1.	Integrating factor = e^{-3x}	B1	
	$\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$	M1	
	$\therefore (ye^{-3x}) = \int xe^{-3x}dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x}dx$	M1	
	$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x}(+c)$	A1	
	$\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	A1ft	5

Notes:

First M for multiplying through by Integrating Factor and evidence of calculus

Second M for integrating by parts 'the right way around'.
Be generous – ignore wrong signs and wrong constants.

Second M dependent on first. Both As dependent on this M.

First A1 for correct expression – constant not required

Second A requires constant for follow through.

If treated as a second order de with errors then send to review.

[5]

2.	(a)	Consider $\frac{(x+3)(x+9)-(3x-5)(x-1)}{(x-1)}$, obtaining $\frac{-2x^2+20x+22}{(x-1)}$	M1A1	
		Factorise to obtain $\frac{-2(x-11)(x+1)}{(x-1)}$.	M1A1	4
	(b)	Identify $x = 1$ and their two other critical values	B1ft	
		Obtain one inequality *as an answer* involving at least one of their critical values	M1	
		To obtain $x < -1$, $1 < x < 11$	A1, A1	4

Notes:

Second M attempt to factorise quadratic expression with 3 terms
(usual rules).

Second A don't require –2 outside but can be part of factors. B1, B1

[8]

3.	(a)	Solve auxiliary equation $3m^2 - m - 2 = 0$ to obtain $m = -\frac{2}{3}$ or 1	M1A1
		C.F is $Ae^{-\frac{2}{3}x} + Be^{x}$	A1ft
		Let PI = $\lambda x^2 + \mu x + v$. Find $y' = 2\lambda x + \mu$, and $y'' = 2\lambda$ and substitute into d.e.	M1
		Giving $\lambda = -\frac{1}{2}$, $\mu = \frac{1}{2}$ and $v = -\frac{7}{4}$,	A1A1A1
		$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^{x}$	A1ft 8

Attempt to solve quadratic expression with 3 terms (usual rules)

Both values required for first accuracy.

Real values only for follow through

Second M 3 term quadratic for PI required

Final A1ft for their CF+ their PI dependent upon at least one M

	(b)	Use boundary conditions:	
		$2 = -\frac{7}{4} + A + B$	M1A1ft
		$y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^{x}$ and $3 = \frac{1}{2} - \frac{2}{3}A + B$	M1A1
		Solve to give $A = 3/4$, $B = 3$ ($\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^{x}$)	M1 A1 6

Second M for attempt to differentiate their y and third M for substitution

[14]

4.	(a)	$a(3 + 2\cos\theta) = 4a$	M1
		Solve to obtain $\cos\theta = \frac{1}{2}$	M1
		$\theta = \pm\frac{\pi}{3}$ and points are ($4a$, $\frac{\pi}{3}$) and ($4a$, $\frac{5\pi}{3}$)	A1, A1 4

First A for $r = 4a$ second for both values in radians.
Accept 1.0471… and 5.2359….2 dp or better for final A

(b) Use area = $\frac{1}{2}\int r^2 d\theta$ to give $\frac{1}{2}a^2\int(3+2\cos\theta)^2 d\theta$ M1

Obtain $\int(9 + 12\cos\theta + 2\cos2\theta + 2)d\theta$ A1

Integrate to give $11\theta + 12\sin\theta + \sin 2\theta$ M1 A1

Use limits $\frac{\pi}{3}$ and π, then double or $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ or theirs M1

Find a third area of circle = $\frac{16\pi a^2}{3}$ B1

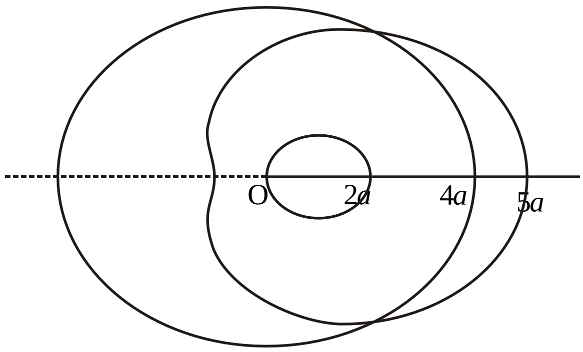
Obtain required area = $\frac{38\pi a^2}{3} - \frac{13\sqrt{3}a^2}{2}$ A1 , A1 8

First M for substitution, expansion and attempt to use double angles.

