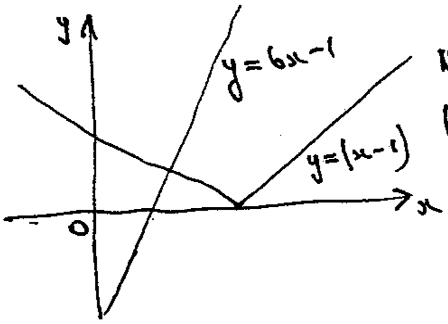
FP2 Mark Schemes from old P4, P5, P6 and FP1, FP2, FP3 papers (back to June 2002)

Please note that the following pages contain mark schemes for questions from past papers.

The standard of the mark schemes is variable, depending on what we still have – many are scanned, some are handwritten and some are typed.

The questions are available on a separate document, originally sent with this one.

1.	$x \geqslant 1$ and $x - 1 > 6x - 1$ $x < 0$ No values	
	OR  No critical value from $y = x - 1$, $y = 6x - 1$	M1 A1
	$\left.\begin{array}{l} y = 1 - x \\ y = 6x - 1 \end{array}\right\} \rightarrow x = \frac{2}{7}$ as critical value	M1 A1
	Solution set $x < \frac{2}{7}$ [Correct final statement needed for A1 here]	A1 cso (5)

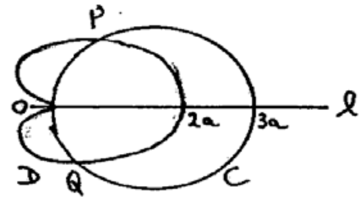
[P4 January 2002 Qn 2]

Question number	Scheme	Marks
2. (a)	$\frac{dv}{dt} - \frac{1}{t}v = 1 \rightarrow \text{I.F.} = e^{-\int \frac{1}{t} dt} = e^{-\ln t} = \frac{1}{t}$	M1 A1 A1
	$\frac{d}{dt}\left(\frac{v}{t}\right) = \frac{1}{t} \rightarrow \frac{v}{t} = \ln t + C$	M1 A1
	$v = t(\ln t + C)$ ⊛	A1 (6)
(b)	$v = 3$ at $t = 2$ so $C = \frac{3}{2} - \ln 2$ or $\cdot 807$	M1 A1
	At $t = 4$, $\frac{v}{4} = \ln 4 + \frac{3}{2} - \ln 2$	M1
	$v = 8.77$	A1 (4)

[P4 January 2002 Qn 6]

3.	(a)	$y = \frac{1}{2}x^2e^x$, $y' = \frac{1}{2}x^2e^x + xe^x$	B1
		$y'' = \frac{1}{2}x^2e^x + 2xe^x + e^x$	B1
		$y'' - 2y' + y = \frac{1}{2}x^2e^x + 2xe^x + e^x - x^2e^x - 2xe^x + \frac{1}{2}x^2e^x$	M1
		$= e^x$	A1 (4)
		OR $ye^{-x} = \frac{1}{2}x^2$, $y'e^{-x} - ye^{-x} = x$ M1, B1	
		$y''e^{-x} - 2y'e^{-x} + ye^{-x} = 1 \Rightarrow y'' - 2y' + y = e^x$ B1 A1	
	(b)	Auxiliary equation $m^2 - 2m + 1 = 0 \Rightarrow m = 1$ repeated	M1, A1
		Complementary function $e^x(A + Bx)$	A1
		General solution $y = e^x(A + Bx) + \frac{1}{2}x^2e^x$	A1 f.t.
		$x = 0,\ y = 1 \Rightarrow A = 1$ (cso)	B1
		$y' = e^x(A + Bx) + Be^x + xe^x + \frac{1}{2}x^2e^x$	M1
		$y' = 2$ at $x = 0 \Rightarrow 2 = A + B \Rightarrow B = 1$	M1 A1
		Specific solution $y = e^x\left(1 + x + \frac{1}{2}x^2\right)$	A1 cso (9)

[P4 January 2002 Qn 7]

Question number	Scheme	Marks
4. (a)	 Circle	B1
	Diameter O → 3a on initial line	B1
	Cardioid cusp at O	B1
	Symmetry on initial line and 2a	B1 (4)
(b)	$3a\cos\theta = a(1+\cos\theta) \rightarrow \cos\theta = \frac{1}{2}$	M1
	$\theta = \pm\frac{\pi}{3} \quad r = \frac{3a}{2}$ at P and Q	A1 A1 (3)
(c)	Area $A_1 = \frac{1}{2}\int a^2(1+\cos\theta)^2\, d\theta$	M1
	$= \frac{1}{2}a^2\int\left[1 + 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right] d\theta$	M1A1
	$= \frac{1}{2}a^2\left[\frac{3\theta}{2} + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]$	(A1, A1, A0)
	Evaluating A1 using limits 0 and $\frac{\pi}{3}$ to get $A_1 = \frac{\pi a^2}{4} + \frac{9\sqrt{3}a^2}{16}$	M1 A1 cao (7)
(d)	Area required $= \frac{9}{4}\pi a^2 - 2A_1 - 2A_2$	M1, B1
	$= \frac{9\pi a^2}{4} - \frac{\pi a^2}{2} - \frac{9\sqrt{3}a^2}{8} - \frac{3\pi a^2}{4} + \frac{9a^2\sqrt{3}}{8}$	M1
	$= \pi a^2$	A1 (4)

[P4 January 2002 Qn 8]

5.	$(x > 0)$ $2x^2 - 5x > 3$ or $2x^2 - 5x = 3$	M1
	$(2x + 1)(x - 3)$, critical values $-½$ and 3	A1, A1
	$x > 3$	A1 ft
	$x < 0$ $2x^2 - 5x < 3$	M1
	Using critical value 0: $-½ < x < 0$	M1, A1 ft
Alt.	$2x - 5 - \frac{3}{x} < 0$ or $(2x - 5)x^2 > 3x$	M1
	$\frac{(2x+1)(x-3)}{x} > 0$ or $x(2x + 1)(x - 3) > 0$	M1, A1
	Critical values $-½$ and 3, $x > 3$	A1, A1 ft
	Using critical value 0, $-½ < x < 0$	M1, A1 ft
		(7 marks)

[P4 June 2002 Qn 4]

6. (*a*)	$\frac{dy}{dx} + y\left(\frac{\sin x}{\cos x}\right) = \cos^2 x$	M1
	Int. factor $e^{\int \tan x\,dx} = e^{-\ln(\cos x)} = \sec x$	M1, A1
	Integrate: $y \sec x = \int \cos x\,dx$	M1 , A1
	$y \sec x = \sin x + C$	A1
	$(y = \sin x \cos x + C \cos x)$	**(6)**
(*b*)	When $y = 0$, $\cos x(\sin x + C) = 0$, $\cos x = 0$	M1
	2 solutions for this $(x = {}^{\pi}/_{2}, {}^{3\pi}/_{2})$	A1 **(2)**
(*c*)	$y = 0$ at $x = 0$: $C = 0$: $y = \sin x \cos x$	M1
	$(y = ½ \sin 2x)$	
	Shape	A1
		A1 **(3)**
	Scales	
		(11 marks)

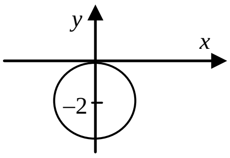
[P4 June 2002 Qn 6]

7.	(*a*)	$2m^2 + 7m + 3 = 0$ $\quad (2m+1)(m+3) = 0$	
		$m = -½, \ -3$	
		C.F. is $y = Ae^{-½t} + Be^{-3t}$	M1, A1
		P.I. $y = at^2 + bt + c$	B1
		$y' = 2at + b, \quad y'' = 2a$	
		$2(2a) + 7(2at + b) + 3(at^2 + bt + c) \equiv 3t^2 + 11t$	M1
		$3a = 3, \ a = 1 \quad 14 + 3b = 11, \ b = -1$	A1
		$4 - 7 + 3c = 0, \ c = 1$	M1, A1
		General solution: $y = Ae^{-½t} + Be^{-3t} + (t^2 - t + 1)$	A1 ft **(8)**
	(*b*)	$y' = -½Ae^{-½t} - 3Be^{-3t} + (2t - 1)$	M1
		$t = 0, \ y' = 1: \ 1 = -1 - ½A - 3B$	
		$t = 0, y = 1: \ 1 = 1 + A + B$ one of these	M1, A1
		Solve: $A + B = 0, \ A + 6B = -4$	
		$A = {}^4/_5, B = -{}^4/_5$	M1
		$y = (t^2 - t + 1) + \frac{4}{5}(e^{-½t} - e^{-3t})$	A1 **(5)**
	(*c*)	$t = 1: \quad y = \frac{4}{5}(e^{-½} - e^{-3}) + 1 \quad (= 1.445\ldots)$	B1 **(1)**
			(14 marks)

[P4 June 2002 Qn 7]

8.	(a)	$y = r\sin\theta = a(3\sin\theta + \sqrt{5}\sin\theta\cos\theta)$	
		$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = a(3\cos\theta + \sqrt{5}\cos 2\theta)$	M1, A1
		$2\sqrt{5}\cos^2\theta + 3\cos\theta - \sqrt{5} = 0$	
		$\cos\theta = \dfrac{-3 \pm \sqrt{9+40}}{4\sqrt{5}}$ **,** $\cos\theta = \dfrac{1}{\sqrt{5}}$	M1, A1
		$\theta = \pm 1.107\ldots$	A1 ft
		$r = 4a$	A1 ft **(6)**
	(b)	$2r\sin\theta = 20$	M1
		$8a\sin\theta = 20$, $a = \dfrac{20}{8\sin\theta} = 2.795\ldots$	M1, A1 **(3)**
	(c)	$(3 + \sqrt{5}\cos\theta)^2 = 9 + 6\sqrt{5}\cos\theta + 5\cos^2\theta$	B1
		Integrate: $9\theta + 6\sqrt{5}\sin\theta + 5\left(\dfrac{\sin 2\theta}{4} + \dfrac{\theta}{2}\right)$	M1, A1
		Limits used: $[\ldots]_0^{2\pi} = 18\pi + 5\pi$ (or upper limit: $9\pi + \dfrac{5\pi}{2}$)	A1
		$\tfrac{1}{2}\int_0^{2\pi} r^2 d\theta = a^2(23\pi) \approx 282\ \mathrm{m}^2$	M1, A1 **(6)**
			(15 marks)

[P4 June 2002 Qn 8]

9.	(*a*)(i)	$\lvert x + (y - 2)\mathrm{i}\rvert = 2\lvert x + (y + \mathrm{i})\rvert$	M1
		$\therefore x^2 + (y - 2)^2 = 4(x^2 + (y + 1)^2)$	
	(ii)	so $3x^2 + 3y^2 + 12y = 0$ any correct from; 3 terms; isw	A1 **(2)**
		 Sketch circle	B1
		Centre (0,–2)	B1
		$r = 2$ or touches axis	B1 **(3)**
	(*b*)	$w = 3(z - 7 + 11\mathrm{i})$	B1
		$= 3z - 21 + 33\mathrm{i}$	B1 **(2)**
			(7 marks)

[P6 June 2002 Qn 3]

10.	(*a*)	$y\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}; + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}; + \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ marks can be awarded in(b)	M1 A1; B1;B1
		$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{-3\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x}}{y}$ or sensible correct alternative	B1 **(5)**
	(*b*)	When $x = 0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2$, and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 5$	M1A1, A1 ft
		$\therefore \quad y = 1 + x - x^2 + \frac{5}{6}x^3 \ldots$	M1, A1 ft **(5)**
	(*c*)	Could use for $x = 0.2$ but not for $x = 50$ as	B1
		approximation is best at values close to $x = 0$	B1 **(2)**
			(12 marks)

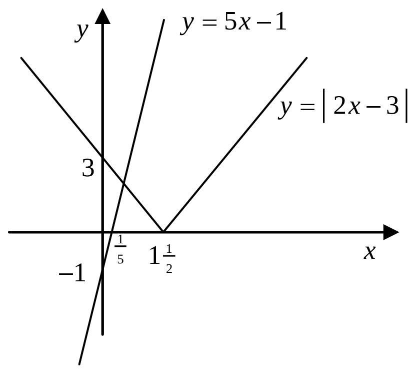
[P6 June 2002 Qn 4]

11.	$zw =$	
	$12\left(\cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3}\right) + 12\mathrm{i}\left(\sin\frac{\pi}{4}\cos\frac{2\pi}{3} + \cos\frac{\pi}{4}\sin\frac{2\pi}{3}\right)$	B1
	$= 12\left[\cos\frac{11\pi}{12} + \mathrm{i}\sin\frac{11\pi}{12}\right]$	M1 A1
		(3 marks)

[P4 January 2003 Qn 1]

12. (*a*)	$\frac{1}{r+1} - \frac{1}{r+3}$	B1 B1 (2)
(*b*)	$\sum_{1}^{n} \frac{1}{r+1} - \frac{1}{r+3} = \frac{1}{2} - \cancel{\frac{1}{4}}$	
	$+ \frac{1}{3} - \frac{1}{5}$	
	$+ \cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}$	M1
	$\vdots$	
	$+ \cancel{\frac{1}{n}} - \frac{1}{n+2}$	
	$+ \cancel{\frac{1}{n+1}} - \frac{1}{n+3}$	
	$= \left(\frac{1}{2} + \frac{1}{3}\right) + \left(-\frac{1}{n+2} - \frac{1}{n+3}\right)$	A1 A1
	$= \frac{5}{6} - \left(\frac{5n^2 + 25n + \cancel{30} - 12n - \cancel{30}}{6(n+2)(n+3)}\right)$	M1
	$= \frac{n(5n+13)}{6(n+2)(n+3)}$ *	A1 cso (5)
		(7 marks)

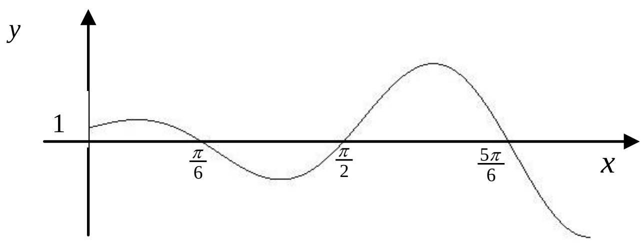
[P4 January 2003 Qn 3]

13.	(*a*) 	shape points on axes	B1 B1 **(2)**
	(*b*) $-2x + 3 = 5x - 1$		M1
	$x = \frac{4}{7}$		A1
	$x > \frac{4}{7}$		A1 ft **(3)**
			(5 marks)

[P4 January 2003 Qn 2]

14. (*a*)	$v + x\frac{\mathrm{d}v}{\mathrm{d}x}$,= $(4 + v)(1 + v)$		M1, M1
	$x\frac{\mathrm{d}v}{\mathrm{d}x} = v^2 + 5v + 4 - v$		A1
	$x\frac{\mathrm{d}v}{\mathrm{d}x} = (v + 2)^2$ *		A1 **(4)**
(*b*)	$\int \frac{1}{(v+2)^2}\,\mathrm{d}v = \int \frac{1}{x}\,\mathrm{d}x$		B1, M1
	$-\frac{1}{2+v} = \ln x + c$	must have + c	M1 A1
	$2 + v = -\frac{1}{\ln x + c}$		M1
	$v = -\frac{1}{\ln x + c} - 2$		A1 **(5)**
(*c*)	$y = -2x - \frac{x}{\ln x + c}$		B1 **(1)**
			(10 marks)

[P4 January 2003 Qn 5]

