Leave blank

**1.**

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2\cos x$$

(a) Find $\dfrac{d^3y}{dx^3}$ in terms of $x$, $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

**(3)**

At $x = 0$, $y = 1$ and $\dfrac{dy}{dx} = 3$

(b) Find the value of $\dfrac{d^3y}{dx^3}$ at $x = 0$

**(1)**

(c) Express $y$ as a series in ascending powers of $x$, up to and including the term in $x^3$.

**(3)**

Leave blank

**2.** (a) Sketch, on the same axes,

(i) $y = |2x - 3|$

(ii) $y = 4 - x^2$

**(3)**

(b) Find the set of values of $x$ for which

$$4 - x^2 > |2x - 3|$$

**(6)**

Leave blank

**3.** $$f(x) = \ln(1 + \sin kx)$$

where $k$ is a constant, $x \in \mathbb{R}$ and $-\dfrac{\pi}{2} < kx < \dfrac{3\pi}{2}$

(a) Find $f'(x)$

**(2)**

(b) Show that $f''(x) = \dfrac{-k^2}{1 + \sin kx}$

**(3)**

(c) Find the Maclaurin series of $f(x)$, in ascending powers of $x$, up to and including the term in $x^3$.

**(4)**

Leave blank

**4.** Find the general solution of the differential equation

$$x\frac{dy}{dx} + (1 + x\cot x)y = \sin x, \qquad 0 < x < \pi$$

giving your answer in the form $y = f(x)$.

**(9)**

Leave blank

**5.** (a) Express $\dfrac{2}{r(r+1)(r+2)}$ in partial fractions.

**(3)**

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$$

**(4)**

Leave blank

**6.** Solve the equation

$$z^5 = -16\sqrt{3} + 16\text{i}$$

giving your answers in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta < \pi$.

**(8)**

Leave blank

**7.** (a) Find the value of the constant $\lambda$ for which $y = \lambda x \mathrm{e}^{2x}$ is a particular integral of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 4y = 6\mathrm{e}^{2x}$$

**(4)**

(b) Hence, or otherwise, find the general solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 4y = 6\mathrm{e}^{2x}$$

**(3)**

Leave blank

**8.** A complex number $z$ is represented by the point $P$ on an Argand diagram.

(a) Given that $|z| = 1$, sketch the locus of $P$.

**(1)**

The transformation $T$ from the $z$-plane to the $w$-plane is given by

$$w = \frac{z + 7\mathrm{i}}{z - 2\mathrm{i}}$$

(b) Show that $T$ maps $|z| = 1$ onto a circle in the $w$-plane.

**(5)**

(c) Show that this circle has its centre at $w = -5$ and find its radius.

**(2)**

Leave blank

**9.**

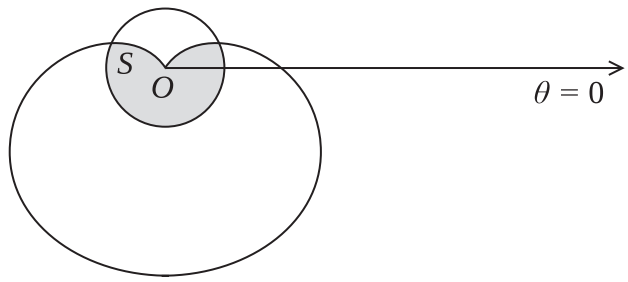


**Figure 1**

Figure 1 shows a sketch of the curves given by the polar equations

$$r = 1 \quad \text{and} \quad r = 2 - 2\sin\theta$$

(a) Find the coordinates of the points where the curves intersect.

**(3)**

The region $S$, between the curves, for which $r < 1$ and for which $r < 2 - 2\sin\theta$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region $S$, giving your answer in the form $a\pi + b\sqrt{3}$, where $a$ and $b$ are rational numbers.

**(8)**