1. Attempt to arrange in correct form $\frac{dy}{dx} + \frac{2}{x} y = \frac{\cos x}{x}$ M1

Integrating Factor: = $e^{\int \frac{2}{x} dx}$, $\left[(= e^{2 \ln x} = e^{\ln x^2}) = x^2\right]$ M1, A1

[$x^2 \frac{dy}{dx}$ + $2xy = x \cos x$ implies M1M1A1]

$\therefore x^2 y = \int x^2 . \frac{\cos x}{x} dx$ or equiv. M1ft

[IF. $y = \int$ *I.F.* (candidate's RHS)dx]

By Parts: $(x^2 y) = x \sin x - \int \sin x \, d\, x$ M1

i.e. $(x^2 y) = x \sin x, + \cos x \; (+ c)$ A1, A1cao

$y = \frac{\sin x}{x} + \frac{\cos x}{x^2} + \frac{c}{x^2}$ A1ft8

First M: At least two terms divided by x.

"By parts" M: Must be complete method, e.g $\int x^{"2} \cos x \, dx$ requires **two** applications

Because of functions involved**, be generous with sign,** but $x \sin x \pm \int \cos x \, d\, x$ is M0 (S.C. "Loop" integral like $\int e^x \cos x \, dx$, *allow M1 if two applications of* "*by parts*", *despite incomplete method*)

Final A ft for dividing all terms by candidates IF., providing "c" used.

[8]

2.	(a)	[($x > -2$)]: Attempt to solve $x^2 - 1 = 3(1 - x)(x + 2)$		M1
		[$4x^2 + 3x - 7 = 0$]		
		$x = 1$, or $-\frac{7}{4}$		B1, A1
		[($x < -2$)]: Attempt to solve $x^2 - 1 = -3(1 - x)(x + 2)$		M1
		Solving $x + 1 = 3x + 6$ ($2x^2 + 3x - 5 = 0$)		M1dep
		$x = -\frac{5}{2}$		A16
		"Squaring"		
		If candidates do not notice the factor of $(x - 1)^2$ they have quartic to solve;		
		Squaring and finding quartic = 0 [$8x^4 + 18x^3 - 25x^2 - 36x + 35 = 0$]		
		Finding one factor and factorising $(x - 1)(8x^3 + 26x^2 + x - 35) = 0$		M1
		Finding one other factor and reducing other factor to quadratic, likely to be $(x - 1)^2(8x^2 + 34x + 35) = 0$		M1
		Complete factorisation $(x - 1)^2(2x + 5)(4x + 7) = 0$		M1
		[Second M1 implies the first, if candidate starts there or cancels $(x - 1)^2$]		
		$x = 1$ B1, $x = -7/4$ A1, $x = -5/2$		A1
		$x = 1$ allowed anywhere, no penalty in (b)		
	(b)	$-\frac{7}{4} < x < 1$	One part	M1
			Both correct and enclosed	A1
		$x < -\frac{5}{2}$ {Must be for $x < -2$ and only one value}		B1ft3
		Correct answers seen with no working is independent of (a) (graphical calculator) mark as scheme. Only allow the accuracy mark if no other interval, in both parts *≤ used penalise first time used*		

[9]

3. (a) $y = x^{-2} \Rightarrow \frac{dy}{dt} = -2x^{-3}\frac{dx}{dt}$ = –2x –3 t [Use of chain rule; need $\frac{dx}{dt}$] M1

$\Rightarrow \frac{d^2y}{dt^2} = -2x^{-3}\frac{d^2x}{dt^2}, \quad + 6x^{-4}\left(\frac{dx}{dt}\right)^2$ A1ft, M1A1

(÷ given d.e. by x^4) $\frac{2}{x^3}\frac{d^2x}{dt^2} - \frac{6}{x^4}\left(\frac{dx}{dt}\right)^2 = \frac{1}{x^2} - 3$

becomes $\left(-\frac{d^2y}{dt^2} = y - 3\right)$ $\frac{d^2y}{dt^2} + y = 3$ **AG** A1 cso5