15.	(*a*)	$y = \lambda x \cos 3x$	
		$\frac{dy}{dx} = \lambda \cos 3x - 3\lambda x \sin 3x$	M1 A1
		$\frac{d^2y}{dx^2} = -3\lambda \sin 3x - 3\lambda \sin 3x - 9\lambda x \cos 3x$	A1
		$\therefore\ -6\lambda \sin 3x - 9\lambda x \cos 3x + 9\lambda x \cos 3x = -12 \sin 3x$	
		$\lambda = 2$ cso	A1 **(4)**
	(*b*)	$\lambda^2 - 9 = 0$	M1
		$\lambda = (\pm)3\text{i}$	A1
		$\therefore\ y = A \sin 3x + B \cos 3x$ form	M1
		$\therefore\ y = A \sin 3x + B \cos 3x + 2x \cos 3x$	A1 ft on λ's **(4)**
	(*c*)	$y = 1, x = 0 \Rightarrow B = 1$	B1
		$\frac{dy}{dx} = 3A \cos 3x - 3B \sin 3x + 2 \cos 3x - 6x \sin 3x$	M1 A1ft on λ's
		$2 = 3A + 2 \Rightarrow A = 0$	
		$\therefore\ y = \cos 3x + 2x \cos 3x$	A1 **(4)**
	(*d*)	 axes shape	B1 B1 **(2)**
			(14 marks)

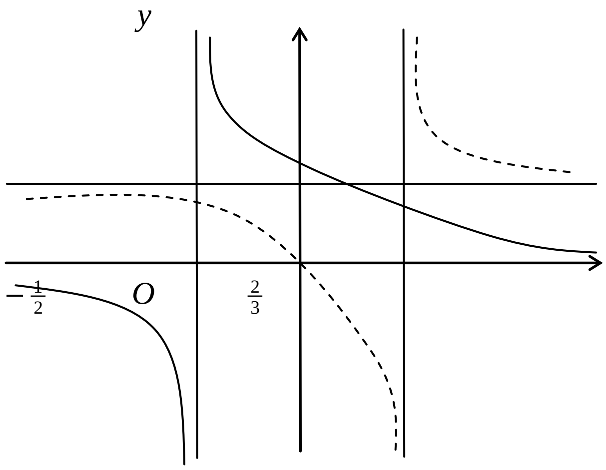
[P4 January 2003 Qn 7]

16. (*a*)	$\frac{1}{2}a^2\int 1+\cos^2\theta+2\cos\theta \quad d\theta$	M1 A1correct with limits
	$=\frac{1}{2}a^2\int 1+\frac{\cos 2\theta+1}{2}+2\cos\theta\ d\theta$	M1 A1
	$= 2\times\frac{1}{2}a^2\left[\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}+2\sin\theta\right]_0^{\pi}$	A1
	$= a^2\left[\frac{3\pi}{2}\right] = \frac{3\pi a^2}{2}$	A1 **(6)**
(*b*)	$x = a\cos\theta + a\cos^2\theta$ $r\cos\theta$	M1
	$\frac{dx}{d\theta} = -a\sin\theta - 2a\cos\theta\sin\theta$	A1
	$\frac{dx}{d\theta} = 0 \Rightarrow \cos\theta = -\frac{1}{2}$ finding θ	M1
	$\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$	
	$r = \frac{a}{2}$ or $r = \frac{a}{2}$ finding r	M1
	A: $r = \frac{a}{2}$, $\theta = \frac{2\pi}{3}$	
	B: $r = \frac{a}{2}$, $\theta = \frac{-2\pi}{3}$ both A and B	A1 **(5)**
(*c*)	$x = -\frac{1}{4}a$ $\therefore WX = 2a + \frac{1}{4}a = 2\frac{1}{4}a$	M1 A1 **(2)**
(*d*)	$WXYZ = \frac{27\sqrt{3}a^2}{8}$	B1 ft **(1)**
(*e*)	Area $= \frac{27\sqrt{3}}{8}\times 100 - \frac{3\pi\times 100}{2} = 113.3\text{ cm}^2$	M1 A1 **(2)**
		(16 marks)

[P4 January 2003 Qn 8]

17.	(*a*)	$\dfrac{r^2-(r-1)^2}{r^2(r-1)^2}=\dfrac{2r-1}{r^2(r-1)^2}$	M1 A1 (2)
	(*b*)	$\displaystyle\sum_{r=2}^{n}\frac{2r-1}{r^2(r-1)^2}=\sum_{r=2}^{n}\frac{1}{(r-1)^2}-\frac{1}{r^2}$	M1
		$=\dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{2^2}-\dfrac{1}{3^2}+\ldots+\dfrac{1}{(n-1)^2}-\dfrac{1}{n^2}$	M1
		$=1-\dfrac{1}{n^2}$ (*)	A1 cso (3)

[P4 June 2003 Qn 1]

18.	Identifying as critical values $-\frac{1}{2}$, $\frac{2}{3}$	B1, B1
	Establishing there are no further critical values	
	Obtaining $2x^2 - 2x + 2$ or equivalent	M1
	$\Delta = 4 - 16 < 0$	A1
	Using exactly two critical values to obtain inequalities	M1
	$-\frac{1}{2} < x < \frac{2}{3}$	A1
		(6 marks)
Graphical alt.	Identifying $x = -\frac{1}{2}$ and $x = \frac{2}{3}$ as vertical asymptotes	B1, B1
	Two rectangular hyperbolae oriented correctly with respect to asymptotes in the correct half-planes.	M1
	Two correctly drawn curves with no intersections	A1
	As above	M1, A1
		

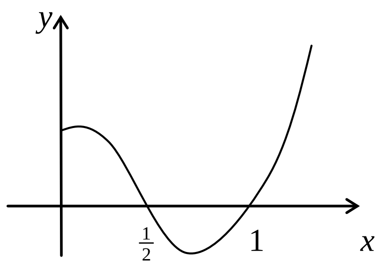
[P4 June 2003 QN n2]

19.	(*a*)	$\frac{dt}{dx} = 2x$ or equivalent	M1
		$I = \frac{1}{2}\int te^{-t}\,dt$ complete substitution	M1
		$= -te^{-t}\,dt + \frac{1}{2}\int e^{-t}\,dt$	M1 A1
		$= -\frac{1}{2}te^{-t} - \frac{1}{2}e^{-t}\ (+c)$	A1
		$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	A1 **(6)**
	(*b*)	I.F. $= e^{\int \frac{3}{x}dx} = x^3$ (or multiplying equation by x^2)	B1
		$\frac{d}{dx}(x^3y) = x^3e^{-x^2}$ or $x^3y = \int x^3e^{-x^2}\,dx$	M1
		$x^3y = -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + \underline{C}$	A1ft <u>A1</u> (4) **(10 marks)**
Alts	(*a*)	(i) mark $t = -x^2$ similarly	M1
		(ii) $\int x^2.(xe^{-x^2})\,dx$ with evidence of attempt at integration by parts	M1
		$= x^2(-\frac{1}{2}e^{-x^2}) + \frac{1}{2}\int 2x.e^{-x^2}\,dx$	M1 A1 + A1
		$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	M1 A1 (6)
		(iii) $u = e^{-x^2}$, $\frac{du}{dx} = -2xe^{-x^2}$	M1
		$x^2 = \ln u$ hence $I = \int \frac{1}{2}\ln u\,du$	M1
		$= \frac{1}{2}u\ln u - \frac{1}{2}\int u.\frac{1}{u}\,du$	M1 A1
		$= \frac{1}{2}u\ln u - \frac{1}{2}u\ (+c)$	A1
		$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2}\ (+c)$	A1 (6)
		(The result $\int \ln u\,du = u\ln u - u$ may be quoted, gaining M1 A1 A1 but must be completely correct.)	

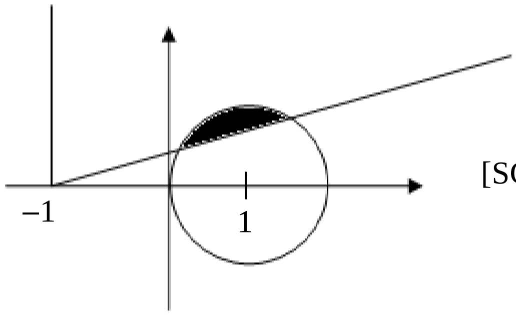
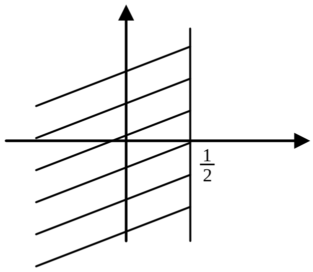
[P4 June 2003 Qn 6]

20. (*a*)	A: $(5a, 0)$ B: $(3a, 0)$	allow on a sketch	B1, B1 (2)
(*b*)	$3 + 2\cos\theta = 5 - 2\cos\theta$		M1
	$\cos\theta = \frac{1}{2}$		M1
	$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$	(allow $-\frac{\pi}{3}$)	A1
	Points are $(4a, \frac{\pi}{3})$, $(4a, \frac{5\pi}{3})$		A1 (4)
(*c*)	$(\frac{1}{2})\int r^2\, d\theta = (\frac{1}{2})\int (5 - 2\cos\theta)^2\, d\theta$		
	$= (\frac{1}{2})\int (25 - 20\cos\theta + 4\cos^2\theta)\, d\theta$		M1
	$= (\frac{1}{2})\int (25 - 20\cos\theta + 2\cos 2\theta + 2)\, d\theta$		M1
	$= (\frac{1}{2})[27\theta - 20\sin\theta + \sin 2\theta]$		A1
	$(\frac{1}{2})\int r^2\, d\theta = (\frac{1}{2})\int (3 + 2\cos\theta)^2\, d\theta$		
	$= (\frac{1}{2})\int (9 + 12\cos\theta + 4\cos^2\theta)\, d\theta$		
	$= (\frac{1}{2})\int (11 + 12\cos\theta + 2\cos 2\theta)\, d\theta$		
	$= (\frac{1}{2})[11\theta + 12\sin\theta + \sin 2\theta]$	2nd integration	A1
	Area $= 2 \times \frac{1}{2}\int(5 - 2\cos\theta)^2\, d\theta + 2 \times \frac{1}{2}\int(3 + 2\cos\theta)^2\, d\theta$	(addition; condone 2/½)	M1
	$= \ldots \int_0^{\frac{\pi}{3}} \ldots + \ldots \int_{\frac{\pi}{3}}^{\pi} \ldots$	correctly identifying limits with ∫s	A1
	$= a^2[27 \times \frac{\pi}{3} - 10\sqrt{3} + \frac{\sqrt{3}}{2}] + a^2[11(\pi - \frac{\pi}{3}) - 6\sqrt{3} - \frac{\sqrt{3}}{2}]$		dM1
	$= a^2[49 - 48\sqrt{3}]$ (*)		A1 cso (8)
			(14 marks)

[P4 June 2003 Qn 7]

21.	(*a*)	$y' = 2kt.e^{3t} + 3kt^2\,e^{3t}$ use of product rule	M1
		$y'' = 2ke^{3t} + 12kt\,e^{3t} + 9t^2\,e^{3t}$ product rule, twice	M1
		substituting $2k + 12kt + 9kt^2 - 12kt - 18kt^2 + 9kt^2 = 4$	M1
		$k = 2$	A1 (4)
	(*b*)	Aux. eqn. (if used) $(m-3)^2 = 0$ $m = 3$, repeated	
		$y_{\text{C.F.}} = (A + Bt)\,e^{3t}$ M1 required form (allow just written down)	M1 A1
		G.S. $y = (A + Bt)\,e^{3t} + 2t^2\,e^{3t}$ (ft on $2t^2\,e^{3t}$)	A1 ft (3)
	(*c*)	$t = 0, y = 3 \Rightarrow A = 3$	B1
		$y' = Be^{3t} + 3(A + Bt)\,e^{3t} + 4te^{3t} + 6t^2e^{3t}$	M1
		$y' = 0, t = 0 \Rightarrow 1 = B + 3A \Rightarrow B = -8$	M1
		$y = (3 - 8t + 2t^2)e^{3t}$	A1 (4)
	(*d*)		
		$\cup$ shape crossing +ve x-axis	B1
		$\frac{1}{2}$, 1	B1
		$y' = (-3 + 4t)e^{3t} + 3(1 - 3t + 2t^2)e^{3t} = 0$	
		$6t^2 - 5t = 0$	M1
		$t = \dfrac{5}{6}$	A1
		$y = -\dfrac{1}{9}e^{2.5}$ (≈ -1.35) awrt -1.35	A1 (5)
			(16 marks)

[P4 June 2003 Qn 8]

22.	(i)(a)  Circle One half line correct Second half line [SC Allow B1 for two “full” lines in correct position]	M1 A1 B1 B1	(4)
	(b) shading correct region	A1 ft	(1)
	(ii)(a) Rearrange $w=\frac{z-1}{z}$ to give $z=\text{f}\left(w\right)$ or $z-1=\text{f}\left(w\right)$	M1	
	$\left(z=\frac{1}{1-w}\;\Rightarrow\;\right)\; z-1=\frac{w}{1-w}$, or $\left\|z-1\right\|=\left\|z\right\|\left\|w\right\|\;\Rightarrow\;\left\|z\right\|\left\|w\right\|=1$	A1	
	Completion $\left(\left\|z-1\right\|=1\;\Rightarrow\;\right)\;\left\|w\right\|=\left\|1-w\right\|=\left\|w-1\right\|$ ∗	A1	(3)
	(b)  Correct line shown Correct shading	M1 A1	(2)
			[10]

[P6 June 2003 Qn 4]

23. (a) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$	M1	
$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$		
$+ 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$	M1 A1	
$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1	
$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-2\cos^2\theta+\cos^4\theta)$	M1	
$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	A1 cso	
		(6)
(b) $\cos 5\theta = -1$ (or 1, or 0)	M1	
$5\theta = (2n \pm 1)180° \Rightarrow \theta = (2n \pm 1)36°$	A1	
$x = \cos\theta = -1, -0.309, 0.809$	M1 A1	
		(4)
	[10]	

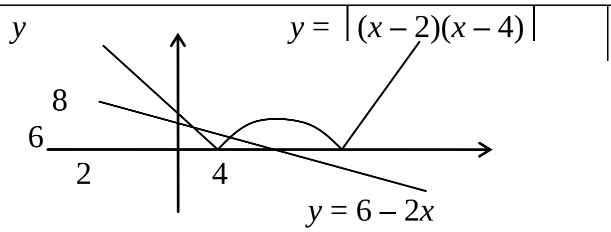
[P6 June 2003 Qn 5]

24.	$\sum_{r=1}^{n}(6r^2+2) = \cancel{2^3} - 0^3$	attempt to use an identity	M1
	$= \cancel{3^3} - 1^3$		
	$\cancel{4^3} - \cancel{2^3}$		
	$\vdots \quad \vdots$		
	$(\cancel{n-1})^3 - (\cancel{n-3})^3$		
	$n^3 - (\cancel{n-2})^3$		
	$(n+1)^3 - (\cancel{n-1})^3$	differences (must see)	M1
	$= (n+1)^3 + n^3 - 1^3$		A1
	$6\sum_{r=1}^{n} r^2 = (n+1)^3 + n^3 - 1 - 2n$	$2n$ or equiv.	B1
	$= 2n^3 + 3n^2 + n$		
	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(2n+1)(n+1)$ **(*)**	Sub. $\Sigma 2$ and $\div$ 6 or equiv. c.s.o.	M1, A1

