FP2 Paper 5 *adapted 2005

1. (a) Sketch the graph of $y = |x - 2a|$, given that $a > 0$.

(2)

(b) Solve $|x - 2a| > 2x + a$, where $a > 0$.

(3)(Total 5 marks)

2. Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y \cot 2x = \sin x, \qquad 0 < x < \frac{\pi}{2},$$

giving your answer in the form $y = f(x)$. **(Total 7 marks)**

3. (a) Show that the transformation $y = xv$ transforms the equation

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 + 9x^2)y = x^5, \qquad \text{I}$$

into the equation

$$\frac{d^2v}{dx^2} + 9v = x^2. \qquad \text{II}$$

(5)

(b) Solve the differential equation II to find v as a function of x.

(6)

(c) Hence state the general solution of the differential equation I.

(1)(Total 12 marks)

4. The curve C has polar equation $\quad r = 6 \cos \theta, \qquad -\frac{\pi}{2} \le \theta < \frac{\pi}{2},$

and the line D has polar equation $\quad r = 3 \sec\left(\frac{\pi}{3} - \theta\right), \quad -\frac{\pi}{6} < \theta < \frac{5\pi}{6}.$

(a) Find a cartesian equation of C and a cartesian equation of D.

(5)

(b) Sketch on the same diagram the graphs of C and D, indicating where each cuts the initial line.

(3)

The graphs of C and D intersect at the points P and Q.

(c) Find the polar coordinates of P and Q.

(5)(Total 13 marks)

5. Find the general solution of the differential equation

$$(x+1)\frac{dy}{dx} + 2y = \frac{1}{x}, \qquad x > 0.$$

giving your answer in the form $y = f(x)$.

(7)(Total 7 marks)

6. (a) On the same diagram, sketch the graphs of $y = |x^2 - 4|$ and $y = |2x - 1|$, showing the coordinates of the points where the graphs meet the axes.

(4)

(b) Solve $|x^2 - 4| = |2x - 1|$, giving your answers in surd form where appropriate.

(5)

(c) Hence, or otherwise, find the set of values of x for which $|x^2 - 4| > |2x - 1|$.

(3)(Total 12 marks)

7. (a) Find the general solution of the differential equation

$$2\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 2x = 2t + 9.$$

(6)

(b) Find the particular solution of this differential equation for which $x = 3$ and $\frac{dx}{dt} = -1$ when $t = 0$.

(4)

The particular solution in part (b) is used to model the motion of a particle P on the x-axis. At time t seconds ($t \geq 0$), P is x metres from the origin O.

(c) Show that the minimum distance between O and P is $\frac{1}{2}(5 + \ln 2)$ m and justify that the distance is a minimum.

(4)(Total 14 marks)

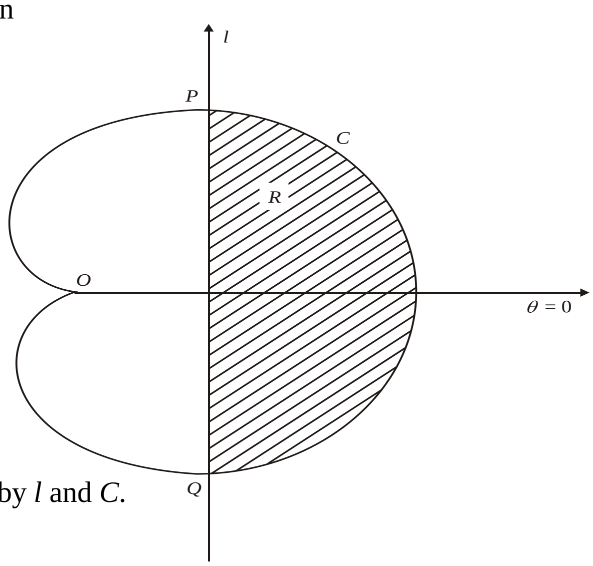
8. The curve C which passes through O has polar equation

$$r = 4a(1 + \cos\theta), \quad -\pi < \theta \leq \pi.$$

The line l has polar equation

$$r = 3a\sec\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

The line l cuts C at the points P and Q, as shown in the diagram.



(a) Prove that $PQ = 6\sqrt{3}a$. (6)

The region R, shown shaded in the diagram, is bounded by l and C.

(b) Use calculus to find the exact area of R.

(7)(Total 13 marks)

9. A complex number z is represented by the point P in the Argand diagram. Given that

$$|z - 3\text{i}| = 3,$$

(a) sketch the locus of P.

2)

(b) Find the complex number z which satisfies both $|z - 3\text{i}| = 3$ and $\arg(z - 3\text{i}) = \frac{3}{4}\pi$.

(4)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{2\text{i}}{z}.$$

(c) Show that T maps $|z - 3\text{i}| = 3$ to a line in the w-plane, and give the cartesian equation of this line.

(5)(Total 11 marks)

10. (a) Given that $z = \text{e}^{\text{i}\theta}$, show that

$$z^n - \frac{1}{z^n} = 2\text{i} \sin n\theta,$$

where n is a positive integer.

(2)

(b) Show that

$$\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta).$$

(5)

(c) Hence solve, in the interval $0 \leq \theta < 2\pi$,

$$\sin 5\theta - 5 \sin 3\theta + 6 \sin \theta = 0.$$

(5)(Total 12 marks)

11. The variable y satisfies the differential equation

$$4(1 + x^2)\frac{\text{d}^2 y}{\text{d}x^2} + 4x\frac{\text{d}y}{\text{d}x} = y.$$

At $x = 0$, $y = 1$ and $\frac{\text{d}y}{\text{d}x} = \frac{1}{2}$.

(a) Find the value of $\frac{\text{d}^2 y}{\text{d}x^2}$ at $x = 0$. **(1)** (c) Find the value of $\frac{\text{d}^3 y}{\text{d}x^3}$ at $x = 0$ **(4)**

(d) Express y as a series, in ascending powers of x, up to and including the term in x^3. **(2)**

(e) Find the value that the series gives for y at $x = 0.1$, giving your answer to 5 decimal places.

(1)(Total 14 marks)