Second M1 is for attempt at product rule. (be generous)
Final A1 requires all working correct and sufficient "substitution" work

(b) Auxiliary equation: $m^2 + 1 = 0$ and produce
Complementary Function $y = \ldots$ M1
$(y) = A\cos t + B\sin t$ A1cao
Particular integral: $y = 3$ B1
$\therefore$ General solution: $(y) = A\cos t + B\sin t + 3$ A1ft4

Answer can be stated; M1 is implied by correct C.F. stated
(allow θ for t)
A1 f.t. for candidates CF + PI
Allow $m^2 + m = 0$ and $m^2 - 1 = 0$ for M1. Marks for (b) can be gained in (c)

(c) $\frac{1}{x^2} = A\cos t + B\sin t + 3$

$x = \frac{1}{2}, t = 0 \Rightarrow (4 = A + 3)\ A = 1$ B1

Differentiating (to include $\frac{dx}{dt}$): $-2x^{-3}\frac{dx}{dt} = -A\sin t + B\cos t$ M1

$\frac{dx}{dt} = 0, t = 0 \Rightarrow (0 = 0 + B)$ $B = 0$ M1

$\therefore \frac{1}{x^2} = 3 + \cos t$ so $x = \frac{1}{\sqrt{3 + \cos t}}$ A1 cao4

Second M : complete method to find other constant
(This may involve solving two equations in A and B)

(d) (Max. value of x when $\cos t = -1$) so max $x = \frac{1}{\sqrt{2}}$ or AWRT 0.707 B11

[14]

4. (a) $x = r\cos\theta = 4\sin\theta\cos^3\theta$ M1

$\frac{dx}{d\theta} = 4\cos^4\theta - 12\cos^2\theta\sin^2\theta$ any correct expression M1A1

Solving $\frac{dx}{d\theta} = 0$ $\left[\frac{dx}{d\theta} = 0 \Rightarrow 4\cos^2\theta(\cos^2\theta - 3\sin^2\theta) = 0\right]$ M1

$\sin\theta = \frac{1}{2}$ or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ AG A1 cso

$r = 4\sin\frac{\pi}{6}\cos^2\frac{\pi}{6} = \frac{3}{2}$ AG A1cso6

So many ways x may be expressing e.g.
$2\sin 2\theta\cos^2\theta$, $\sin 2\theta(1 + \cos 2\theta)$, $\sin 2\theta + (1/2)\sin 4\theta$
leading to many results for $\frac{dx}{d\theta}$
Some relevant equations in solving
$[(1 - 4\sin^2\theta) = 0, (4\cos^2\theta - 3) = 0, (1 - 3\tan^2\theta) = 0, \cos 3\theta = 0]$
Showing that $\theta = \frac{\pi}{6}$ satisfies $\frac{dx}{d\theta} = 0$, allow M1 A1
providing $\frac{dx}{d\theta}$ correct
Starting with $x = r\sin\theta$ can gain M0M1M1

(b) $A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} r^2 d\theta = \frac{1}{2}.16\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2\theta\cos^4\theta d\theta$

$8\sin^2\theta\cos^4\theta = 2\cos^2\theta(4\sin^2\theta\cos^2\theta) = 2\cos^2\theta\sin^2 2\theta$ M1

$= (\cos 2\theta + 1)\sin^2 2\theta$ M1

$= \cos 2\theta\sin^2 2\theta + \frac{1 - \cos 4\theta}{2}$ = Answer AG A1 cso3

First M1 for use of double angle formula for sin 2A
Second M1 for use of $\cos 2A = 2\cos^2 A - 1$
Answer given: must be intermediate step, as shown, and
no incorrect work

(c) Area = $\left[\frac{1}{6}\sin^3 2\theta + \frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{\left(\frac{\pi}{6}\right)}^{\left(\frac{\pi}{4}\right)}$ ignore limits M1A1