[P4 January 2004 Qn 1]

25. (*a*)	IF = $e^{\int 1+\frac{3}{x}\,dx}$		M1
	$= e^{x+3\ln x}$		A1
	$= e^x e^{\ln x^3}$	must see	
	$= x^3 e^x$		A1 (3)
(*b*)	$x^3 e^x y = \int x^3 e^x \frac{1}{x^2}\,dx$		M1
	$= \int x e^x\,dx$		
	$= xe^x - e^x + c$	∫ by parts	M1 A1
	$y = \frac{1}{x^2} - \frac{1}{x^3} + \frac{c}{x^3}e^{-x}$	o.e.	A1 (4)
(*c*)	$I = ce^{-1}$ ∴ $c = e^1$		M1
	$y = \frac{1}{4} - \frac{1}{8} + \frac{e.e^{-2}}{8}$		M1
	$= \frac{1}{8}(1 + e^{-1})$		A1 (3)
	or = 0.171	0.171 or better	
			(10 marks)

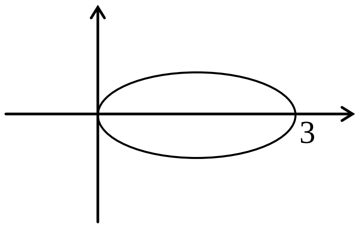
[P4 January 2004 Qn4]

26. (*a*)		Line crosses axes Curve shape Axes contacts 6, 8, 3 Cusps at 2 and 4	B1 B1 B1 B1 (4)
(*b*)	$6 - 2x = (x-2)(x-4)$ and $-6 + 2x = (x-2)(x-4)$		M1, M1
	$x^2 - 4x + 2 = 0$ $x^2 - 8x + 14 = 0$	either	M1
	$x = \frac{4 \pm \sqrt{16-8}}{2}$ $x = \frac{8 \pm \sqrt{64-56}}{2}$		
	$= 2 - \sqrt{2}$ $= 4 - \sqrt{2}$		A1, A1 (5)
(c)	$2 - \sqrt{2} < x < 4 - \sqrt{2}$		M1, A1 (2)
			(11 marks)

[P4 January 2004 Qn5]

27. (*a*)	$m^2 + 4m + 5 = 0$		M1
	$m = \frac{-4 \pm \sqrt{-4}}{2}$		
	$= -2 \pm \mathrm{i}$		A1
	$y = \mathrm{e}^{-2x}(A\cos x \pm B\sin x)$		M1
	$\text{PI} = \lambda\sin 2x + \mu\cos 2x$	PI & attempt diff.	M1
	$y' = 2\lambda\cos 2x - 2\mu\sin 2x$		
	$y'' = -4\lambda\sin 2x - 4\mu\cos 2x$		A1
	$\therefore\ -4\lambda - 8\mu + 5\lambda = 65$		
	$-4\mu + 8\lambda + 5\mu = 0$	subst. in eqn. & equate	M1
	$\lambda - 8\mu = 65$		
	$8\lambda + \mu = 0$	solving sim. eqn.	M1
	$64\lambda + 8\mu = 0$		
	$65\lambda = 65$		
	$\lambda = 1,\ \mu = -8$		A1
	$y = \mathrm{e}^{-2x}(A\cos x + B\sin x) + \sin 2x - 8\cos 2x$	ft on their λ and μ	A1ft (9)
(*b*)	As $x \to \infty$, $\mathrm{e}^{-2x} \to 0\ \therefore\ y \to \sin 2x - 8\cos 2x$		B1ft
	$y \to R\sin(2x + \alpha)$		M1
	$R = \sqrt{65}$		
	$\alpha = \tan^{-1}-8 = -1.446$ or $-82.9°$		A1 (3)
			(12marks)

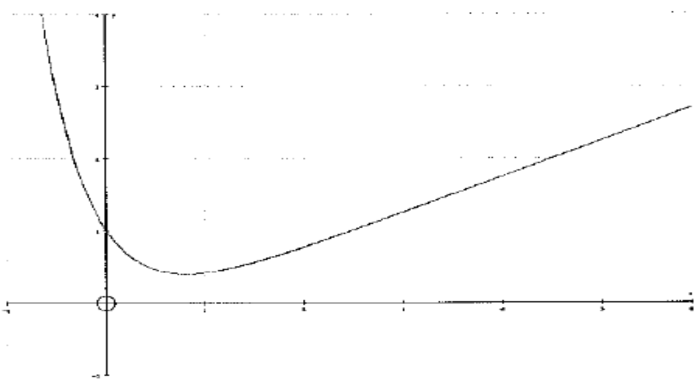
[P4 January 2004 Qn6]

28. (*a*)		Shape + horiz. axis	B1	
		3	B1	(2)
(*b*)	Area = $\frac{1}{2}\int r^2\,d\theta$			
	$= \frac{1}{2}\int 9\cos^2 2\theta\,d\theta$	use of $\frac{1}{2}\int r^2$	M1	
	$= \frac{9}{2}\int \frac{\cos 4\theta + 1}{2}\,d\theta$	use of $\cos 4\theta = 2\cos^2 2\theta - 1$	M1	
	$= \frac{9}{2}\left[\frac{\sin 4\theta}{8} + \frac{\theta}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	$\int$	dM1, A1	
	$= \frac{9}{2}\left[\frac{\pi}{8} - \frac{\sqrt{3}}{16} - \frac{\pi}{12}\right]$	subst. $\frac{\pi}{4}$ and $\frac{\pi}{6}$	M1	
	$= \frac{9}{2}\left[\frac{\pi}{24} - \frac{\sqrt{3}}{16}\right]$ or 0.103		A1	(6)
(*c*)	$r\sin\theta = 3\sin\theta\cos 2\theta$			
	$\frac{dy}{d\theta} = 3\cos\theta\cos 2\theta - 6\sin\theta\sin 2\theta$	diff. $r\sin\theta$	M1, A1	
	$\frac{dy}{d\theta} = 0 \Rightarrow\ 6\cos^2\theta - 3\cos\theta - 12\sin^2\theta\cos\theta = 0$	use of $\frac{dy}{d\theta} = 0$	M1	
	$6\cos^2\theta - 3\cos\theta - 12(1 - \cos^2\theta)\cos\theta = 0$	use double angle formula	M1	
	$18\cos^3\theta - 15\cos\theta = 0$	solving	M1	
	$\cos\theta = 0$ or $\cos^2\theta = \frac{5}{6}$ or $\tan^2\theta = \frac{1}{5}$ or $\sin^2\theta = \frac{1}{6}$		A1	
	$\therefore\ r = 3(2 \times \frac{5}{6}) - 1$			
	$= 2$			
	$\therefore r\sin\theta = 2\sqrt{\frac{1}{6}}$	use of $d = 2r\sin\theta$	M1	
	$d = \frac{2\sqrt{6}}{3}$		A1	(8)
				(16 marks)

[P4 January 2004 Qn 7]

29.	Solves $x^2 - 2 = 2x$ by valid method	**M1**
	Obtains $x = 1 \pm \sqrt{3}$ or equivalent (may only obtain relevant root if graph is used)	**A1**
	Solves $2 - x^2 = 2x$	**M1**
	Obtains $x = -1 \pm \sqrt{3}$	**A1**
	Rejects two of these roots and obtains (or uses graph and obtains)	**dM1**
	$x > 1 + \sqrt{3}, \quad x < -1 + \sqrt{3}$	**A1, A1**
		(7)
	Special case:	
	Squares both sides to obtain quadratic in x^2 and solve to obtain $x^2 = 4 \pm 2\sqrt{3}$	***M1 A1***
	Obtains $x = 1 \pm \sqrt{3}$ or $x = -1 \pm \sqrt{3}$	***M1A1***
	Last three marks as before.	***dM1A1A1***
		(7)

[P4 June 2004 Qn 4]

30.	**(a)**	Integrating Factor $= e^{2x}$ $\frac{d}{dx}(ye^{2x}) = xe^{2x}$ $ye^{2x} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$ $= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$ $\therefore y = \frac{1}{2}x - \frac{1}{4} + ce^{-2x}$		**B1** **M1** **M1** **A1** **A1** **(5)**
	(b)	$1 = c - \frac{1}{4} \to c = \frac{5}{4}$ $\therefore y = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$ and $\frac{dy}{dx} = \frac{1}{2} - \frac{5}{2}e^{-2x}$ When $y' = 0$, $e^{-2x} = \frac{1}{5}$ $\therefore 2x = \ln 5$ $x = \frac{1}{2}\ln 5,\ y = \frac{1}{4}\ln 5$ at minimum point.		**M1** **M1** **M1** **A1** **(4)**
	(c)		Min point and passing through (0,1) shape	**B1** **B1** **(2)**

[P4 June 2004 Qn 6]

31.	(a)	Auxiliary equation: $m^2+2m+2=0 \to m=-1\pm i$	M1
		Complementary Function is $y=e^{-t}(A\cos t+B\sin t)$	M1A1
		Particular Integral is $y=\lambda e^{-t}$, *with* $y'=-\lambda e^{-t}$, *and* $y''=\lambda e^{-t}$	M1
		$\therefore (\lambda-2\lambda+2\lambda)e^{-t}=2e^{-t} \to \lambda=2$	A1
		$\therefore y=e^{-t}(A\cos t+B\sin t+2)$	B1 **(6)**
	(b)	Puts $1=A+2$ and solves to obtain $A=-1$	M1,
		$y'=e^{-t}(-A\sin t+B\cos t)-e^{-t}(A\cos t+B\sin t+2)$	M1 A1ft
		Puts $1=B-A-2$ and uses value for A to obtain B	M1
		B=2	A1cso
		$\therefore y=e^{-t}(2\sin t-\cos t+2)$	A1cso **(6)**

[P4 June 2004 Qn 7]

32.	**(a)**	$3a(1-\cos\theta) = a(1+\cos\theta)$ $2a = 4a\cos\theta \to \cos\theta = \frac{1}{2} \quad \therefore \theta = \frac{\pi}{3}\ or -\frac{\pi}{3}$ $r = \frac{3a}{2}$ [Co-ordinates of points are $(\frac{3a}{2}, \frac{\pi}{3})$ and $(\frac{3a}{2}, -\frac{\pi}{3})$]	**M1** **M1** **A1 A1** **(4)**
	(b)	$AB = 2r\sin\theta = \dfrac{3a\sqrt{3}}{2}$	**M1A1** **(2)**
	(c)	$\text{Area} = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} r^2 d\theta$ $= \frac{1}{2}\int[a^2(1+\cos\theta)^2 - 9a^2(1-\cos\theta)^2]d\theta$ $= \frac{a^2}{2}\int[1 + 2\cos\theta + \cos^2\theta - 9(1 - 2\cos\theta + \cos^2\theta)]d\theta$ $= \frac{a^2}{2}\int[-8 + 20\cos\theta - 8\cos^2\theta]d\theta$ $= \text{k}[-8\theta + 20\sin\theta \ldots$ $\ldots\ldots -2\sin 2\theta - 4\theta]$ Uses limits $\dfrac{\pi}{3}$ and $-\dfrac{\pi}{3}$ correctly or uses twice smaller area and uses limits $\dfrac{\pi}{3}$ and 0 correctly.(Need not see 0 substituted) $= a^2[-4\pi + 10\sqrt{3} - \sqrt{3}]$ or $= a^2[-4\pi + 9\sqrt{3}]$ or $3.022a^2$	 **M1 M1** **A1** **B1** **B1** **M1** **A1** **(7)**
	(d)	$3a\dfrac{\sqrt{3}}{2} = 4.5 \to a = \sqrt{3}$ $\therefore \text{Area} = 3[9\sqrt{3} - 4\pi], \quad = 9.07\ \text{cm}^2$	**B1** **M1, A1** **(3)**

[P4 June 2004 Qn 8]

33. (a) $f'(x) = \sec^2 x$ $\quad f''(x) = 2\sec x(\sec x \tan x)$ (or equiv.) M1 A1

$f'''(x) = 2\sec^2 x(\sec^2 x) + 2\tan x(2\sec^2 x \tan x)$ (or equiv.) A1 (3)

$(2\sec^2 x + 6\sec^2 x \tan^2 x)$

$(2\sec^4 x + 4\sec^2 x \tan^2 x)$, $(6\sec^4 x - 4\sec^2 x)$, $(2 + 8\tan^2 x + 6\tan^4 x)$

(b) $\tan\frac{\pi}{4} = 1$ or $\sec\frac{\pi}{4} = \sqrt{2}$ (1, 2, 4, 16) B1

$\tan x = f\left(\frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)f'\left(\frac{\pi}{4}\right) + \frac{1}{2}\left(x - \frac{\pi}{4}\right)^2 f''\left(\frac{\pi}{4}\right) + \frac{1}{6}\left(x - \frac{\pi}{4}\right)^3 f'''\left(\frac{\pi}{4}\right)$ M1

$= 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3$ (Allow equiv. fractions A1(cso) (3)

(c) $x = \frac{3\pi}{10}$, so use $\left(\frac{3\pi}{10} - \frac{\pi}{4}\right)$ $\quad \left(= \frac{\pi}{20}\right)$ M1

$\tan\frac{3\pi}{10} \approx 1 + \frac{\pi}{10} + \left(2 \times \frac{\pi^2}{400}\right) + \left(\frac{8}{3} \times \frac{\pi^3}{8000}\right) = 1 + \frac{\pi}{10} + \frac{\pi^2}{200} + \frac{\pi^3}{3000}$ (*) A1(cso) (2)

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[P6 June 2004 Qn 2]

34. (a) $n = 1$: $\frac{d}{dx}\left(e^x \cos x\right) = e^x \cos x - e^x \sin x$ (Use of product rule) M1

$\cos\left(x + \frac{\pi}{4}\right) = \cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\left(\cos x - \sin x\right)$ M1

$\frac{d}{dx}\left(e^x \cos x\right) = 2^{1/2} e^x \cos\left(x + \frac{\pi}{4}\right)$ True for $n = 1$ (cso + comment) A1

Suppose true for $n = k$.