Second M for integrating expression of the form $a + b\cos\theta + c\cos2\theta$

Lose final A only if a^2 missing in last line

(c)



correct shape B1

5a and 4a marked B1

2a marked and passes through O B1 3

First B for approximately symmetrical shape about initial line, only 1 loop which is convex strictly within shaded region

[15]

5. (a) $m^2 + 4m + 3 = 0$ $m = -1,\ m = -3$ M1A1

C.F. $(x =)Ae^{-t} + Be^{-3t}$ must be function of t, not x A1

P.I. $x = pt + q$ (**or** $x = at^2 + bt + c$) B1

$4p + 3(pt + q) = kt + 5$ $3p = k$ (Form at least one eqn. in p and/or q) M1

$4p + 3q = 5$

$p = \frac{k}{3},\ q = \frac{5}{3} - \frac{4k}{9}\left(= \frac{15-4k}{9}\right)$ A1

General solution: $x = Ae^{-t} + Be^{-3t} + \frac{kt}{3} + \frac{15-4k}{9}$

must include $x =$ and be function of t A1ft 7

M1 for auxiliary equation substantially correct

B1 not awarded for $x = kt$ + constant

(b) When $k = 6$, $x = 2t - 1$ M1 A1cao

M mark for using $k = 6$ to derive a linear expression in t.
(cf must have involved negative exponentials only)
so e.g. $y = 2t - 1$ is M1 A0

[9]

6. (a) $\frac{4}{x} = \frac{x}{2} + 3$ $x^2 + 6x - 8 = 0$ $x = ..., \left(\frac{-6 \pm \sqrt{68}}{2}\right)$ $-3 \pm \sqrt{17}$ M1, A1

– root not needed

$-\frac{4}{x} = \frac{x}{2} + 3,\ x^2 + 6x + 8 = 0$ $x = -4$ and -2 M1, A1

Three correct solutions (and no extras): $-4, -2, -3 + \sqrt{17}$ A1 5

<u>Alternative using squaring method</u>

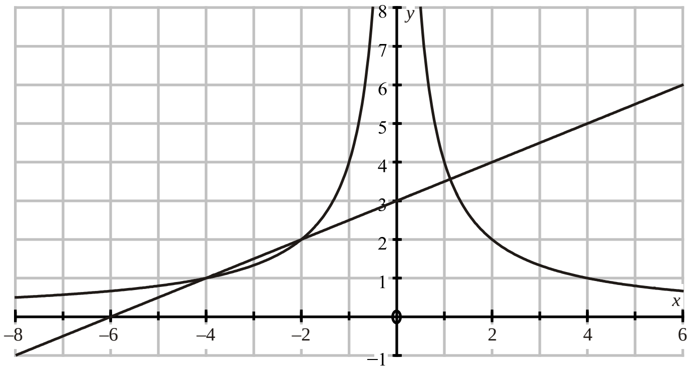
Square both sides and attempt to find roots M1

$x^4 + 12x^3 + 36x^2 - 64 = 0$ gives $x = -2$ and $x = -4$ A1

Obtain quadratic factor, divide find solutions of quadratic
and obtain $(-3 \pm \sqrt{17})$ M1 A1

Last mark as before

(b)



Line through point on –ve x axis and + y axis B1

Curve B1

3 Intersections in correct quadrants B1 3

(c) $-4 < x < -2$, $x > -3 + \sqrt{17}$ o.e. B1, B1 2

Use of $\leq$ instead of $<$ lose last B1 Extra inequalities lose last B1

[10]

7. (a) $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ B1

$\left(v + x\dfrac{dv}{dx}\right) = \dfrac{x}{vx} + \dfrac{3vx}{x} \Rightarrow x\dfrac{dv}{dx} = 2v + \dfrac{1}{v}$ (*) M1A1 3

B1 for statement printed or for $\dfrac{dy}{dx} = (x + v\dfrac{dx}{dv})\dfrac{dv}{dx}$

First M1 is for RHS of equation only but for A1 need whole answer correct .

(b) $\int \frac{v}{1+2v^2}\,\mathrm{d}v = \int \frac{1}{x}\mathrm{d}x$ M1

$\frac{1}{4}\ln(1+2v^2), = \ln x (+C)$ dM1A1, B1

$Ax^4 = 1 + 2v^2$ dM1

$Ax^4 = 1 + 2\left(\frac{y}{x}\right)^2$ so $y = \sqrt{\frac{Ax^6 - x^2}{2}}$ or $y = x\sqrt{\frac{Ax^4 - 1}{2}}$

or $y = x\sqrt{\left(\frac{1}{2}e^{4\ln x + 4c} - \frac{1}{2}\right)}$ M1A1 7

First M1 accept $\int \frac{1}{2v + \frac{1}{v}}\mathrm{d}v = \int \frac{1}{x}\mathrm{d}x$

Second M1 requires an integration of correct form ¼ may be missing

A1 for LHS correct with ¼ and B1 is independent and is for ln*x*

Third M1 is **dependent** and needs correct application of log laws

Fourth M1 is independent and merely requires return *to y/x* for *v*

N.B. There is an IF method possible after suitable rearrangement – see note.