$= \left(\frac{1}{6}\sin^3\frac{\pi}{2} + \frac{\pi}{8} - \frac{\sin\pi}{8}\right) - \left(\frac{1}{6}\sin^3\frac{\pi}{3} + \frac{\pi}{12} - \frac{\sin\frac{2\pi}{3}}{8}\right)$ (sub. limits) M1

$= \left(\frac{1}{6} + \frac{\pi}{8}\right) - \left(\frac{\sqrt{3}}{16} + \frac{\pi}{12} - \frac{\sqrt{3}}{16}\right) = \frac{1}{6}, \ + \frac{\pi}{24}$ both cao A1, A15

For first M, of the form $a\sin^3 2\theta + \frac{\theta}{2} \pm b\sin 4\theta$ (Allow if two of correct form)

On ePen the order of the As in answer is as written

[14]

5. $1\frac{1}{2}$ and 3 are 'critical values', e.g. used in solution, or both seen as asymptotes. B1

$(x + 1)(x - 3) = 2x - 3 \Rightarrow x(x - 4) = 0$

$x = 4, x = 0$ M1A1, A1

M1: Attempt to find at least one other critical value

$0 < x < 1\frac{1}{2}, 3 < x < 4$ M1A1, A17

M1: An inequality using $1\frac{1}{2}$ or 3

First M mark can be implied by the two correct values, but otherwise a method must be seen. (The method may be graphical, but either (x =) 4 or (x =) 0 needs to be clearly written or used in this case). Ignore 'extra values' which might arise through 'squaring both sides' methods.

$\leq$ appearing: maximum one A mark penalty (final mark).

[7]

6. Integrating factor $e^{\int -\tan x \, dx} = e^{\ln(\cos x)}$ (or $e^{-\ln(\sec x)}$), $= \cos x \left(\text{or } \frac{1}{\sec x}\right)$ — M1, A1

$\left(\cos x \frac{dy}{dx} - y \sin x = 2\sec^2 x\right)$

$y \cos x = \int 2\sec^2 x \, dx$ (or equiv.) $\left(\text{Or: } \frac{d}{dx}(y\cos x) = 2\sec^2 x\right)$ — M1A1(ft)

$y \cos x = 2\tan x\ (+C)$ (or equiv.) — A1

$y = 3$ at $x = 0$: $C = 3$ — M1

$y = \frac{2\tan x + 3}{\cos x}$ (Or equiv. in the form $y = f(x)$) — A17

1st M: Also scored for $e^{\int \tan x \, dx} = e^{-\ln(\cos x)}$ (or $e^{\ln(\sec x)}$), then A0 for $\sec x$.

2nd M: Attempt to use their integrating factor (requires one side of the equation 'correct' for their integrating factor).

2nd A: The follow-through is allowed <u>only</u> in the case where the integrating factor used is $\sec x$ or $-\sec x$. $\left(y \sec x = \int 2\sec^4 x \, dx\right)$

3rd M: Using $y = 3$ at $x = 0$ to find a value for C (dependent on an integration attempt, however poor, on the RHS).

<u>Alternative</u>

1st M: Multiply through the given equation by $\cos x$.

1st A: Achieving $\cos x \frac{dy}{dx} - y \sin x = 2\sec^2 x$. (Allowing the possibility of integrating by inspection).