$\left[\frac{d^{k+1}}{dx^{k+1}}\left(e^x \cos x\right)\right] = \frac{d}{dx}\left(2^{\frac{1}{2}k} e^x \cos\left(x + \frac{k\pi}{4}\right)\right)$ M1

$= 2^{\frac{1}{2}k}\left[e^x \cos\left(x + \frac{k\pi}{4}\right) - e^x \sin\left(x + \frac{k\pi}{4}\right)\right]$ A1

$= 2^{\frac{1}{2}k} e^x \sqrt{2} \cos\left(x + \frac{k\pi}{4} + \frac{\pi}{4}\right) = 2^{\frac{1}{2}(k+1)} e^x \cos\left(x + (k+1)\frac{\pi}{4}\right)$ M1 A1

$\therefore$ True for $n = k + 1$, so true (by induction) for all n. (≥ 1) A1(cso) (8)

(b) $1 + \left(\sqrt{2}\cos\frac{\pi}{4}\right)x + \frac{1}{2}\left(2\cos\frac{\pi}{2}\right)x^2 + \frac{1}{6}\left(2\sqrt{2}\cos\frac{3\pi}{4}\right)x^3 + \frac{1}{24}\left(4\cos\pi\right)x^4$ M1

(1) (0) (–2) (–4)

$e^x \cos x = 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4$ (or equiv. fractions) A2(1,0) (3)

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[P6 June 2004 Qn 4]

35. (*a*) $\arg z = \frac{\pi}{4} \Rightarrow z = \lambda + \lambda\text{i}$ (or putting *x* and *y* equal at some stage) B1

$w = \frac{(\lambda+1)+\lambda\text{i}}{\lambda+(\lambda+1)\text{i}}$, and attempt modulus of numerator or denominator. M1

(Could still be in terms of *x* and *y*)

$|(\lambda+1)+\lambda\text{i}| = |\lambda+(\lambda+1)\text{i}| = \sqrt{(\lambda+1)^2+\lambda^2}$, $\therefore |w| = 1\,(*)$ A1, A1cso (4)

(*b*) $w = \frac{z+1}{z+\text{i}} \Rightarrow zw + w\text{i} = z+1 \Rightarrow z = \frac{1-w\text{i}}{w-1}$ M1

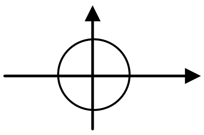
$|z| = 1 \Rightarrow |1-w\text{i}| = |w-1|$ M1 A1

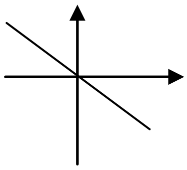
For $w = a+\text{i}b$, $|(1+b)-a\text{i}| = |(a-1)+\text{i}b|$ M1

$\sqrt{(1+b)^2+a^2} = \sqrt{(a-1)^2+b^2}$ M1

$b = -a$ Image is (line) $y = -x$ A1 (6)

(c)





B1 B1 (2)

(d) $z = \text{i}$ marked (*P*) on *z*-plane sketch. B1

$z = \text{i} \Rightarrow w = \frac{1+\text{i}}{2\text{i}} = \frac{\text{i}-1}{-2} = \frac{1}{2} - \frac{1}{2}\text{i}$ marked (*Q*) on *w*-plane sketch. B1 (2)

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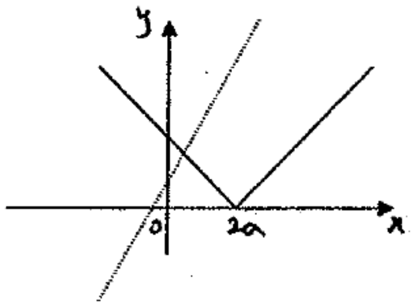
[P6 June 2004 Qn 7]

36.	(a)  Shape, vertex on x-axis — B1 At least 2a seen on positive x-axis — B1 (2) (b) Attempting to solve $-(x-2a) = 2x + a$ anywhere — M1 Completely correct method — dep M1 [e.g. solving $-(x-2a) > 2x + a$; if finding two "solutions" needs to be evidence for giving "correct" result] $x < \frac{1}{3}a$ — A1 (3) [5]

[FP1/P4 January 2005 Qn 1]

37.	I.F. = $e^{\int 2\cot 2x\,dx}$; = $\sin 2x$ — M1A1 Multiplying **throughout** by IF. — M1 * y x (IF) = integral of candidate's RHS — M1 $= \int 2\sin^2 x \cos x\,dx$ or $\int -\left(\frac{\cos 3x - \cos x}{2}\right)dx$ — M1 [This M gained when in position to complete integration, dep on M *] $= \frac{2}{3}\sin^3 x (+C)$ or $-\frac{1}{6}\sin 3x + \frac{1}{2}\sin x + c$ — A1 $y = \frac{2\sin^3 x}{3\sin 2x} + \frac{C}{\sin 2x}$ or $-\frac{\sin 3x}{6\sin 2x} + \frac{\sin x}{2\sin 2x} + \frac{c}{\sin 2x}$ or equiv. — A1√ [7]

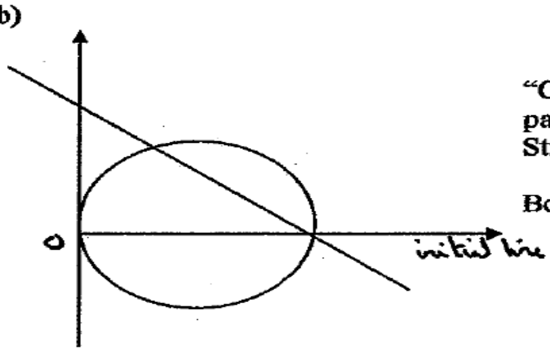
[FP1/P4 January 2005 Qn 3]

38.	(a) $\frac{1}{r(r+2)} \equiv \frac{A}{r} + \frac{B}{r+2} \equiv \frac{A(r+2)+Br}{r(r+2)}$ and attempt to find A and B	M1
	$\equiv \frac{1}{2r} - \frac{1}{2(r+2)}$	A1 (2)
	(b) $\sum \frac{4}{r(r+2)} \equiv 2\left[\frac{1}{r} - \frac{1}{r+2}\right]$	
	$\sum_{1}^{n}\left[\frac{1}{r} - \frac{1}{r+2}\right] = \{1 - \frac{1}{3}\} + \left\{\frac{1}{2} - \frac{1}{4}\right\} + \left\{\frac{1}{3} - \frac{1}{5}\right\} + \ldots\ldots$ $+ \left\{\frac{1}{n-1} - \frac{1}{n+1}\right\} + \left\{\frac{1}{n} - \frac{1}{n+2}\right\}$	M1A1
	[If A and B incorrect, allow A1√ here only, providing still differences]	
	$= \frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$	A1
	Forming single fraction: $\frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{2(n+1)(n+2)}$	M1
	Deriving given answer $\frac{n(3n+5)}{(n+1)(n+2)}$, cso	A1 (5)
	(c) Using S(100) − S(49) = $\frac{100 \text{ x } 305}{101 \text{ x } 102} - \frac{49 \text{ x} 152}{50 \text{ x } 51}$	M1A1
	[= 2.96059… − 2.92078…]	
	= 0.0398 (4 d.p.)	A1 (3) [10]
	[Allow S(100) − S(50), (⇒ 0.0383) for M1]	

[FP1/P4 January 2005 Qn 5]

39.	(a) $\frac{dy}{dx} = x\frac{dv}{dx} + v, \quad \frac{d^2y}{dx^2} = x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}$	M1A1
	[M1 for diff. product, A1 both correct]	
	$\therefore\ x^2\left(x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}\right) - 2x\left(x\frac{dv}{dx} + v\right) + (2 + 9x^2)vx = x^5$	M1
	$x^3\frac{d^2v}{dx^2} + 2x^2\frac{dv}{dx} - 2x^2\frac{dv}{dx} - 2vx + 2vx + 9vx^3 = x^5$	A1
	$[x^3\frac{d^2v}{dx^2} + + 9vx^3 = x^5]$	
	Given result: $\frac{d^2v}{dx^2} + 9v = x^2$ cso	A1 (5)
	(b) CF: $v = A\sin 3x + B\cos 3x$ (may just write it down)	M1A1
	Appropriate form for P1: $v = \lambda x^2 + \mu$ (or $ax^2 + bx + c$)	M1
	Complete method to find λ and μ	M1
	$v = A\sin 3x + B\cos 3x + \frac{1}{9}x^2 - \frac{2}{81}$ [f.t. only on wrong CF]	M1A1√ (6)
	(c) $\therefore\ y = Ax\sin 3x + Bx\cos 3x + \frac{1}{9}x^3 - \frac{2}{81}x$	B1√ (1) [12]
	[f.t. for $y = x$ (candidate's CF + PI), providing two arbitary constants]	

[FP1/P4 January 2005 Qn 6]

40.	(a) For C: Using polar/ cartesian relationships to form Cartesian equation	M1
	so $x^2 + y^2 = 6x$	A1
	[Equation in any form: e.g. $(x-3)^2 + y^2 = 9$ from sketch. or $\sqrt{x^2+y^2} = \frac{6x}{\sqrt{x^2+y^2}}$]	
	For D: $r\cos\left(\frac{\pi}{3} - \theta\right) = 3$ and attempt to expand	M1
	$\frac{x}{2} + \frac{\sqrt{3}y}{2} = 3$ (any form)	M1A1 (5)
	(b) 	
	"Circle", symmetric in initial line passing through pole	B1
	Straight line	B1
	Both passing through (6, 0)	B1 (3)
	(c) Polars: Meet where $6\cos\theta\cos\left(\frac{\pi}{3} - \theta\right) = 3$	M1
	$\sqrt{3}\sin\theta\cos\theta = \sin^2\theta$	M1
	$\sin\theta = 0$ or $\tan\theta = \sqrt{3}$ $\quad [\theta = 0$ or $\frac{\pi}{3}]$	M1
	Points are $(6, 0)$ and $(3, \frac{\pi}{3})$	B1,A1 (5) [13]

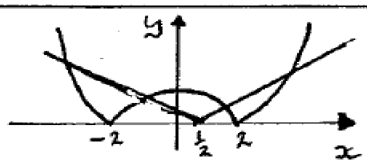
[FP1/P4 January 2005 Qn 7]

41.(a)	$\frac{2}{4r^2-1} = \frac{1}{2r-1} - \frac{1}{2r+1}$	B1
	$\sum_{r=1}^{n} \frac{2}{4r^2-1} = \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \cdots \frac{1}{2n-1} - \frac{1}{2n+1}$ Attempt Method of Differences	M1
	$= 1 - \frac{1}{2n+1}$ (*)	A1 c.s.o. (3)
(b)	Sum $= \left(\frac{1}{2}\right)\left[f(20) - f(10)\right]$ Use of (a) and $\sum_{1}^{20} - \sum_{1}^{10}$	M1
	$= \frac{1}{2}\left[1 - \frac{1}{41} - 1 + \frac{1}{21}\right] = \frac{10}{21 \times 41}$ or $\frac{10}{861}$	A1 cao (2) (5)

[FP1/P4 June 2005 Qn 1]

42.	$\frac{dy}{dx} + \frac{2}{1+x} y = \frac{1}{x(x+1)}$ Attempt $y' + Py = Q$ form	M1
	I.F. $= e^{\int \frac{2}{1+x} dx} = e^{2\ln(1+x)}, = (1+x)^2$	M1, A1
	$\therefore \quad y(1+x)^2 = \int \left(\frac{x+1}{x}\right) dx$ OR $\frac{d}{dx}\left(y(1+x)^2\right) = \frac{x+1}{x}$	M1 ($\surd$ I.F.)
	i.e. $\left(y(1+x)^2 =\right) x + \ln x + (C)$	M1 A1
	$y = \frac{x + \ln x + C}{(1+x)^2}$	A1 c.a.o. (7)

[FP1/P4 June 2005 Qn 3]

43.(a)	  Shape - Symmetric about y-axis	B1
	V Shape. Vertex on positive x-axis	B1
	−2, 2	B1
	$\frac{1}{2}$	B1 (4)
(b)	$x^2 - 4 = 2x - 1$	M1
	$x^2 - 2x - 3 = 0 \quad \Rightarrow \quad x = \underline{3, -1}$	A1
	$x^2 - 4 = -(2x-1)$	M1
	$x^2 + 2x - 5 = 0, \quad \Rightarrow \quad x = \frac{-2 \pm \sqrt{4 + 20}}{2}$ Correct 3 term Quadratic = 0	A1,
	$x = \underline{-1 \pm \sqrt{6}}$	A1 (5)
(c)	$x < -1 - \sqrt{6}$; $-1 < x < \sqrt{6} - 1$; $x > 3$ ($\surd$surds) Accept 3 s.f.	B1$\surd$; B1$\surd$; B1 (3) (12)

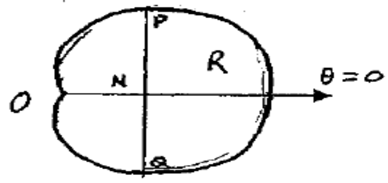
[FP1/P4 June 2005 Qn 6]

44.(a)	$2m^2+5m+2=0$ Attempt aux eqn $\to m=$	M1
	$\Rightarrow \quad m=-\frac{1}{2}, -2$	
	$\therefore \quad x_{CF} = Ae^{-2t}+Be^{-\frac{1}{2}t}$ C.F.	A1
	Particular Integral: $x = pt+q$ P.I.	B1
	$\dot{x}=p$, $\ddot{x}=0$ and sub.	M1
	$\Rightarrow \quad 5p+2q+2pt = 2t+9 \to \underline{p=1,\ q=2}$	A1
	General solution $\quad x = \underline{Ae^{-2t}+Be^{-\frac{1}{2}t}+t+2}$	A1$\int$ ($\int$ ms, p, q) (6)
(b)	$x=3, t=0 \Rightarrow 3 = A+B+2 \quad$ (or $A+B=1$)	M1
	$\dot{x} = -2Ae^{-2t}-\frac{1}{2}Be^{-\frac{1}{2}t}+1$ Attempt $\dot{x}$	M1
	$\dot{x}=1, t=0 \Rightarrow -1 = -2A-\frac{1}{2}B+1 \quad$ (or $4A+B=4$) 2 correct eqns	A1
	Solving $\to A=1, B=0$ and $\underline{x = e^{-2t}+t+2}$	A1 (4)
(c)	$\dot{x} = -2e^{-2t}+1 = 0$ $\dot{x}=0$	M1
	$\Rightarrow \quad t = \frac{1}{2}\ln 2$	A1
	$\ddot{x} = 4e^{-2t} > 0\ (\forall t) \therefore$ min	M1
	Min $\quad x = e^{-\ln 2}+\frac{1}{2}\ln 2+2$	
	$= \frac{1}{2}+\frac{1}{2}\ln 2+2$	
	$= \underline{\frac{1}{2}(5+\ln 2)}$ (*)	A1 cso (4)
		(14)

[FP1/P4 June 2005 Qn 7]

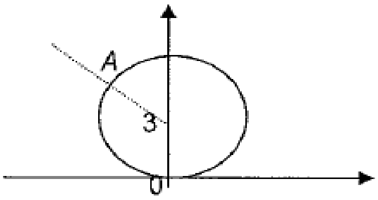
45.