(c) $x = 1$ at $y = 3$: $3 = \sqrt{\frac{A-1}{2}}$ $A = \ldots$ M1

$y = \sqrt{\frac{19x^6 - x^2}{2}}$ or $y = x\sqrt{\frac{19x^4 - 1}{2}}$ A1 2

[12]

8. (a) $r\cos\theta = 4(\cos\theta - \cos^2\theta)$ or $r\cos\theta = 4\cos\theta - 2\cos 2\theta - 2$ B1

$\frac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta} = 4(-\sin\theta + 2\cos\theta\sin\theta)$ or $\frac{\mathrm{d}(r\cos\theta)}{\mathrm{d}\theta} = 4(-\sin\theta + \sin 2\theta)$ M1A1

$4(-\sin\theta + 2\cos\theta\sin\theta) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ which is

satisfied by $\theta = \frac{\pi}{3}$ and $r = 2$(*) dM1A1 5

<u>Alternative for first 3 marks</u>:

$\frac{\mathrm{d}r}{\mathrm{d}\theta} = 4\sin\theta$ B1

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -r\sin\theta + \cos\theta\frac{\mathrm{d}r}{\mathrm{d}\theta} = -4\sin\theta + 8\sin\theta\cos\theta$ M1A1

Substituting r = 2 and 0 = f into **original equation** scores 0 marks.

(b) $\frac{1}{2}\int r^2 \mathrm{d}\theta = (8)\int (1 - 2\cos\theta + \cos^2\theta)\mathrm{d}\theta$ M1

$= (8)\left[\theta - 2\sin\theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2}\right]$ M1A1

$8\left[\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi/2} = 8\left(\left(\frac{3\pi}{4} - 2\right) - \left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right)\right) = 2\pi - 16 + 7\sqrt{3}$ M1

Triangle: $\frac{1}{2}(r\cos\theta)(r\sin\theta) = \frac{1}{2} \times 1 \times \sqrt{3} = \frac{\sqrt{3}}{2}$ M1A1

Total area: $\left(2\pi - 16 + 7\sqrt{3}\right) + \frac{\sqrt{3}}{2} = (2\pi - 16) + \frac{15\sqrt{3}}{2}$ (A1)A1 8

M1 needs attempt to expand $(1 - \cos\theta)^2$ giving three terms (allow slips)
Second M1 needs integration of $\cos^2\theta$ using $\cos 2\theta \pm 1$
Third M1 needs correct limits– may evaluate two areas and subtract
M1 needs attempt at area of triangle and A1 for cao
Next A1 is for value of area within curve, then final A1 is cao,
must be exact but allow 4 terms and isw for incorrect collection of terms

Special case for use of $r\sin\theta$ gives B0M1A0M0A0

[13]

9. (a) $(x^2+1)\dfrac{d^3y}{dx^3}+2x\dfrac{d^2y}{dx^2}=4y\dfrac{dy}{dx}+(1-2x)\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}$ M1A1

$(x^2+1)\dfrac{d^3y}{dx^3}=(1-4x)\dfrac{d^2y}{dx^2}+(4y-2)\dfrac{dy}{dx}$ (*) A1 3

M: Use of product rule (at least once) and implicit differentiation (at least once).

(b) $\left(\dfrac{d^2y}{dx^2}\right)_0=3$ B1

$\left(\dfrac{d^3y}{dx^3}\right)_0=5$ Follow through: $\dfrac{d^3y}{dx^3}=\dfrac{d^2y}{dx^2}+2$ B1ft

$y = 1 + x + \dfrac{3}{2}x^2+\dfrac{5}{6}x^3...$ M1A1 4

M: Use of series expansion with values for the derivatives (can be allowed without the first term 1, and can also be allowed if final term uses 3 rather than 3!)