[7]

7. C.F. $m^2 + 3m + 2 = 0$ $\quad m = -1$ and $m = -2$ — M1

$y = Ae^{-x} + Be^{-2x}$ — A12

P.I. $y = cx^2 + dx + e$ — B1

$\frac{dy}{dx} = 2cx + d, \frac{d^2y}{dx^2} = 2c$ $\quad 2c + 3(2cx + d) + 2(cx^2 + dx + e) \equiv 2x^2 + 6x$ — M1

$2c = 2$ $\quad c = 1$ (One correct value) — A1

$6c + 2d = 6$ $\quad d = 0$

$2c + 3d + 2e = 0$ $\quad e = -1$ (Other two correct values) — A1

General soln: $y = Ae^{-x} + Be^{-2x} + x^2 - 1$ (Their C.F. + their P.I.) — A1ft5

$x = 0$, $y = 1$: $1 = A + B - 1$	$(A + B = 2)$	M1
$\frac{dy}{dx} = -Ae^{-x} - 2Be^{-2x} + 2x$, $x = 0$, $\frac{dy}{dx} = 1$	$1 = -A - 2B$	M1
Solving simultaneously: $A = 5$ and $B = -3$		M1A1
Solution: $y = 5e^{-x} - 3e^{-2x} + x^2 - 1$		A15

1st M: Attempt to solve auxiliary equation.

2nd M: Substitute their $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the D>E> to form an identity in x with unknown constants.

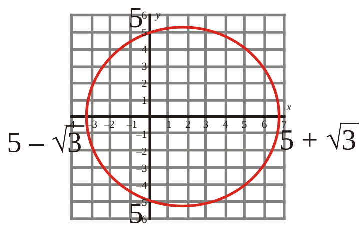
3rd M: Using $y = 1$ at $x = 0$ in their general solution to find an equation in A and B.

4th M: Differentiating their general solution (condone 'slips', but the <u>powers</u> of each term must be correct) and using $\frac{dy}{dx} = 1$ at $x = 0$ to find an equation in A and B.

5th M: Solving simultaneous equations to find both a value of A and a value of B.

[12]

8. (a)



Shape (close curve, approx. symmetrical about the initial line, in all 'quadrants' and 'centred' to the right of the pole/origin). B1

Shape (at least one correct 'intercept' r value... shown on sketch or perhaps seen in a table). B12

(Also allow awrt 3.27 or awrt 6.73).

(b) $y = r\sin\theta = 5\sin\theta + \sqrt{3}\sin\theta\cos\theta$ M1

$\frac{dy}{d\theta} = 5\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta \; (= 5\cos\theta + \sqrt{3}\cos 2\theta)$ A1

$5\cos\theta - \sqrt{3}(1 - \cos^2\theta) + \sqrt{3}\cos^2\theta = 0$ M1

$2\sqrt{3}\cos^2\theta + 5\cos\theta - \sqrt{3} = 0$

$(2\sqrt{3}\cos\theta - 1)(\cos\theta + \sqrt{3}) = 0$ $\cos\theta = \ldots\ (0.288\ldots)$ $\left(\text{Also allow } \pm\arccos\frac{1}{2\sqrt{3}}\right)$ M1

$\theta = 1.28$ and 5.01 (awrt) (Allow ± 1.28 awrt) A1

$r = 5 + \sqrt{3}\left(\frac{1}{2\sqrt{3}}\right) = \frac{11}{2}$ (Allow awrt 5.50) A16

2nd M: Forming a quadratic in $\cos\theta$.

3rd M: Solving a 3 term quadratic to find a value of $\cos\theta$ (even if called θ).

Speacial case: Working with $r\cos\theta$ instead of $r\sin\theta$:
1st M1 for $r\cos\theta = 5\cos\theta + \sqrt{3}\cos^2\theta$
1st A1 for derivative $-5\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$, then no further marks.

(c) $r^2 = 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta$ B1

$\int 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta \, d\theta = \underline{\frac{53\theta}{2} + 10\sqrt{3}\sin\theta} + \underline{3\left(\frac{\sin 2\theta}{4}\right)}$ M1 A1ft A1ft

(ft for integration of $(a + b\cos\theta)$ and $c\cos 2\theta$ respectively)

$\frac{1}{2}\left[25\theta + 10\sqrt{3}\sin\theta + \frac{3\sin 2\theta}{4} + \frac{3\theta}{2}\right]_0^{2\pi} = \ldots\ldots$ M1

$= \frac{1}{2}(50\pi + 3\pi) = \frac{53\pi}{2}$ or equiv. in terms of π. A16

1st M: Attempt to integrate at least one term.