(a)



$4a(1+\cos\theta) = \dfrac{3a}{\cos\theta}$ or $r = 4a\left(1+\dfrac{3a}{r}\right)$		M1
$4\cos^2\theta + 4\cos\theta - 3 = 0$ or $r^2 - 4ar - 12a^2 = 0$	dep	A1
$(2\cos\theta - 1)(2\cos\theta + 3) = 0$ or $(r-6a)(r+2a) = 0$		M1
$\cos\theta = \dfrac{1}{2}, \left(\theta = \dfrac{\pi}{3}\right)$ or $r = 6a$		A1
Note $ON = 3a$		
$PQ = 2 \times ON \tan\dfrac{\pi}{6} = 6\sqrt{3}a$ *	cso	M1 A1 6
or $PQ = 2\times\sqrt{\left[(6a)^2 - (3a)^2\right]} = 2\sqrt{(27a^2)} = 6\sqrt{3}a$ *	cso	
or any complete equivalent		
(b) $2\times\dfrac{1}{2}\int_0^{\pi/3} r^2\,d\theta = \ldots\int 16a^2(1+\cos\theta)^2\,d\theta$	$\int r^2 d\theta$	M1
$= \ldots\int\left(1 + 2\cos\theta + \dfrac{1}{2} + \dfrac{1}{2}\cos 2\theta\right)d\theta$	$\cos^2\theta \to \cos 2\theta$	M1
$= \ldots\left[\dfrac{3}{2}\theta + 2\sin\theta + \dfrac{1}{4}\sin 2\theta\right]$		A1
$= 16a^2\left[\dfrac{\pi}{2} + \sqrt{3} + \dfrac{\sqrt{3}}{8}\right]$ $\left(= 2a^2\left[4\pi + 9\sqrt{3}\right] \approx 56.3a^2\right)$		use of their $\frac{\pi}{3}$ M1 A1
Area of Δ POQ $= \frac{1}{2} 6\sqrt{3}a \times 3a$ or $9a^2\sqrt{3}$		B1
$R = a^2(8\pi + 9\sqrt{3})$	cao	A1 7 (13)

[FP1/P4 June 2005 Qn 7]

46.	(a)  Circle Correct circle. (centre (0, 3), radius 3)	M1 A1 (2)
	(b) Drawing correct **half**-line passing as shown	B1
	Find either *x* or *y* coord of *A*.	M1A1
	$z = -\frac{3\sqrt{2}}{2} + (3 + \frac{3\sqrt{2}}{2})\text{i}$ [Algebraic approach, i.e. using $y = 3 - x$ and equation of circle will only gain M1A1, unless the second solution is ruled out, when B1 can be given by implication, and final A1, if correct]	A1 (4)
	(c) $\lvert z - 3\text{i}\rvert = 3 \rightarrow \quad \left\lvert \frac{2\text{i}}{\omega} - 3\text{i} \right\rvert = 3$	M1
	$\Rightarrow \frac{\lvert 2\text{i} - 3\text{i}\omega \rvert}{\lvert\omega\rvert} = 3$	A1
	$\Rightarrow \lvert \omega - {}^2/_3 \rvert = \lvert \omega \rvert$	M1A1
	Line with equation $u = {}^1/_3 \quad (x = {}^1/_3)$	A1 (5)
	Some alternatives:	**[11]**
	(i) $\omega = \frac{2\text{i}}{x + \text{i}y} = \frac{2\text{i}(x - \text{i}y)}{x^2 + y^2} \Rightarrow u = \frac{2y}{x^2 + y^2},\ v = \frac{2x}{x^2 + y^2}$ M1A1 As $x^2 + y^2 - 6y = 0, \quad u = \frac{1}{3},$ M1,A1A1 (ii) $\omega = \frac{2\text{i}}{3\cos\theta + 3\text{i}(1 + \sin\theta)} = \frac{2\text{i}\{\cos\theta - \text{i}(1 + \sin\theta)\}}{3\{\cos^2\theta + (1 + \sin\theta)^2\}}$ M1A1 $= \frac{2}{3}\frac{(1 + \sin\theta) + \text{i}\cos\theta}{2 + 2\sin\theta}, = \frac{1}{3} + \text{i}\frac{\cos\theta}{1 + \sin\theta},$ M1A1 So locus is line $u = \frac{1}{3}$ A1	

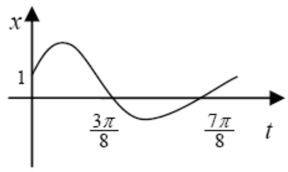
[FP3/P6 June 2005 Qn 4]

47.	(a) $z^n = e^{in\theta} = (\cos n\theta + i\sin n\theta)$, $z^{-n} = e^{-in\theta} = (\cos n\theta - i\sin n\theta)$	M1
	Completion (needs to be convincing) $z^n - \frac{1}{z^n} = 2i\sin n\theta$ (*)AG	A1 (2)
	(b) $\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$	M1A1
	$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$	
	$(2i\sin\theta)^5 = 32i\sin^5\theta = 2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta$	M1A1
	$\Rightarrow \sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$ (*) AG	A1 (5)
	(c) Finding $\sin^5\theta = ¼\sin\theta$	M1
	$\theta = 0, \pi$ (both)	B1
	$(\sin^4\theta = ¼)$ $\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$	M1
	$\theta = \frac{\pi}{4}, \frac{3\pi}{4}; \quad \frac{5\pi}{4}, \frac{7\pi}{4}$	A1;A1 (5)
		[12]

[FP3/P6 June 2005 Qn 5]

48.	2 is a 'critical value', e.g. used in solution, or $x = 2$ seen as an asymptote	B1
	$x^2 = 2x^2 - 4x \Rightarrow x^2 - 4x = 0$	
	$x = 0,\ x = 4$ M1: two other critical values	M1 A1
	$x < 0$	B1
	$2 < x < 4$ M1: An inequality using the critical value 2	M1 A1 (6)
		Total 6 marks

[FP1/P4 January 2006 Qn 2]

49.	(a) $m^2+2m+5=0 \Rightarrow m=-1\pm 2\mathrm{i}$	M1 A1	
	$x=\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)$ M: Correct form (needs the two different constants)	M1 A1	(4)
	(b) (1, 0) $\Rightarrow A=1$	dB1	
	$\dot{x}=-\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)+\mathrm{e}^{-t}\left(-2A\sin 2t+2B\cos 2t\right)$ M: Product diff. attempt	dM1	
	With $A=1$, $\mathrm{e}^{-t}\{\cos 2t(-1+2B)+\sin 2t(-B-2)\}$		
	$\dot{x}=1, t=0 \Rightarrow 1=-A+2B$	M1	
	$B=1$ $\left(x=\mathrm{e}^{-t}\left(\cos 2t+\sin 2t\right)\right)$ M: Use value of A to find B.	dM1 A1cso	(5)
	(c)  'Single oscillation' between 0 and π	B1	
	Decreasing amplitude (dep. on a turning point)	B1ft	
	Initially increasing to maximum	B1ft	
	Any <u>one</u> correct intercept, whether in terms of π or not: 1 or $\frac{3\pi}{8}$ or $\frac{7\pi}{8}$	B1	(4)
	(Allow degrees: 67.5° or 157.5°) (Allow awrt 0.32π or 1.18 or 2.75)	**Total 13 marks**	

[FP1/ P4 January 2006 Qn 4]

50.	(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=v+x\frac{\mathrm{d}v}{\mathrm{d}x}$	B1	
	$v+x\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{3x-4vx}{4x+3vx}$ (All in terms of v and x)	M1	
	$x\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{3-4v-v(4+3v)}{4+3v}$ (Requires $x\frac{\mathrm{d}v}{\mathrm{d}x}=\mathrm{f}(v)$, 2 terms over common denom.)	M1	
	$x\frac{\mathrm{d}v}{\mathrm{d}x}=-\frac{3v^2+8v-3}{3v+4}$ (*)	A1 cso	(4)
	(b) $\frac{3v+4}{3v^2+8v-3}\mathrm{d}v=-\frac{1}{x}\mathrm{d}x$ Separating variables	M1	
	$\pm\ln x$	B1	
	$\frac{1}{2}\ln(3v^2+8v-3)$ M: $k\ln(3v^2+8v-3)$	M1 A1	
	$\frac{1}{2}\ln\left(\frac{3y^2}{x^2}+\frac{8y}{x}-3\right)=-\ln x+C$ Or any equivalent form	A1	(5)
	(c) $\frac{3y^2}{x^2}+\frac{8y}{x}-3=\frac{A}{x^2}$ Removing ln's correctly at any stage, dep. on having C.	M1	
	Using (1, 7) to form an equation in A (need not be $A=\ldots$)	M1	
	(1, 7) $\Rightarrow 3\times 49+56-3=A \Rightarrow A=200$ (or equiv., can still be ln)	A1	
	$3y^2+8yx-3x^2=200$ $(3y-x)(y+3x)=200$ (M dependent on the 2 previous M's) (*)	M1 A1 cso	(5)
		Total 14 marks	

[FP1/P4 January 2006 Qn 6]

51.	(a)(i) $r^2\sin^2\theta = a^2\cos 2\theta\sin^2\theta = a^2\left(1-2\sin^2\theta\right)\sin^2\theta$	B1 (1)
	$\left(=a^2\left(\sin^2\theta-2\sin^4\theta\right)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(a^2\left(\sin^2\theta-2\sin^4\theta\right)\right)=a^2\left(2\sin\theta\cos\theta-8\sin^3\theta\cos\theta\right)$, $=0$	M1 A1, M1
	$2=8\sin^2\theta$ (Proceed to $a\sin^2\theta=b$)	M1
	$\sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$, $r=\frac{a}{\sqrt{2}}$ (*)	A1, A1 cso (6)
	(b) $\frac{a^2}{2}\int\cos 2\theta\,\mathrm{d}\theta=\frac{a^2}{4}\sin 2\theta$ M: Attempt $\frac{1}{2}\int r^2\,\mathrm{d}\theta$, to get $k\sin 2\theta$	M1 A1
	$\left[\ldots\right]_{\pi/6}^{\pi/4}=\frac{a^2}{4}\left[1-\frac{\sqrt{3}}{2}\right]$ M: Using correct limits	M1 A1
	$\Delta=\frac{1}{2}\left(\frac{a}{\sqrt{2}}.\frac{1}{2}\right)\times\left(\frac{a}{\sqrt{2}}.\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}a^2}{16}$ M: Full method for rectangle or triangle	M1 A1
	$R=\frac{\sqrt{3}a^2}{16}-\frac{a^2}{4}\left[1-\frac{\sqrt{3}}{2}\right]=\frac{a^2}{16}\left(3\sqrt{3}-4\right)$ M: Subtracting, either way round (*)	dM1 A1 cso (8)
		Total 15 marks

[FP1/P4 January 2006 Qn 7]

52.	$\cos\frac{\pi}{2}+\mathrm{i}\sin\frac{\pi}{2}$	B1
	$\cos\frac{\pi}{10}+\mathrm{i}\sin\frac{\pi}{10}$	B1
	$\cos\left(\frac{(4k+1)\pi}{10}\right)+\mathrm{i}\sin\left(\frac{(4k+1)\pi}{10}\right)$, $k=$ 2, 3, 4(or equiv.)	M1A2,1,0
		(5)
	[$\cos\left(\frac{9\pi}{10}\right)+\mathrm{i}\sin\left(\frac{9\pi}{10}\right)$, $\cos\left(\frac{13\pi}{10}\right)+\mathrm{i}\sin\left(\frac{13\pi}{10}\right)$, $\cos\left(\frac{17\pi}{10}\right)+\mathrm{i}\sin\left(\frac{17\pi}{10}\right)$]	
	[Degrees : 18, 90, 162, 234, 306]	
		Total 5 marks

[FP3/P6 January 2006 Qn 1]

53.	(a) Correct method for producing 2nd order differential equation e.g. $\frac{d}{dx}\left\{(1+2x)\frac{dy}{dx}\right\} = \frac{d}{dx}\{x+4y^2\}$ attempted	M1
	$(1+2x)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 1 + 8y\frac{dy}{dx}$ seen + conclusion AG	A1* (2)
	(b) Differentiating again w.r.t. x: $(1+2x)\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} = 8y\frac{d^2y}{dx^2} + 8\left(\frac{dy}{dx}\right)^2 - 2\frac{d^2y}{dx^2}$ or equiv.	M1A2,1,0 (3)
	[e.g. $(1+2x)\frac{d^3y}{dx^3} = 8\left(\frac{dy}{dx}\right)^2 + 4(2y-1)\frac{d^2y}{dx^2}$	

3

	(c) $\frac{dy}{dx}$ (at $x = 0$) = 1	B1
	Finding $\frac{d^2y}{dx^2}$ (at $x = 0$) (= 3)	M1
	Finding $\frac{d^3y}{dx^3}$, at $x = 0$; = 8 [A1 f.t. is on part (c) values only]	M1A1√
	$y = \frac{1}{2} + x + \frac{3}{2}x^2 + \frac{4}{3}x^3 + \ldots$	M1A1 (6)
		Total 11 marks
	[Alternative (c): Polynomial for y: $y = ½ + ax + bx^2 + cx^3 + \ldots$	M1
	In given d.e.: $(1+2x)(a + 2bx + 3cx^2 + \ldots) \equiv x + 4(½ + ax + bx^2 + cx^3 + \ldots)^2$	M1A1
	a = 1 B1, Complete method for other coefficients M1, answer	A1

[FP3/P6 January 2006 Qn 6]

54.

(a) Relating lines and angle (generous)

[*angle between* $\pm 2\text{i}$ *to* P *and* ± 2 *to* P] — M1

Angle between correct lines is $\frac{\pi}{2}$ — A1

Circle — M1

Selecting correct ("top half") semi-circle . — A1

[If algebraic approach:

Method for finding Cartesian equation — M1

Correct equation, any form, $\Rightarrow x(x+2)+y(y-2)=0$ — A1

Sketch: showing circle — M1

Correct circle { centre (–1, 1)}, choosing only "top half" — A1]

(4)

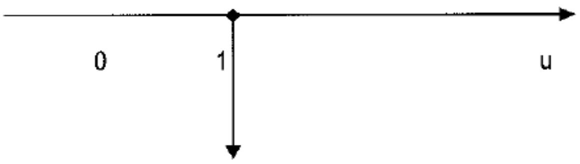
(b) $|z+1-\text{i}|$ is radius; $=\sqrt{2}$ — M1A1

(2)

(c) $z=\frac{2(1+\text{i})-2\omega}{\omega}\quad\left(=\frac{2(1+\text{i})}{\omega}-2\right)$ — M1

$\frac{z-2\text{i}}{z+2}=\frac{2(1+\text{i})-2(1+\text{i})\omega}{2(1+\text{i})}\quad(=1-\omega)$ — M1A1

Arg $(1-\omega)=\frac{\pi}{2}$ is line segment, passing through (1,0) — A1,A1



A1

(6)

Total 12 marks

Alt ©: $u+iv=\frac{2+2i}{(x+2)+iy}=\frac{(2x+2y+4)+i(x+2-y)}{(x+2)^2+y^2}$ M1

$x=-1+\sqrt{2}\cos\theta,\ y=1+\sqrt{2}\sin\theta$ M1

$\Rightarrow w=\frac{(2\sqrt{2}\cos\theta+2\sqrt{2}\sin\theta+4)+i.....}{(2\sqrt{2}\cos\theta+2\sqrt{2}\sin\theta+4)}\quad\{=1+i\,\text{f}(\theta)\}$ A1,

$\Rightarrow$ part of line $u=1$, show lower "half" of line A1,A1

[FP3/P6 January 2006 Qn 8]

55.	Use of $\frac{1}{2}\int r^2\,d\theta$	B1
	Limits are $\frac{\pi}{8}$ and $\frac{\pi}{4}$	B1
	$16a^2\cos^2 2\theta = 8a^2(1+\cos 4\theta)$	M1
	$\int(1+\cos 4\theta)\,d\theta = \theta + \frac{\sin 4\theta}{4}$	M1 A1
	$A = 4a^2\left[\theta + \frac{\sin 4\theta}{4}\right]_{\pi/8}^{\pi/4}$	
	$= a^2\left[4\left(\frac{\pi}{4}-\frac{\pi}{8}\right)+(0-1)\right]$	M1
	$= a^2\left(\frac{\pi}{2}-1\right) = \frac{1}{2}a^2(\pi-2)$ ∗ cso	A1 (7)
		[7]

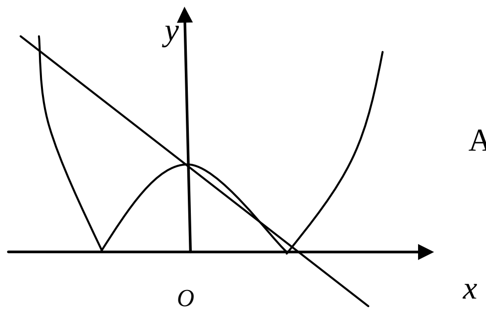
[FP1 June 2006 Qn 2]

