1. (a) $(r+1)^3 - (r-1)^3 = (r^3 + 3r^2 + 3r + 1) - (r^3 - 3r^2 + 3r - 1)$ M1

$= \underline{6r^2 + 2}$ A1 2

(b) $\sum_{r=1}^{n}(6r^2 + 2) = 2^3 - 0^3$ (attempt to use an identity) M1

$= 3^3 - 1^3$

$4^3 - 2^3$

.

.

.

$(n-1)^3 - (n-3)^3$

$n^3 - (n-2)^3$

$(n+1)^3 - (n-1)^3$ differences (must see) M1

$= (n+1)^3 + n^3 - 1^3$ A1

$6\sum_{r=1}^{n} r^2 = (n+1)^3 + n^3 - 1 - \underline{2n}$ $2n$ or equiv. B1

$= 2n^3 + 3n^2 + n$

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(2n+1)(n+1)$ (*) Sub. Σ2 and ÷ 6 or equiv. c.s.o. M1, A1 6

[8]

2. (a) IF $= e^{\int 1+\frac{3}{x}\,dx}$ M1

$= e^{x+3\ln x}$ A1

$= e^x e^{\ln x^3}$ must see

$= \underline{x^3 e^x}$ A1 3

(b) $x^3 e^x y = \int x^3 e^x \frac{1}{x^2}\,dx$ M1

$= \int xe^x$

$= xe^x - e^x + c$ ∫ by parts M1 A1

$y = \frac{1}{x^2} - \frac{1}{x^3} + \frac{c}{x^3}e^{-x}$ o.e. A1 4

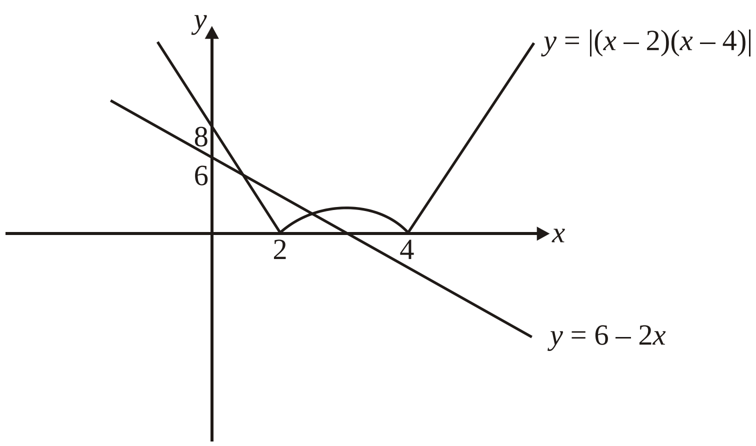
(c) $I = ce^{-1} \therefore c = e^1$ M1

$y = \frac{1}{4} - \frac{1}{8} + \frac{e.e^{-2}}{8}$ M1

$= \underline{\frac{1}{8}(1 + e^{-1})}$

or $= \underline{0.171}$ (0.171 or better) A1 3

3. (a)



Line crosses axes	B1	
Curve shape	B1	
Axes contacts 6, 8, 3	B1	
Cusps at 2 and 4	B1	4

(b) $6 - 2x = (x - 2)(x - 4)$ and $-6 + 2x = (x - 2)(x - 4)$ M1, M1

$x^2 - 4x + 2 = 0$ $\quad$ $x^2 - 8x + 14 = 0$ either M1

$x = \dfrac{4 \pm \sqrt{16 - 8}}{2}$ $\quad$ $x = \dfrac{8 \pm \sqrt{64 - 56}}{2}$

$= 2 - \sqrt{2}$ $\quad$ $= 4 - \sqrt{2}$ A1, A1 5

(c) $2 - \sqrt{2} < x < 4 - \sqrt{2}$ M1, A1 2

[11]

4. (a) $m^2 + 4m + 5 = 0$ M1

$m = \dfrac{-4 \pm \sqrt{-4}}{2}$

$= \underline{-2 \pm i}$ A1

$y = e^{-2x}(A\cos x \pm B\sin x)$ M1

PI $= \lambda \sin 2x + \mu \cos 2x$ PI & attempt diff. *M1*

$y' = 2\lambda \cos 2x - 2\mu \sin 2x$

$y'' = -4\lambda \sin 2x - 4\mu \cos 2x$ A1

$\therefore -4\lambda - 8\mu + 5\lambda = 65$

$-4\mu + 8\lambda + 5\mu = 0$ subst. in eqn. & equate M1

$\lambda - 8\mu = 65$

$8\lambda + \mu = 0$ solving sim. eqn. M1

$64\lambda + 8\mu = 0$

$65\lambda = 65$

$\underline{\lambda = 1}$, $\underline{\mu = -8}$ A1

$\therefore y = e^{-2x}(A\cos x + B\sin x) + \sin 2x - 8 \cos 2x$ on their λ and μ A1ft 9