2nd M: Requires use of the $\frac{1}{2}$, correct limits (which could be 0 to 2π, or $-\pi$ to π, or 'double' 0 to π), and subtraction (which could be implied).

[14]

9. (a) $\frac{y_1 - 0.2}{0.1} \approx \left(\frac{dy}{dx}\right)_0 = 0.2 \times e^0 \; (= 0.2)$ M1

$y_1 \approx 0.22$ A12

	(b)	$\left(\frac{dy}{dx}\right)_{0.1} \approx 0.22 \times e^{0.01} \approx 0.2222...$		B1
		$\frac{y_2 - 0.2}{0.2} \approx 0.2222...$		M1
		$y_2 \approx 0.2444$	cao	A13
				[5]
10.	(a)	$(1-x^2)\frac{d^3y}{dx^3} - 2x\frac{d^2y}{dx^2} - x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2\frac{dy}{dx} = 0$		M1
		At $x = 0$, $\frac{d^3y}{dx^3} = -\frac{dy}{dx} = 1$		M1A1cso3
	(b)	$\left(\frac{d^2y}{dx^2}\right)_0 = -4$	Allow anywhere	B1
		$y = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + ...$		
		$= 2 - x - 2x^2, \; + \frac{1}{6}x^3 + ...$		M1A1ft, A1 (dep)4
				[7]
11.	(a)	$z^n = (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$		
		$z^{-n} = (\cos\theta + i\sin\theta)^{-n} = \cos(-n\theta) + i\sin(-n\theta) = \cos n\theta - i\sin n\theta$	both	M1
		Adding $z^n + \frac{1}{z^n} = 2\cos n\theta$ *	cso	A12
	(b)	$\left(z + \frac{1}{z}\right)^6 = z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$		M1
		$= z^6 + z^{-6} + 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) + 20$		M1
		$64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$		M1
		$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$		A1, A1
		$(p = 1, q = 6, r = 15, s = 10)$	A1 any two correct	5

(c) $\int \cos^6 \theta \, d\theta = \left(\frac{1}{32}\right)\int (\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)\,d\theta$

$= \left(\frac{1}{32}\right)\left[\frac{\sin 6\theta}{6} + \frac{6\sin 4\theta}{4} + \frac{15\sin 2\theta}{2} + 10\theta\right]$ M1A1ft

$[.....]_0^{\frac{\pi}{3}} = \frac{1}{32}\left[-\frac{3}{2}\times\frac{\sqrt{3}}{2} + \frac{15}{2}\times\frac{\sqrt{3}}{2} + \frac{10\pi}{3}\right] = \frac{5\pi}{48} + \frac{3\sqrt{3}}{32}$ M1A14

or exact equivalent

[11]

12. (a) Let $z = \lambda + \lambda i$; $w = \frac{\lambda + (\lambda+1)i}{\lambda(1+i)}$ M1

$= \frac{\lambda + (\lambda+1)i}{\lambda(1+i)} \times \frac{1-i}{1-i}$ M1

$u + iv = \frac{(2\lambda+1)+i}{2\lambda}$ A1

$u = 1 + \frac{1}{2\lambda}, v = \frac{1}{2\lambda}$ M1

Eliminating λ gives a line with equation $v = u - 1$ or equivalent A15

(b) Let $z = \lambda - (\lambda + 1)i$: $w = \frac{\lambda - \lambda i}{\lambda - (\lambda+1)i}$ M1

$= \frac{\lambda - \lambda i}{\lambda - (\lambda+1)i} \times \frac{\lambda + (\lambda+1)i}{\lambda + (\lambda+1)i}$ M1