56.	(a) $y' = 3\sin 2x + 6x\cos 2x$	M1
	$y'' = 12\cos 2x - 12x\sin 2x$	A1
	Substituting $12\cos 2x - 12x\sin 2x + 12x\sin 2x = k\cos 2x$	M1
	$k = 12$	A1 (4)
	(b) General solution is $y = A\cos 2x + B\sin 2x + 3x\sin 2x$	B1
	$(0, 2) \Rightarrow A = 2$	B1
	$\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = B + \frac{3\pi}{4} \Rightarrow B = -\frac{\pi}{4}$	M1
	$y = 2\cos 2x - \frac{\pi}{4}\sin 2x + 3x\sin 2x$ Needs $y = \ldots$	A1 (4)
		[8]

[FP1 June 2006 Qn 3]

57.	(a) $(2r+1)^3 = 8r^3 + 12r^2 + 6r + 1$ $(2r-1)^3 = 8r^3 - 12r^2 + 6r - 1$ $(2r+1)^3 - (2r-1)^3 = 24r^2 + 2 \qquad (A = 24,\ B = 2)$	M1 A1 (2)
	Accept $r = 0 \Rightarrow B = 2$ and $r = 1 \Rightarrow A + B = 26 \Rightarrow A = 24$ M1 for both	
	(b) $\cancel{3^3} - 1^3 = 24 \times 1^2 + 2$ $5^3 - \cancel{3^3} = 24 \times 2^2 + 2$ M $(2n+1)^3 - \cancel{(2n-1)^3} = 24 \times n^2 + 2$	
	$(2n+1)^3 - 1^3 = 24\sum_{r=1}^{n} r^2 + \underline{2n}$ $\underline{\text{ft}}$ their B	M1 A1 $\underline{\text{A1ft}}$
	$\sum_{r=1}^{n} r^2 = \frac{8n^3 + 12n^2 + 4n}{24}$	M1
	$= \frac{1}{6}n(2n^2 + 3n + 1) = \frac{1}{6}n(n+1)(2n+1)$ ✱ cso	A1 (5)
	(c) $\sum_{r=1}^{40}(3r-1)^2 = \sum_{r=1}^{40}(9r^2 - 6r + 1)$	M1
	$= 9 \times \frac{1}{6} \times 40 \times 41 \times 81 - 6 \times \frac{1}{2} \times 40 \times 41 + 40$	M1
	$= 194380$	A1 (3) [10]

[FP1 June 2006 Qn 5]

58.	(a) $2x^2 + x - 6 = 6 - 3x$	M1
	Leading to $x^2 + 2x - 6 = 0$	
	$(x+1)^2 = 7 \Rightarrow x = -1 \pm \sqrt{7}$ surds required	M1 A1
	$-2x^2 - x + 6 = 6 - 3x$	M1
	Leading to $2x^2 - 2x = 0 \Rightarrow x = 0, 1$	A1, A1 (6)
	(b) Accept if parts (a) and (b) done in reverse order	
	 Curved shape	B1
	Line	B1
	At least 3 intersections	B1 (3)
	(c) Using all 4 CVs and getting all into inequalities	M1
	$x > \sqrt{7} - 1, \quad x < -\sqrt{7} - 1$ both	A1ft
	ft their greatest positive and their least negative CVs	
	$0 < x < 1$	A1 (3)
		[12]

[FP1 June 2006 Qn 7]

59.	(a) $\int \frac{2}{120-t}\,\mathrm{d}t = -2\ln(120-t)$	B1	
	$\mathrm{e}^{-2\ln(120-t)} = (120-t)^{-2}$	M1 A1	
	$\frac{1}{(120-t)^2}\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{2S}{(120-t)^3} = \frac{1}{4(120-t)^2}$		
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{S}{(120-t)^2}\right) = \frac{1}{4(120-t)^2}$ or integral equivalent	M1	
	$\frac{S}{(120-t)^2} = \frac{1}{4(120-t)}(+C)$	M1 A1	
	$(0, 6) \Rightarrow 6 = 30 + 120^2 C \Rightarrow C = -\frac{1}{600}$	M1	
	$S = \frac{120-t}{4} - \frac{(120-t)^2}{600}$ accept C = awrt -0.0017	A1	(8)
	(b) $\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{1}{4} + \frac{2(120-t)}{600}$	M1	
	$\frac{\mathrm{d}S}{\mathrm{d}t} = 0 \Rightarrow t = 45$	M1 A1	
	Substituting $S = 9\frac{3}{8}$ (kg)	A1	(4)
			[12]

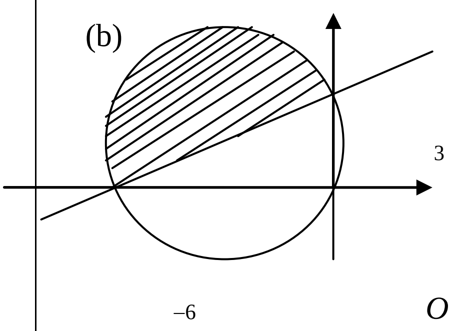
[FP1 June 2006 Qn 8]

60. (a)	$\mathrm{f}(x)=\cos 2x$, $\quad \mathrm{f}\left(\frac{\pi}{4}\right)=0$		
	$\mathrm{f}'(x)=-2\sin 2x$, $\quad \mathrm{f}'\left(\frac{\pi}{4}\right)=-2$	M1	
	$\mathrm{f}''(x)=-4\cos 2x$, $\quad \mathrm{f}''\left(\frac{\pi}{4}\right)=0$		
	$\mathrm{f}'''(x)=8\sin 2x$, $\quad \mathrm{f}'''\left(\frac{\pi}{4}\right)=8$	A1	
	$\mathrm{f}^{(\mathrm{iv})}(x)=16\cos 2x$, $\quad \mathrm{f}^{(\mathrm{iv})}\left(\frac{\pi}{4}\right)=0$		
	$\mathrm{f}^{(\mathrm{v})}(x)=-32\sin 2x$, $\quad \mathrm{f}^{(\mathrm{v})}\left(\frac{\pi}{4}\right)=-32$	A1	
	$\cos 2x=\mathrm{f}\left(\frac{\pi}{4}\right)+\mathrm{f}'\left(\frac{\pi}{4}\right)\left(x-\frac{\pi}{4}\right)+\frac{\mathrm{f}''\left(\frac{\pi}{4}\right)}{2}\left(x-\frac{\pi}{4}\right)^2+\frac{\mathrm{f}'\left(\frac{\pi}{4}\right)}{3!}\left(x-\frac{\pi}{4}\right)^3+\ldots$ Three terms are sufficient to establish method	M1	
	$\cos 2x=-2\left(x-\frac{\pi}{4}\right)+\frac{4}{3}\left(x-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(x-\frac{\pi}{4}\right)^5+\ldots$	A1	**(5)**
(b)	Substitute $x = 1$ $\quad \left(1-\frac{\pi}{4}\approx 0.21460\right)$	B1	
	$\cos 2=-2\left(1-\frac{\pi}{4}\right)+\frac{4}{3}\left(1-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(1-\frac{\pi}{4}\right)^5+\ldots$ ≈ -0.416147 cao	M1 A1	**(3)**
			[8]

[FP3 June 2006 Qn 2]

61.	(a) In this solution $\cos\theta = c$ and $\sin\theta = s$	
	$\cos 5\theta + \mathrm{i}\sin 5\theta = (c + \mathrm{i}s)^5$	M1
	$\left(= c^5 + 5c^4\,\mathrm{i}s + 10c^3(\mathrm{i}s)^2 + 10c^2(\mathrm{i}s)^3 + 5c(\mathrm{i}s)^4 + (\mathrm{i}s)^5\right)$	
	$\Im \quad \sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$ equate	M1 A1
	$= 5c^4 s - 10c^2(1-c^2)s + (1-c^2)^2 s$ $\quad s^2 = 1 - c^2$	M1
	$= s(16c^4 - 12c^2 + 1)$ ✱	A1 **(5)**
	(b) $\sin\theta(16\cos^4\theta - 12\cos^2\theta + 1) + 2\cos^2\theta\sin\theta = 0$	M1
	$\sin\theta = 0 \Rightarrow \theta = 0$	B1
	$16c^4 - 10c^2 + 1 = (8c^2 - 1)(2c^2 - 1) = 0$	M1
	$c = \pm\frac{1}{2\sqrt{2}}, \quad c = \pm\frac{1}{\sqrt{2}}$ any two	A1
	$\theta \approx 1.21, 1.93; \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}$ any two	A1
	all four	A1 **(6)**
	accept awrt 0.79, 1.21,1.93,2.36	**[11]**
	Ignore any solutions out of range.	

[FP3 June 2006 Qn 3]

62.	(a) Let $z = x + \mathrm{i}y$		
	$(x-6)^2 + (y+3)^2 = 9\left[(x+2)^2 + (y-1)^2\right]$	M1	
	Leading to $8x^2 + 8y^2 + 48x - 24y = 0$	M1 A1	
	This is a circle; the coefficients of x^2 and y^2 are the same and there is no *xy* term.		
	Allow equivalent arguments and ft their $\mathrm{f}(x, y)$ if appropriate.	A1ft	
	$(x^2 + 6x + y^2 - 3y = 0)$		
	Leading to $(x+3)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{45}{4}$	M1	
	Centre: $\left(-3, \frac{3}{2}\right)$	A1	
	Radius: $\frac{3}{2}\sqrt{5}$ or equivalent	A1	**(7)**
	(b) 		
	Circle	B1	
	centre in correct quadrant	B1 ft	
	through origin	B1	
	Line cuts –ve *x* and +ve *y* axes	B1	
	intersects with circle on axes and all correct	B1	**(5)**
	(c) Shading inside circle	B1	
	and above line with all correct	B1	**(2)**
			[14]

[FP3 June 2006 Qn 6]

63.	Attempt to arrange in correct form $\frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x}$	M1
	Integrating Factor: $= e^{\int \frac{2}{x}dx}$, $[(= e^{2\ln x} = e^{\ln x^2}) = x^2$	M1,A1
	[$x^2\frac{dy}{dx} + 2xy = x\cos x$ implies M1M1A1]	
	$\therefore \quad x^2 y = \int x^2 . \frac{\cos x}{x} dx$ or equiv. [I.F. y $= \int I.F.(candidate' sRHS)dx$]	M1√
	By Parts: $(x^2 y) = x\sin x - \int \sin x \, dx$	M1
	i.e. $(x^2 y) = x\sin x, + \cos x \ (+c)$	A1, A1cao
	$y = \frac{\sin x}{x} + \frac{\cos x}{x^2} + \frac{c}{x^2}$	A1√ **[8]**

[FP1 January 2007 Qn 2]

64.	Working from RHS: (a) Combining $\frac{1}{r}-\frac{1}{r+1}$ $\quad[\frac{1}{r(r+1)}]$ Forming single fraction : $\frac{r(r-1)(r+1)+(r+1)-r}{r(r+1)}$ $= \frac{r(r^2-1)+1}{r(r+1)} = \frac{r^3-r+1}{r(r+1)}$ **AG** Note: For A1, must be intermediate step, as shown Working from LHS: (a) $\frac{r(r^2-1)+1}{r(r+1)} = \frac{r(r+1)(r-1)+1}{r(r+1)} = r-1+\frac{1}{r(r+1)}$ M1 Splitting $\frac{1}{r(r+1)}$ into partial fractions M1 Showing $= \frac{r(r^2-1)+1}{r(r+1)} = r-1+\frac{1}{r}-\frac{1}{r+1}$ no incorrect working seen A1	M1 M1 A1cso (3)
	Notes: In first method, second M needs all necessary terms, allowing for sign errors In second method first M is for division: Second method mark is for method shown (allow "cover up" rule stated) If long division, allow reasonable attempt which has remainder constant or linear function of r. Setting $\frac{r(r^2-1)+1}{r(r+1)} = \frac{A}{r}+\frac{B}{r+1}$ is M0 If 3 or 4 constants used in a correct initial statement, M1 for finding 2 constants; M1 for complete method to find remaining constant(s)	

[FP1 Jan 2007 Qn 4]

65.	(a) $[(x > -2)]$: Attempt to solve $x^2 - 1 = 3(1-x)(x+2)$	M1
	$[4x^2 + 3x - 7 = 0]$	
	$x = 1, \quad or \quad -\frac{7}{4}$	B1, A1
	$[(x < -2)]$: Attempt to solve $x^2 - 1 = -3(1-x)(x+2)$	M1
	Solving $x + 1 = 3x + 6 \quad (2x^2 + 3x - 5 = 0)$	M1dep
	$x = -\frac{5}{2}$	A1 **(6)**
	(b) $-\frac{7}{4} < x < 1$ One part	M1
	Both correct and enclosed	A1
	$x < -\frac{5}{2}$ { Must be for x < −2 and only one value}	B1 √ **(3)** **[9]**

FP1 January 2007 Qn 5]

66.	(a) $y = x^{-2} \Rightarrow \quad \frac{dy}{dt} = -2x^{-3}\frac{dx}{dt}$ [Use of chain rule; need $\frac{dx}{dt}$]	M1
	$\Rightarrow \frac{d^2y}{dt^2} = -2x^{-3}\frac{d^2x}{dt^2}, \quad +6x^{-4}\left(\frac{dx}{dt}\right)^2$	A1√, M1A1
	($\div$ given d.e. by x^4) $\quad \frac{2}{x^3}\frac{d^2x}{dt^2} - \frac{6}{x^4}\left(\frac{dx}{dt}\right)^2 = \frac{1}{x^2} - 3$	
	becomes $\left(-\frac{d^2y}{dt^2} = y - 3\right) \quad \frac{d^2y}{dt^2} + y = 3$ **AG**	A1 cso **(5)**
	(b) Auxiliary equation: $m^2 + 1 = 0$ and produce Complementary Function $y = \ldots$	M1
	$(y) = A\cos t + B\sin t$	A1cao
	Particular integral: $y = 3$	B1
	$\therefore$ General solution: $(y) = A\cos t + B\sin t + 3$	A1√ **(4)**
	(c) $\frac{1}{x^2} = A\cos t + B\sin t + 3$	
	$x = \frac{1}{2}, t = 0 \quad \Rightarrow \quad (4 = A + 3) \quad A = 1$	B1
	Differentiating (to include $\frac{dx}{dt}$): $\quad -2x^{-3}\frac{dx}{dt} = -A\sin t + B\cos t$	M1
	$\frac{dx}{dt} = 0, t = 0 \quad \Rightarrow \quad (0 = 0 + B) \quad B = 0$	M1
	$\therefore \frac{1}{x^2} = 3 + \cos t \quad$ *so* $\quad x = \frac{1}{\sqrt{3 + \cos t}}$	A1 cao **(4)**
	(d) (Max. value of x when $\cos t = -1$) so max $x = \frac{1}{\sqrt{2}}$ or AWRT 0.707	B1 **(1)** **[14]**

[FP1 January 2007 Qn 7]