(b) As $x \to \infty$, $e^{-2x} \to 0$ $\therefore y \to \sin 2x - 8 \cos 2x$ B1ft

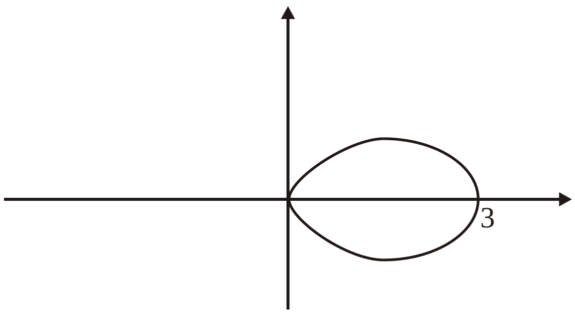
$y \to R \sin(2x + \alpha)$ M1

$R = \sqrt{65}$

$\alpha = \tan^{-1} -8 = -1.446$ or $-82.9°$ A1 3

[12]

5. (a)



Shape + horiz. axis B1

3 B1 2

(b) Area = $\frac{1}{2}\int r^2\,d\theta$			
$= \frac{1}{2}\int 9\cos^2 2\theta\,d\theta$	use of $\frac{1}{2}\int r^2$	M1	
$= \frac{9}{2}\int \frac{\cos 4\theta + 1}{2}\,d\theta$	use of $\cos 4\theta = 2\cos^2 2\theta - 1$	M1	
$= \frac{9}{2}\left[\frac{\sin 4\theta}{8} + \frac{\theta}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$		M1, A1	
$= \frac{9}{2}\left[\frac{\pi}{8} - \frac{\sqrt{3}}{16} - \frac{\pi}{12}\right]$	subst. $\frac{\pi}{4}$ and $\frac{\pi}{6}$	M1	
$= \frac{9}{2}\left[\frac{\pi}{24} - \frac{\sqrt{3}}{16}\right]$ or 0.103		A1	6
(c) $r\sin\theta = 3\sin\theta\cos 2\theta$			
$\frac{d'y'}{d\theta} = 3\cos\theta\cos 2\theta - 6\sin\theta\sin 2\theta$	(diff. $r\sin\theta$)	M1, A1	
$\frac{dy}{d\theta} = 0 \Rightarrow 6\cos^2\theta - 3\cos\theta - 12\sin^2\theta\cos\theta = 0$	use of $\frac{dy}{d\theta} = 0$	M1	
$6\cos^2\theta - 3\cos\theta - 12(1 - \cos^2\theta)\cos\theta = 0$	use double angle formula	M1	
$18\cos^3\theta - 15\cos\theta = 0$	solving	M1	
$\cos\theta = 0$ or $\cos^2\theta = \frac{5}{6}$ or $\tan^2\theta = \frac{1}{5}$ or $\sin^2\theta = \frac{1}{6}$		A1	
$\therefore r = 3(2 \times \frac{5}{6}) - 1$			
$= 2$			
$\therefore r\sin\theta = 2\sqrt{\frac{1}{6}}$	use of $d = 2r\sin\theta$	M1	
$\Rightarrow d = \frac{2\sqrt{6}}{3}$		A1	8
		[16]	

6. Solves $x^2 - 2 = 2x$ by valid method		M1	
Obtains $x = 1 \pm \sqrt{3}$ or equivalent		A1	
	(may only obtain relevant root if graph is used)		
Solves $2 - x^2 = 2x$		M1	
Obtains $x = -1 \pm \sqrt{3}$		A1	
Rejects two of these roots and obtains (or uses graph and obtains)		dM1	
$x > 1 + \sqrt{3}$, $x < -1 + \sqrt{3}$		A1, A1	7

Special case:

Squares both sides to obtain quadratic in x^2 and solve to obtain $x^2 = 4 \pm 2\sqrt{3}$ — *M1A1*

Obtains $x = 1 \pm \sqrt{3}$ or $x = -1 \pm \sqrt{3}$ — *M1A1*

Last three marks as before. — *dM1A1A1*

[7]

7. (a) Integrating Factor = e^{2x} — B1

$\frac{\mathrm{d}}{\mathrm{d}x}(ye^{2x}) = xe^{2x}$ — M1

$ye^{2x} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,\mathrm{d}x$ — M1

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$ — A1

Min point and passing through (0, 1)