$u + iv = \frac{\lambda(2\lambda+1) + \lambda i}{2\lambda^2 + 2\lambda + 1}$ A1

$u = \frac{\lambda(2\lambda+1)}{2\lambda^2 + 2\lambda + 1}, v = \frac{\lambda}{2\lambda^2 + 2\lambda + 1}$ M1

$\frac{u}{v} = 2\lambda + 1$

$v = \frac{2\lambda}{4\lambda^2 + 4\lambda + 2} = \frac{(2\lambda+1)-1}{(2\lambda+1)^2+1} = \frac{\frac{u}{v}-1}{\left(\frac{u}{v}\right)^2+1}$ M1

Reducing to the circle with equation $u^2 + v^2 - u + v = 0$ * cso M1A17

Alternative 1

Let z = $\lambda - (\lambda + 1)$i: $w = \dfrac{\lambda - \lambda \mathrm{i}}{\lambda - (\lambda+1)\mathrm{i}}$ M1

$= \dfrac{\lambda - \lambda \mathrm{i}}{\lambda - (\lambda+1)\mathrm{i}} \times \dfrac{\lambda + (\lambda+1)\mathrm{i}}{\lambda + (\lambda+1)\mathrm{i}}$ M1

$u + \mathrm{i}v = \dfrac{\lambda(2\lambda+1) + \lambda \mathrm{i}}{2\lambda^2 + 2\lambda + 1}$ A1

$u = \dfrac{\lambda(2\lambda+1)}{2\lambda^2 + 2\lambda + 1}, v = \dfrac{\lambda}{2\lambda^2 + 2\lambda + 1}$ M1

$$u^2 + v^2 - u + v = \left(\frac{\lambda(2\lambda+1)}{2\lambda^2 + 2\lambda + 1}\right)^2 + \left(\frac{\lambda}{2\lambda^2 + 2\lambda + 1}\right)^2 - \frac{\lambda(2\lambda+1)}{2\lambda^2 + 2\lambda + 1} + \frac{\lambda}{2\lambda^2 + 2\lambda + 1}$$

$= \dfrac{(4\lambda^4 + 4\lambda^3 + \lambda^2) + \lambda^2 - 2\lambda^2(2\lambda^2 + 2\lambda + 1)}{(2\lambda^2 + 2\lambda + 1)^2}$ M1

= 0* M1A1

Alternative 2

Let z = $\lambda - (\lambda + 1)$i: $u + \mathrm{i}v = \dfrac{\lambda - \lambda \mathrm{i}}{\lambda - (\lambda+1)\mathrm{i}}$ M1

$(u + \mathrm{i}v)(\lambda - (\lambda + 1)\mathrm{i}) = \lambda - \lambda \mathrm{i}$ M1

$u\lambda + v(\lambda + 1) + [v\lambda - u(\lambda + 1)]\mathrm{i} = \lambda - \lambda 1$ A1

Equating real & imaginary parts

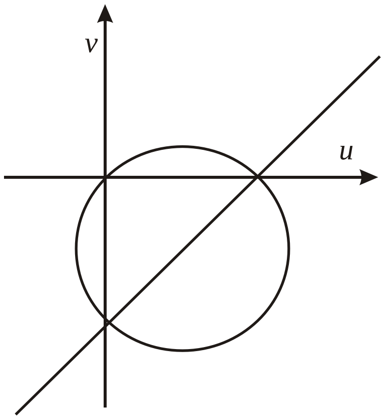
$u\lambda + v(\lambda + 1) = \lambda$ (i) $v\lambda - \lambda u - u = -\lambda$ (ii) M1

From (i) $\lambda = \dfrac{v}{1 - u - v}$ From (ii) $\lambda = \dfrac{u}{1 - u + v}$

$\dfrac{v}{1-u-v} = \dfrac{u}{1-u+v}$ M1

Reducing to the circle with equation $u^2 + v^2 - u + v = 0$ * M1A1

(c)



ft their line	B1ft	
Circle through origin, centre in correct quadrant	B1	
Intersection correctly placed	B13	
		[15]