67.	(a) $x = r\cos\theta = 4\sin\theta\cos^3\theta$	M1
	$\frac{dx}{d\theta} = 4\cos^4\theta - 12\cos^2\theta\sin^2\theta$ any correct expression	M1A1
	Solving $\frac{dx}{d\theta} = 0$ $\quad [\frac{dx}{d\theta} = 0 \Rightarrow 4\cos^2\theta(\cos^2\theta - 3\sin^2\theta) = 0]$	M1
	$\sin\theta = \frac{1}{2}$ or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \theta = \frac{\pi}{6}$ AG	A1 cso
	$r = 4\sin\frac{\pi}{6}\cos^2\frac{\pi}{6} = \frac{3}{2}$ AG	A1 cso **(6)**
	(b) $A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} r^2\, d\theta = \frac{1}{2}.16\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2\theta\cos^4\theta\, d\theta$	
	$8\sin^2\theta\cos^4\theta = 2\cos^2\theta(4\sin^2\theta\cos^2\theta) = 2\cos^2\theta\sin^2 2\theta$	M1
	$= (\cos 2\theta + 1)\sin^2 2\theta$	M1
	$= \cos 2\theta\sin^2 2\theta + \frac{1-\cos 4\theta}{2}$ =Answer AG	A1 cso **(3)**
	(c) Area $= \left[\frac{1}{6}\sin^3 2\theta + \frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{\left(\frac{\pi}{6}\right)}^{\left(\frac{\pi}{4}\right)}$ (ignore limits)	M1A1
	$= \left(\frac{1}{6}\sin^3\frac{\pi}{2} + \frac{\pi}{8} - \frac{\sin\pi}{8}\right) - \left(\frac{1}{6}\sin^3\frac{\pi}{3} + \frac{\pi}{12} - \frac{\sin\frac{2\pi}{3}}{8}\right)$ (sub. limits)	M1
	$= \left(\frac{1}{6} + \frac{\pi}{8}\right) - \left(\frac{\sqrt{3}}{16} + \frac{\pi}{12} - \frac{\sqrt{3}}{16}\right) = \frac{1}{6}, + \frac{\pi}{24}$ both cao	A1,A1 **(5)** **[14]**

[FP1 January 2007 Qn 8]

68.	$1\frac{1}{2}$ and 3 are 'critical values', e.g. used in solution, or both seen as asymptotes	B1
	$(x+1)(x-3) = 2x-3 \Rightarrow x(x-4) = 0$	
	$x = 4,\ x = 0$ M1: attempt to find at least one other critical value	M1 A1, A1
	$0 < x < 1\frac{1}{2}$, $3 < x < 4$ M1: An inequality using $1\frac{1}{2}$ or 3	M1 A1, A1 (7)
		7

[FP1 June 2007 Qn 1]

69.	Integrating factor $e^{\int -\tan x\,dx} = e^{\ln(\cos x)}\ \left(\text{or } e^{-\ln(\sec x)}\right)$, $= \cos x \left(\text{or } \frac{1}{\sec x}\right)$	M1, A1
	$\left(\cos x\frac{dy}{dx} - y\sin x = 2\sec^2 x\right)$	
	$y\cos x = \int 2\sec^2 x\,dx$ (or equiv.) $\left(\text{Or: } \frac{d}{dx}(y\cos x) = 2\sec^2 x\right)$	M1 A1(ft)
	$y\cos x = 2\tan x\ (+C)$ (or equiv.)	A1
	$y = 3$ at $x = 0$: $C = 3$	M1
	$y = \frac{2\tan x + 3}{\cos x}$ (Or equiv. in the form $y = f(x)$)	A1 (7)
		7
	1st M: Also scored for $e^{\int \tan x\,dx} = e^{-\ln(\cos x)}\ \left(\text{or } e^{\ln(\sec x)}\right)$, then A0 for $\sec x$. 2nd M: Attempt to use their integrating factor (requires one side of the equation 'correct' for their integrating factor). 2nd A: The follow-through is allowed <u>only</u> in the case where the integrating factor used is $\sec x$ or $-\sec x$. $\left(y\sec x = \int 2\sec^4 x\,dx\right)$ 3rd M: Using $y = 3$ at $x = 0$ to find a value for C (dependent on an integration attempt, however poor, on the RHS). <u>Alternative</u> 1st M: Multiply through the given equation by $\cos x$. 1st A: Achieving $\cos x\frac{dy}{dx} - y\sin x = 2\sec^2 x$. (Allowing the possibility of integrating by inspection).	

[FP1 June 2007 Qn 2]

70.	(a) $(r+1)^3 = r^3 + 3r^2 + 3r + 1$ and $(r-1)^3 = r^3 - 3r^2 + 3r - 1$	M1
	$(r+1)^3 - (r-1)^3 = 6r^2 + 2$ (*)	A1cso (2)
	(b) $r = 1:\ 2^3 - 0^3 = 6(1^2) + 2$ $r = 2:\ 3^3 - 1^3 = 6(2^2) + 2$ $: \ : \ : \ : \ : \ : \ : \ : \ : \ : \ :$	
	$r = n:\ (n+1)^3 - (n-1)^3 = 6n^2 + 2$ M: Differences: at least first, last and one other.	M1 A1
	Sum: $(n+1)^3 + n^3 - 1 = 6\sum r^2 + 2n$ M: Attempt to sum at least one side. $\left(6\sum r^2 = 2n^3 + 3n^2 + n\right)$	M1 A1
	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ (Intermediate steps are not required) (*)	A1cso (5)
	(c) $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2, \ = \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2n-1)$	M1, A1
	$= \frac{1}{6}n\left(\left(16n^2 + 12n + 2\right) - \left(2n^2 - 3n + 1\right)\right)$	M1
	$= \frac{1}{6}n(n+1)(14n+1)$	A1 (4)
		11
	(b) 1st A: Requires first, last and one other term correct on both LHS and RHS (but condone 'omissions' if following work is convincing). (c) 1st M: Allow also for $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$. 2nd M: Taking out (at some stage) factor $\frac{1}{6}n$, and multiplying out brackets to reach an expression involving n^2 terms.	

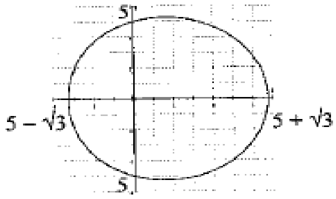
[FP1 June 2007 Qn 3]

71.	C.F. $m^2+3m+2=0$ $\quad m=-1$ and $m=-2$	M1	
	$y=Ae^{-x}+Be^{-2x}$	A1	(2)
	P.I. $y=cx^2+dx+e$	B1	
	$\dfrac{dy}{dx}=2cx+d,\ \dfrac{d^2y}{dx^2}=2c \qquad 2c+3(2cx+d)+2(cx^2+dx+e)\equiv 2x^2+6x$	M1	
	$2c=2 \qquad c=1$ (One correct value)	A1	
	$6c+2d=6 \qquad d=0$		
	$2c+3d+2e=0 \qquad e=-1$ (Other two correct values)	A1	
	General soln: $y=Ae^{-x}+Be^{-2x}+x^2-1$ (Their C.F. + their P.I.)	A1ft	(5)
	$x=0,\ y=1:\ \ 1=A+B-1 \qquad (A+B=2)$	M1	
	$\dfrac{dy}{dx}=-Ae^{-x}-2Be^{-2x}+2x,\quad x=0,\ \dfrac{dy}{dx}=1: \qquad 1=-A-2B$	M1	
	Solving simultaneously: $A=5$ and $B=-3$	M1 A1	
	Solution: $y=5e^{-x}-3e^{-2x}+x^2-1$	A1	(5)
			12

[FP1 June 2007 Qn 5]

72.

(a)



Shape (closed curve, approx. symmetrical about the initial line, in all 'quadrants' and 'centred' to the right of the pole/origin). **B1**

Scale (at least one correct 'intercept' r value… shown on sketch or perhaps seen in a table). (Also allow awrt 3.27 or awrt 6.73). **B1** (2)

(b) $y = r\sin\theta = 5\sin\theta + \sqrt{3}\sin\theta\cos\theta$ **M1**

$\frac{dy}{d\theta} = 5\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta \quad \left(= 5\cos\theta + \sqrt{3}\cos 2\theta\right)$ **A1**

$5\cos\theta - \sqrt{3}\left(1-\cos^2\theta\right) + \sqrt{3}\cos^2\theta = 0$ **M1**

$2\sqrt{3}\cos^2\theta + 5\cos\theta - \sqrt{3} = 0$

$\left(2\sqrt{3}\cos\theta - 1\right)\left(\cos\theta + \sqrt{3}\right) = 0 \qquad \cos\theta = \ldots\ (0.288\ldots)$ **M1**

$\theta = 1.28$ and 5.01 (awrt) (Allow ±1.28 awrt) $\left(\text{Also allow} \pm \arccos\frac{1}{2\sqrt{3}}\right)$ **A1**

$r = 5 + \sqrt{3}\left(\frac{1}{2\sqrt{3}}\right) = \frac{11}{2}$ (Allow awrt 5.50) **A1** (6)

(c) $r^2 = 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta$ **B1**

$\int 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta \, d\theta = \frac{53\theta}{2} + 10\sqrt{3}\sin\theta + 3\left(\frac{\sin 2\theta}{4}\right)$ **M1 A1ft A1ft**

(ft for integration of $(a + b\cos\theta)$ and $c\cos 2\theta$ respectively)

$\frac{1}{2}\left[25\theta + 10\sqrt{3}\sin\theta + \frac{3\sin 2\theta}{4} + \frac{3\theta}{2}\right]_0^{2\pi} = \ldots\ldots$ **M1**

$= \frac{1}{2}(50\pi + 3\pi) = \frac{53\pi}{2}$ or equiv. in terms of π. **A1** (6)

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(b) 2nd M: Forming a quadratic in $\cos\theta$.
3rd M: Solving a 3 term quadratic to find a value of $\cos\theta$ (even if called θ).

Special case: Working with $r\cos\theta$ instead of $r\sin\theta$:
1st M1 for $r\cos\theta = 5\cos\theta + \sqrt{3}\cos^2\theta$
1st A1 for derivative $-5\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$, then no further marks.

(c) 1st M: Attempt to integrate at least one term.
2nd M: Requires use of the $\frac{1}{2}$, correct limits (which could be 0 to 2π, or $-\pi$ to π, or 'double' 0 to π), and subtraction (which could be implied).

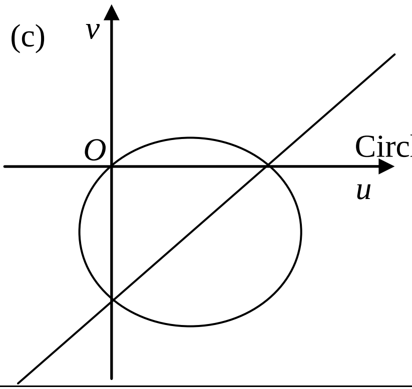
[FP1 June 2007 Qn 7]

73.	(a)	$\left(1-x^2\right)\frac{d^3y}{dx^3}-2x\frac{d^2y}{dx^2}-x\frac{d^2y}{dx^2}-\frac{dy}{dx}+2\frac{dy}{dx}=0$	M1
		At $x=0$, $\frac{d^3y}{dx^3}=-\frac{dy}{dx}=1$	M1 A1 cso **(3)**
	(b)	$\left(\frac{d^2y}{dx^2}\right)_0=-4$ Allow anywhere	B1
		$y=f(0)+f'(0)x+\frac{f''(0)}{2}x^2+\frac{f'''(0)}{6}x^3+\ \ldots$	
		$=2-x-2x^2,+\frac{1}{6}x^3+\ldots$	M1 A1ft, A1 (dep) **(4)**
			[7]

[FP3 June 2007 Qn 2]

74.	(a) $z^n=(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$ $z^{-n}=(\cos\theta+i\sin\theta)^{-n}=\cos(-n\theta)+i\sin(-n\theta)=\cos n\theta-i\sin n\theta$ both	M1
	Adding $z^n+\frac{1}{z^n}=2\cos n\theta$ ∗ cso	A1 **(2)**
	(b) $\left(z+\frac{1}{z}\right)^6=z^6+6z^4+15z^2+20+15z^{-2}+6z^{-4}+z^{-6}$	M1
	$=z^6+z^{-6}+6\left(z^4+z^{-4}\right)+15\left(z^2+z^{-2}\right)+20$	M1
	$64\cos^6\theta=2\cos 6\theta+12\cos 4\theta+30\cos 2\theta+20$	M1
	$32\cos^6\theta=\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10$	A1, A1
	$(p=1,\ q=6,\ r=15,\ s=10)$ A1 any two correct	**(5)**
	(c) $\int\cos^6\theta\,d\theta=\left(\frac{1}{32}\right)\int\left(\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10\right)d\theta$	
	$=\left(\frac{1}{32}\right)\left[\frac{\sin 6\theta}{6}+\frac{6\sin 4\theta}{4}+\frac{15\sin 2\theta}{2}+10\theta\right]$	M1 A1ft
	$[\ \ldots.\ ]_0^{\frac{\pi}{3}}=\frac{1}{32}\left[-\frac{3}{2}\times\frac{\sqrt{3}}{2}+\frac{15}{2}\times\frac{\sqrt{3}}{2}+\frac{10\pi}{3}\right]=\frac{5\pi}{48}+\frac{3\sqrt{3}}{32}$ or exact equivalent	M1 A1 **(4)**
		[11]

[FP3 June 2007 Qn 4]

75.	(a) Let $z = \lambda + \lambda\text{i}$; $\quad w = \dfrac{\lambda + (\lambda+1)\text{i}}{\lambda(1+\text{i})}$	M1
	$= \dfrac{\lambda + (\lambda+1)\text{i}}{\lambda(1+\text{i})} \times \dfrac{1-\text{i}}{1-\text{i}}$	M1
	$u + \text{i}v = \dfrac{(2\lambda+1)+\text{i}}{2\lambda}$	A1
	$u = 1 + \dfrac{1}{2\lambda}, \quad v = \dfrac{1}{2\lambda}$	M1
	Eliminating λ gives a line with equation $v = u - 1$ or equivalent	A1 **(5)**
	(b) Let $z = \lambda - (\lambda+1)\text{i}$; $\quad w = \dfrac{\lambda - \lambda\text{i}}{\lambda - (\lambda+1)\text{i}}$	M1
	$= \dfrac{\lambda - \lambda\text{i}}{\lambda - (\lambda+1)\text{i}} \times \dfrac{\lambda + (\lambda+1)\text{i}}{\lambda + (\lambda+1)\text{i}}$	M1
	$u + \text{i}v = \dfrac{\lambda(2\lambda+1)+\lambda\text{i}}{2\lambda^2+2\lambda+1}$	A1
	$u = \dfrac{\lambda(2\lambda+1)}{2\lambda^2+2\lambda+1}, \quad v = \dfrac{\lambda}{2\lambda^2+2\lambda+1}$	M1
	$\dfrac{u}{v} = 2\lambda + 1$	
	$v = \dfrac{2\lambda}{4\lambda^2+4\lambda+2} = \dfrac{(2\lambda+1)-1}{(2\lambda+1)^2+1} = \dfrac{\frac{u}{v}-1}{\left(\frac{u}{v}\right)^2+1}$	M1
	Reducing to the circle with equation $u^2 + v^2 - u + v = 0$ ∗ cso	M1 A1 **(7)**
	(c) 	
	ft their line	B1ft
	Circle through origin, centre in correct quadrant	B1
	Intersections correctly placed	B1 **(3)**
		[15]

[FP3 June 2007 Qn 8]

76.	Integrating factor = e^{-3x}	B1
	$\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$	M1
	$\therefore (ye^{-3x}) = \int xe^{-3x}dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x}dx$	M1
	$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x}(+c)$	A1
	$\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	A1ft
		[5]

[FP1 January 2008 Qn 1]

77.(a)	Consider $\frac{(x+3)(x+9)-(3x-5)(x-1)}{(x-1)}$, obtaining $\frac{-2x^2+20x+22}{(x-1)}$	M1 A1
	Factorise to obtain $\frac{-2(x-11)(x+1)}{(x-1)}$.	M1 A1
		(4)
(b)	Identify $x = 1$ and their two other critical values	B1ft
	Obtain one inequality *as an answer* involving at least one of their critical values	M1
	To obtain $x < -1$, $1 < x < 11$	A1, A1
		(4)
		[8]