$\therefore\ y = \frac{1}{2}x - \frac{1}{4} + ce^{-2x}$ — A1 — 5

shape

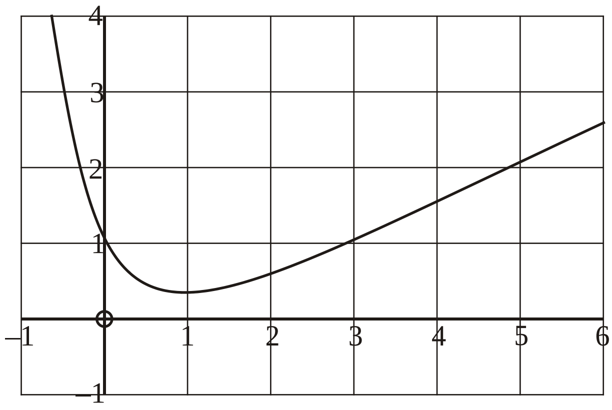
(b) $1 = c - \frac{1}{4} \rightarrow c = \frac{5}{4}$ — M1

$\therefore y = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$ and $\frac{\mathrm{d}}{\mathrm{d}x} = \frac{1}{2} - \frac{5}{2}e^{-2x}$ — M1

When $y' = 0$, $e^{-2x} = \frac{1}{5}$ $\therefore 2x = \ln 5$ — M1

$x = \frac{1}{2}\ln 5$, $y = \frac{1}{4}\ln 5$ at minimum point. — A1 — 4

(c)



B1B1 2

[11]

8. (a) Auxiliary equation: $m^2 + 2m + 2 = 0 \rightarrow m = -1 \pm i$ M1

Complementary Function is $y = e^{-1}\,(A \cos t + B \sin t)$ M1A1

Particular Integral is $y = \lambda e^{-1}$, with $y' = -\lambda e^{-1}$, and $y'' = \lambda e^{-1}$ M1

$\therefore (\lambda - 2\lambda + 2\lambda)e^{-1} = 2e^{-1} \rightarrow \lambda = 2$ A1

$\therefore y = e^{-1}(A \cos t + B \sin t + 2)$ B1 6

(b) Puts $1 = A + 2$ and solves to obtain $A = -1$

$y' = e^{-1}(-A \sin t + B \cos t) - e^{-1}(A \cos t + B \sin t + 2)$ M1 A1ft

Puts $1 = B - A - 2$ and uses value for A to obtain B M1

$B = 2$ A1cso

$\therefore y = e^{-t}(2 \sin t - \cos t + 2)$ A1cso 6

[12]

9. (a) $3a(1 - \cos\theta) = a(1 + \cos\theta)$ M1

$2a = 4a \cos\theta \rightarrow \cos\theta = \dfrac{1}{2} \quad \therefore \theta = \dfrac{\pi}{3} \text{ or } -\dfrac{\pi}{3}$ M1

$r = \dfrac{3a}{2}$ A1A1 4

[Co-ordinates of points are $\left(\frac{3a}{2}, \frac{\pi}{3}\right)$ *and* $\left(\frac{3a}{2}, -\frac{\pi}{3}\right)$ *]*

(b) $AB = 2r\sin\theta = \dfrac{3a\sqrt{3}}{2}$ M1A1 2

Area = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} r^2 \mathrm{d}\theta$

$= \frac{1}{2}\int [a^2(1+\cos\theta)^2 - 9a^2(1-\cos\theta)^2]\mathrm{d}\theta$ M1 M1

$= \frac{a^2}{2}\int [1 + 2\cos\theta + \cos^2\theta - 9(1 - 2\cos\theta + \cos^2\theta)]\mathrm{d}\theta$ A1

$= \frac{a^2}{2}\int [-8 + 20\cos\theta - 8\cos^2\theta)]\mathrm{d}\theta$

$= k[-8\theta + 20\sin\theta \ldots$ B1

$\ldots\ldots - 2\sin 2\theta - 4\theta]$ B1

Uses limits $\frac{\pi}{3}$ *and* $-\frac{\pi}{3}$ *correctly or uses twice smaller area*

and uses limits $\frac{\pi}{3}$ *and 0 correctly.(Need not see 0 substituted)*

$= a^2[-4\pi + 10\sqrt{3} - \sqrt{3}]$ or $= a^2[-4\pi + 9\sqrt{3}]$ or $3.022a^2$ A1 7

(d) $3a\frac{\sqrt{3}}{2} = 4.5 \rightarrow a = \sqrt{3}$ B1

$\therefore$ Area $= 3[9\sqrt{3} - 4\pi]$, $= 9.07$ cm^2 M1, A1 3

[16]