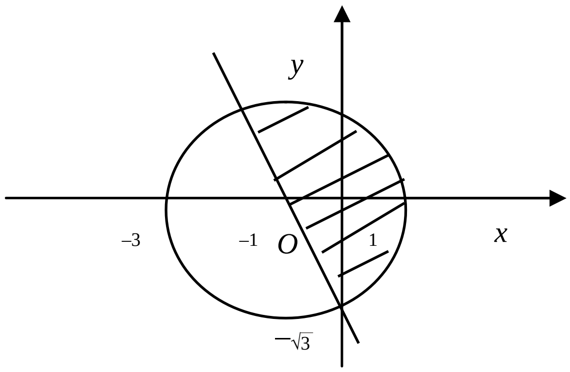
(c) $x = -0.5$, $y \approx 1 - 0.5 + 0.375 - 0.104166\ldots$
$= 0.77$ (2 d.p.) [awrt 0.77] B1 1
[8]

10. (a) $|(x-3) + \mathrm{i}y| = 2|x + \mathrm{i}y| \Rightarrow (x-3)^2 + y^2 = 4x^2 + 4y^2$ M1A1
$\therefore x^2 + y^2 + 2x - 3 = 0$
$(x+1)^2 + y^2 = 4$ M1
Centre $(-1, 0)$, .radius 2 A1, A1 5

1st M: Use $z = x + \mathrm{i}y$, and attempt square of modulus of each side.
Not squaring the 2 on the RHS would be M1 A0.

2nd M: Attempting to express in the form $(x-a)^2 + (y-b)^2 = k$,
or attempting centre and radius from the form
$x^2 + y^2 + 2gx + 2fy + c = 0$

(b)



Circle, centre on x-axis B1
$C\,(-1, 0)$, $r = 2$ dB1ft
Follow through centre and radius, but dependent on first B1.
There must be indication of their '–3', '–1' or '1' on the x-axis
and no contradictory evidence for their radius.
Straight line B1
Straight line through $(-1, 0)$, or perp. bisector of $(-3, 0)$ and
$(0, \sqrt{3})$. B1
Straight line through point of int. of circle & –ve y-axis, or
through $(0, -\sqrt{3})$ B1 5

(c) Shading (only) inside circle B1
Inside correct circle and all of the correct side of the correct line…
this mark is dependent on <u>all</u> the previous B marks in parts (b)
and (c). dB1 2
[12]

11.	(a)	$(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta$ $\therefore$ true for $n = 1$	B1	
		Assume true for $n = k$, $(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$		
		$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1	
		$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	M1	
		(Can be achieved either from the line above or the line below)		
		$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1	
		Requires full justification of $(\cos\theta + \mathrm{i}\sin\theta)^{k+1}$		
		$= \cos(k+1)\theta + \mathrm{i}\,sin(k+1)\theta$		
		($\therefore$ true for $n = k+1$ if true for $n = k$) $\therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1cso	5
		<u>Alternative</u>:		
		For the 2nd M mark: $(e^{\mathrm{i}k\theta})(e^{\mathrm{i}\theta}) = e^{\mathrm{i}\theta(k+1)}$		
	(b)	$\cos 5\theta = \mathrm{Re}[(\cos\theta + \mathrm{i}\sin\theta)^5]$		
		$= \cos^5\theta + 10\cos^3\theta\,\mathrm{i}^2\sin^2\theta + 5\cos\theta\,\mathrm{i}^4\sin^4\theta$	M1A1	
		$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1	
		$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$	M1	
		$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	A1cso	5
		<u>Alternative</u>:		
		$\left(z+\frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z}\right)^2 + 10z^2\left(\frac{1}{z}\right)^3 + 5z\left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5$	M1	
		$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	A1	
		$(2\cos\theta)^5 =$ ....and attempt to put $\cos 3\theta$ in powers of $\cos\theta$	M1	
		Correct method (or formula) for putting $\cos 3\theta$ in powers of $\cos\theta$	M1	
		$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	A1cso	
	(c)	$\frac{\cos 5\theta}{\cos\theta} = 0 \Rightarrow \cos 5\theta = 0$	M1	
		$5\theta = \frac{\pi}{2}$ $\quad \theta = \frac{\pi}{10}$	A1	
		$x = 2\cos\theta$, $x = 2\cos\frac{\pi}{10}$ is a root (*)	A1	3

Alternatives:

(i) Substitute given root into $x^4 - 5x^2 + 5$:

$$\left(2\cos\frac{\pi}{10}\right)^4 - 5\left(2\cos\frac{\pi}{10}\right)^2 + 5 = 2^4\left(\cos\frac{\pi}{10}\right)^4 - 5\times 2^2\left(\cos\frac{\pi}{10}\right)^2 + 5 \quad \text{M1}$$

'Multiply by cos θ' and use result from part (b): ... = $\cos\frac{5\pi}{10}$ A1

= 0 and conclusion A1

(ii) Use $5\theta = \frac{\pi}{2}$ in result from part (b) M1

$16\left(\cos\frac{\pi}{10}\right)^5 - 20\left(\cos\frac{\pi}{10}\right)^3 + 5\left(\cos\frac{\pi}{10}\right)$ and divide by cos θ A1

= 0 and conclusion A1

[13]