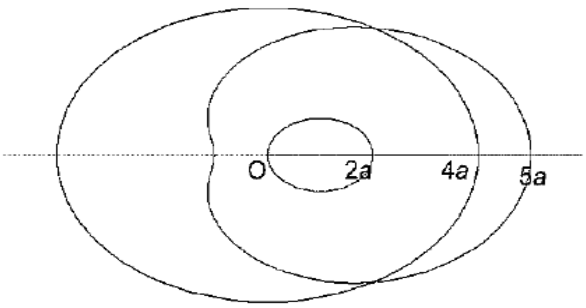
[FP1 January 2008 Qn 3]

78.(a)	Method to obtain partial fractions e.g. $5r+4 = A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ And equating coefficients, or substituting values for x.	M1
	$A = 2$, $B = 1$, $C = -3$ or $\frac{2}{r} + \frac{1}{r+1} - \frac{3}{r+2}$	A1 A1 A1
		(4)
(b)	$\sum_{r=1}^{n} \ldots = \frac{2}{1} + \frac{1}{2} - \frac{3}{3}$ $+ \frac{2}{2} + \frac{1}{3} - \frac{3}{4}$ $+ \frac{2}{3} + \frac{1}{4} - \frac{3}{5}$ $= 2 + \frac{3}{2}, -\frac{2}{n+1} - \frac{3}{n+2}$ or equivalent $+ \ldots$ $+ \frac{2}{n-1} + \frac{1}{n} - \frac{3}{n+1}$ $+ \frac{2}{n} + \frac{1}{n+1} - \frac{3}{n+2}$	M1 A1, A1
	$= \frac{7(n+1)(n+2) - 4(n+2) - 6(n+1)}{2(n+1)(n+2)} = \frac{7n^2 + 11n}{2(n+1)(n+2)}$ *	M1 A1
		(5)

[FP1 January 2008 Qn 5]

79.(a)	Solve auxiliary equation $3m^2 - m - 2 = 0$ to obtain $m = -\frac{2}{3}$ or 1	M1 A1
	C.F is $Ae^{-\frac{2}{3}x} + Be^x$	A1ft
	Let PI $= \lambda x^2 + \mu x + \nu$. Find $y' = 2\lambda x + \mu$, and $y'' = 2\lambda$ and substitute into d.e. Giving $\lambda = -\frac{1}{2}$, $\mu = \frac{1}{2}$ and $\nu = -\frac{7}{4}$	M1 A1 A1A1
	$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^x$	A1ft **(8)**
(b)	Use boundary conditions: $2 = -\frac{7}{4} + A + B$	M1A1ft
	$y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^x$ and $3 = \frac{1}{2} - \frac{2}{3}A + B$	M1 M1
	Solve to give A=3/4, $B = 3$ $(\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^x)$	M1 A1 **(6)** **[14]**

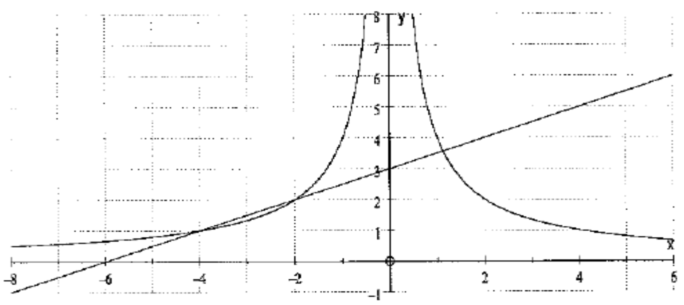
[FP1 January 2008 Qn 7]

80.(a)	$a(3 + 2\cos\theta) = 4a$	M1
	Solve to obtain $\cos\theta = \frac{1}{2}$	M1
	$\theta = \pm\frac{\pi}{3}$ and points are $(4a, \frac{\pi}{3})$ and $(4a, \frac{5\pi}{3})$	A1, A1 (4)
(b)	Use area $= \frac{1}{2}\int r^2 d\theta$ to give $\frac{1}{2}a^2\int(3 + 2\cos\theta)^2 d\theta$	M1
	Obtain $\int(9 + 12\cos\theta + 2\cos 2\theta + 2)d\theta$	A1
	Integrate to give $11\theta + 12\sin\theta + \sin 2\theta$	M1 A1
	Use limits $\frac{\pi}{3}$ and π, then double or $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ or theirs	M1
	Find a third area of circle $= \frac{16\pi a^2}{3}$	B1
	Obtain required area $= \frac{38\pi a^2}{3} - \frac{13\sqrt{3}a^2}{2}$	A1 , A1 (8)
(c)	 correct shape 5a and 4a marked 2a marked and passes through O	B1 B1 B1 (3) **[15]**

[FP1 January 2008 Qn 8]

81.	(a) $m^2+4m+3=0$ $\quad m=-1,\ m=-3$	M1 A1	
	C.F. $(x=)Ae^{-t}+Be^{-3t}$ must be function of t, not x	A1	
	P.I. $x=pt+q$ (**or** $x=at^2+bt+c$)	B1	
	$4p+3(pt+q)=kt+5$ $\quad 3p=k$ (Form at least one eqn. in p and/or q) $4p+3q=5$	M1	
	$p=\frac{k}{3},\quad q=\frac{5}{3}-\frac{4k}{9}\left(=\frac{15-4k}{9}\right)$	A1	
	General solution: $x=Ae^{-t}+Be^{-3t}+\frac{kt}{3}+\frac{15-4k}{9}$ must include x = and be function of t	A1 ft	(7)
	(b) When $k=6$, $\quad x=2t-1$	M1 A1cao	(2)
			9

[FP1 June 2008 Qn 4]

82.	(a) $\frac{4}{x}=\frac{x}{2}+3$ $\quad x^2+6x-8=0$ $x=\ldots,\left(\frac{-6\pm\sqrt{68}}{2}\right)$ $\quad -3\pm\sqrt{17}$ - root not needed	M1, A1	
	$-\frac{4}{x}=\frac{x}{2}+3,$ $\quad x^2+6x+8=0$ $\quad x=-4$ and -2	M1, A1	
	Three correct solutions (and no extras): $-4,\ -2,\ -3+\sqrt{17}$	A1	(5)
	(b)  Line through point on –ve x axis and + y axis	B1	
	Curve	B1	
	3 Intersections in correct quadrants	B1	(3)
	(c) $-4<x<-2,$ $\quad x>-3+\sqrt{17}$ $\quad$ o.e.	B1, B1	(2)
			10
	(a) <u>Alternative using squaring method</u> Square both sides and attempt to find roots	M1	
	$x^4+12x^3+36x^2-64=0$ gives x = -2 and x = -4	A1	
	Obtain quadratic factor, divide find solutions of quadratic and obtain $\left(-3\pm\sqrt{17}\right)$	M1 A1	
	Last mark as before		
	(c) Use of $\le$ instead of $<$ lose last B1 $\quad$ Extra inequalities lose last B1		

[FP1 June 2008 Qn 5]

83.	(a) $\frac{2}{(r+1)(r+3)}=\frac{1}{r+1}-\frac{1}{r+3}$ M: $\frac{2}{(r+1)(r+3)}=\frac{A}{r+1}+\frac{B}{r+3}$	M1 A1 (2)
	(b) $r=1$: $\left(\frac{2}{2\times 4}\right)=\frac{1}{2}-\frac{1}{4}$	M1
	$r=2$: $\left(\frac{2}{3\times 5}\right)=\frac{1}{3}-\frac{1}{5}$	
	$\ldots r=n-1$: $\left(\frac{2}{n(n+2)}\right)=\frac{1}{n}-\frac{1}{n+2}$	
	$r=n$: $\left(\frac{2}{(n+1)(n+3)}\right)=\frac{1}{n+1}-\frac{1}{n+3}$	A1 ft
	Summing: $\sum=\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}$	M1 A1
	$=\frac{5(n+2)(n+3)-6(n+3)-6(n+2)}{6(n+2)(n+3)}=\frac{n(5n+13)}{6(n+2)(n+3)}$	**d** M1 A1cso (6)
	(c) $\sum_{21}^{30}=\sum_{1}^{30}-\sum_{1}^{20}=\frac{30\times 163}{6\times 32\times 33}-\frac{20\times 113}{6\times 22\times 23}$, $=0.02738$	M1A1ft,A1cso (3)
		(11)

[FP1 June 2008 Qn 6]

84.	(a) $\frac{dy}{dx}=v+x\frac{dv}{dx}$	B1
	$\left(v+x\frac{dv}{dx}\right)=\frac{x}{vx}+\frac{3vx}{x} \Rightarrow x\frac{dv}{dx}=2v+\frac{1}{v}$ (*)	M1 A1 (3)
	(b) $\int\frac{v}{1+2v^2}dv=\int\frac{1}{x}dx$	M1
	$\frac{1}{4}\ln(1+2v^2)$, $=\ln x$ $(+C)$	dM1 A1, B1
	$Ax^4=1+2v^2$	d M1
	$Ax^4=1+2\left(\frac{y}{x}\right)^2$ so $y=\sqrt{\frac{Ax^6-x^2}{2}}$ or $y=x\sqrt{\frac{Ax^4-1}{2}}$ or $y=x\sqrt{\left(\frac{1}{2}e^{4\ln x+4c}-\frac{1}{2}\right)}$	M1 A1 (7)
	(c) $x=1$ at $y=3$: $3=\sqrt{\frac{A-1}{2}}$ $A=\ldots$	M1
	$y=\sqrt{\frac{19x^6-x^2}{2}}$ or $y=x\sqrt{\frac{19x^4-1}{2}}$	A1 (2) **12**

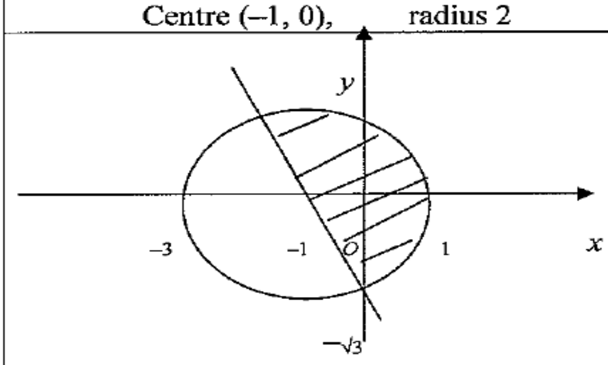
[FP1 June 2008 Qn 7]

85.	(a) $r\cos\theta = 4(\cos\theta - \cos^2\theta)$ or $r\cos\theta = 4\cos\theta - 2\cos 2\theta - 2$	B1
	$\frac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta} = 4(-\sin\theta + 2\cos\theta\sin\theta)$ or $\frac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta} = 4(-\sin\theta + \sin 2\theta)$	M1 A1
	$4(-\sin\theta + 2\cos\theta\sin\theta) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ which is satisfied by $\theta = \frac{\pi}{3}$ and $r = 2$(*)	d M1 A1 (5)
	(b) $\frac{1}{2}\int r^2\,\mathrm{d}\theta = (8)\int(1 - 2\cos\theta + \cos^2\theta)\mathrm{d}\theta$	M1
	$= (8)\left[\theta - 2\sin\theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2}\right]$	M1 A1
	$8\left[\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi/2} = 8\left(\left(\frac{3\pi}{4} - 2\right) - \left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right)\right) = 2\pi - 16 + 7\sqrt{3}$	M1
	Triangle: $\frac{1}{2}(r\cos\theta)(r\sin\theta) = \frac{1}{2}\times 1\times\sqrt{3} = \frac{\sqrt{3}}{2}$	M1 A1
	Total area: $(2\pi - 16 + 7\sqrt{3}) + \frac{\sqrt{3}}{2} = (2\pi - 16) + \frac{15\sqrt{3}}{2}$	(A1) A1 (8)
		13

[FP1 June 2008 Qn 8]

86.	(a)	$(x^2+1)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4y\frac{\mathrm{d}y}{\mathrm{d}x} + (1-2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1
		$(x^2+1)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = (1-4x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (4y-2)\frac{\mathrm{d}y}{\mathrm{d}x}$ (*)	A1 (3)
	(b)	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 = 3$	B1
		$\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0 = 5$ Follow through: $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2$	B1ft
		$y = 1 + x + \frac{3}{2}x^2 + \frac{5}{6}x^3 \ldots.$	M1 A1 (4)
	(c)	$x = -0.5,\quad y \approx 1 - 0.5 + 0.375 - 0.104166\ldots$ $= 0.77$ (2 d.p.) [awrt 0.77]	B1 (1)
			(8)

[FP3 June 2008 QN 3]

87.	(a)	$\lvert (x-3)+\mathrm{i}y\rvert = 2\lvert x+\mathrm{i}y\rvert \Rightarrow (x-3)^2+y^2 = 4x^2+4y^2$ $\therefore\ x^2+y^2+2x-3=0$ $(x+1)^2+y^2=4$ Centre $(-1, 0)$, radius 2	M1 A1 M1 A1, A1 (5)
	(b)	 Circle, centre on x-axis B1 $C(-1, 0)$, $r=2$ dB1ft Follow through centre and radius, but dependent on first B1. There must be indication of their '–3', '–1' or '1' on the x-axis and no contradictory evidence for their radius. Straight line B1 Straight line through $(-1, 0)$, or perp. bisector of $(-3, 0)$ and $(0, \sqrt{3})$. B1 Straight line through point of int. of circle & –ve y-axis, or through $(0, -\sqrt{3})$ B1	B1 dB1 B1 B1 B1 (5)

[FP3 June 2008 Qn 4]

88.	(a)	$(\cos\theta+\mathrm{i}\sin\theta)^1 = \cos\theta+\mathrm{i}\sin\theta \quad \therefore$ true for $n=1$ Assume true for $n=k$, $(\cos\theta+\mathrm{i}\sin\theta)^k = \cos k\theta+\mathrm{i}\sin k\theta$ $(\cos\theta+\mathrm{i}\sin\theta)^{k+1} = (\cos k\theta+\mathrm{i}\sin k\theta)(\cos\theta+\mathrm{i}\sin\theta)$ $= \cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta+\cos k\theta\sin\theta)$ (Can be achieved either from the line above or the line below) $= \cos(k+1)\theta+\mathrm{i}\sin(k+1)\theta$ Requires full justification of $(\cos\theta+\mathrm{i}\sin\theta)^{k+1} = \cos(k+1)\theta+\mathrm{i}\sin(k+1)\theta$ ($\therefore$ true for $n=k+1$ if true for $n=k$) $\therefore$ true for $n\in\mathbb{Z}^+$ by induction	B1 M1 M1 A1 A1cso (5)
	(b)	$\cos 5\theta = \mathrm{Re}\left[(\cos\theta+\mathrm{i}\sin\theta)^5\right]$ $= \cos^5\theta + 10\cos^3\theta\,\mathrm{i}^2\sin^2\theta + 5\cos\theta\,\mathrm{i}^4\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	M1 A1 M1 M1 A1cso (5)
	(c)	$\dfrac{\cos 5\theta}{\cos\theta} = 0 \Rightarrow \cos 5\theta = 0$ $5\theta = \dfrac{\pi}{2}$ $\theta = \dfrac{\pi}{10}$ $x = 2\cos\theta$, $x = 2\cos\dfrac{\pi}{10}$ is a root (*)	M1 A1 A1 (3) **(13)**

[FP3 June 2008 Qn 6]