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3. (a) Express $\frac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

(b) Hence show that

$$\sum_{r=1}^n \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

(4)

(c) Evaluate $\sum_{r=10}^{100} \frac{2}{(r+1)(r+3)}$, giving your answer to 3 significant figures.

(2)



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5. (a) Find, in the form $y = f(x)$, the general solution of the equation

$$\frac{dy}{dx} + 2y \tan x = \sin 2x, \quad 0 < x < \frac{\pi}{2}$$

(6)

Given that $y = 2$ at $x = \frac{\pi}{3}$

(b) find the value of y at $x = \frac{\pi}{6}$, giving your answer in the form $a + k \ln b$, where a and b are integers and k is rational.

(4)



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6. The complex number $z = e^{i\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2 \cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5 \cos 3\theta + 12 \cos \theta = 0$$

in the interval $0 \leq \theta < 2\pi$

(4)



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Question 6 continued

Lined area for writing the answer to Question 6.



8.

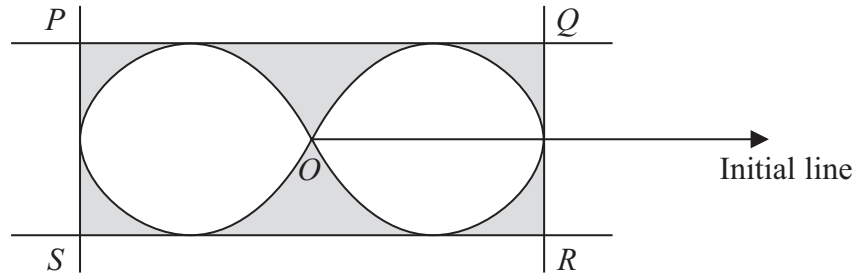


Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \quad \text{where } -\frac{\pi}{4} < \theta \leq \frac{\pi}{4}, \quad \frac{3\pi}{4} < \theta \leq \frac{5\pi}{4}$$

The lines PQ , SR , PS and QR are tangents to C , where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

- (a) Find the total area enclosed by the curve C , shown unshaded inside the rectangle in Figure 1. (4)

- (b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1. (9)



