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2.

$$\frac{d^2y}{dx^2} = e^x \left(2y \frac{dy}{dx} + y^2 + 1 \right)$$

(a) Show that

$$\frac{d^3y}{dx^3} = e^x \left[2y \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx} \right)^2 + ky \frac{dy}{dx} + y^2 + 1 \right],$$

where k is a constant to be found.

(3)

Given that, at $x=0$, $y=1$ and $\frac{dy}{dx} = 2$,

(b) find a series solution for y in ascending powers of x , up to and including the term in x^3 .

(4)



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4. Given that

$$(2r + 1)^3 = Ar^3 + Br^2 + Cr + 1,$$

(a) find the values of the constants A , B and C .

(2)

(b) Show that

$$(2r + 1)^3 - (2r - 1)^3 = 24r^2 + 2$$

(2)

(c) Using the result in part (b) and the method of differences, show that

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(5)



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Question 4 continued

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6.

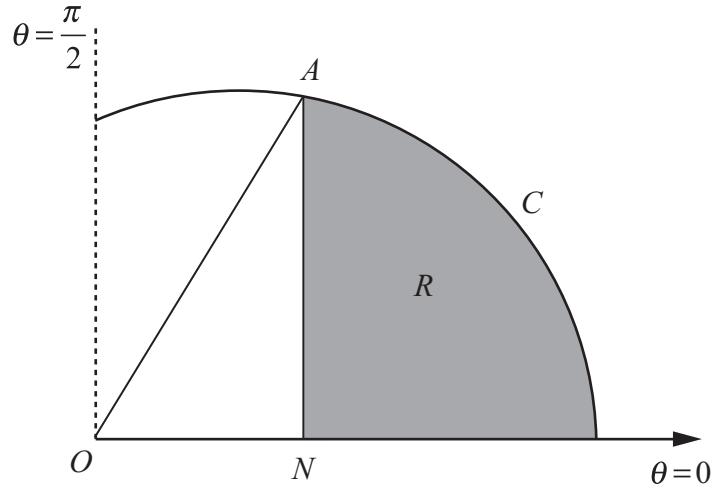


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

At the point A on C , the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the initial line and the line AN .

Find the exact area of the shaded region R .

(9)



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Question 8 continued

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Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

