, \quad x > 0 \tag{I}$$

into the equation

$$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z \tag{II}$$

(7)

(b) Find the general solution of the differential equation (II).

(6)

(c) Hence obtain the general solution of the differential equation (I) giving your answer in the form $y = f(x)$.

(1)