10. (a) $f'(x) = \sec^2 x$ $f''(x) = 2\sec x(\sec x\tan x)$ (or equiv.) M1 A1

$f'''(x) = 2\sec^2 x(\sec^2 x) + 2\tan x(2\sec^2 x\tan x)$ (or equiv.) A1 3

$(2\sec^2 x + 6\sec^2 x\tan^2 x)$

$(2\sec^4 x + 4\sec^2 x\tan^2 x)$, $(6\sec^4 x - 4\sec^2 x)$, $(2 + 8\tan^2 x + 6\tan^4 x)$

(b) $\tan\frac{\pi}{4} = 1$ or $\sec\frac{\pi}{4} = \sqrt{2}$ (1, 2, 4, 16) B1

$\tan x = f\left(\frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)f'\left(\frac{\pi}{4}\right) + \frac{1}{2}\left(x - \frac{\pi}{4}\right)^2 f''\left(\frac{\pi}{4}\right) + \frac{1}{6}\left(x - \frac{\pi}{4}\right)^3 f'''\left(\frac{\pi}{4}\right)$ M1

$= 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3$ A1(cso)3

(Allow equiv. fractions)

(c) $x = \dfrac{3\pi}{10}$, so use $\left(\dfrac{3\pi}{10} - \dfrac{\pi}{4}\right)$ $\left(= \dfrac{\pi}{20}\right)$ M1

$\tan\dfrac{3\pi}{10} \approx 1 + \dfrac{\pi}{10} + \left(2\times\dfrac{\pi^2}{400}\right) + \left(\dfrac{8}{3}\times\dfrac{\pi^3}{8000}\right) = 1 + \dfrac{\pi}{10} + \dfrac{\pi^2}{200} + \dfrac{\pi^3}{3000}$ (*) A1(cso)2

[8]

11. (a) $n = 1$: $\dfrac{d}{dx}(e^x \cos x) = e^x \cos x - e^x \sin x$ M1

(Use of product rule)

$\cos\left(x + \dfrac{\pi}{4}\right) = \cos x \cos\dfrac{\pi}{4} - \sin x \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}(\cos x - \sin x)$ M1

$\dfrac{d}{dx}\left(e^x \cos x\right) = 2^{1/2} e^x \cos\left(x + \dfrac{\pi}{4}\right)$ True for $n = 1$ (c.s.o. + comment) A1

Suppose true for $n = k$.

$\left[\dfrac{d^{k+1}}{dx^{k+1}}\left(e^x \cos x\right)\right] = \dfrac{d}{dx}\left(2^{\frac{1}{2}k} e^x \cos\left(x + \dfrac{k\pi}{4}\right)\right)$ M1

$= 2^{\frac{1}{2}k}\left[e^x \cos\left(x + \dfrac{k\pi}{4}\right) - e^x \sin\left(x + \dfrac{k\pi}{4}\right)\right]$ A1

$= 2^{\frac{1}{2}k} e^x \sqrt{2} \cos\left(x + \dfrac{k\pi}{4} + \dfrac{\pi}{4}\right) = 2^{\frac{1}{2}(k+1)} e^x \cos\left(x + (k+1)\dfrac{\pi}{4}\right)$ M1 A1

$\therefore$ True for $n = k + 1$, so true (by induction) for all n. (≥ 1) A1(cso)8

(b) $1 + \left(\sqrt{2}\cos\dfrac{\pi}{4}\right)x + \dfrac{1}{2}\left(2\cos\dfrac{\pi}{2}\right)x^2 + \dfrac{1}{6}\left(2\sqrt{2}\cos\dfrac{3\pi}{4}\right)x^3 + \dfrac{1}{24}\left(4\cos\pi\right)x^4$ M1

(1) (0) (–2) (–4)

$e^x \cos x = 1 + x - \dfrac{1}{3}x^3 - \dfrac{1}{6}x^4$ (or equiv. fractions) A2(1,0) 3

[11]

12. (a) $\arg z = \dfrac{\pi}{4} \Rightarrow z = \lambda + \lambda i$ (or putting x and y equal at some stage) B1

$w = \dfrac{(\lambda + 1) + \lambda i}{\lambda + (\lambda + 1)i}$, and attempt modulus of numerator or denominator. M1

(Could still be in terms of x and y)

$|(\lambda + 1) + \lambda i| = |\lambda + (\lambda + 1)i| = \sqrt{(\lambda + 1)^2 + \lambda^2}$, $\therefore |w| = 1$ (*) A1, A1cso

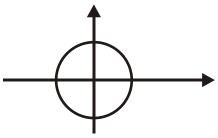
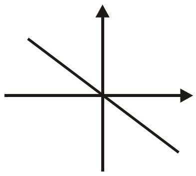
(b) $w = \dfrac{z+1}{z+\text{i}} \Rightarrow zw + w\text{i} = z + 1 \Rightarrow z = \dfrac{1 - w\text{i}}{w-1}$ M1

$|z| = 1 \Rightarrow |1 - w\text{i}| = |w - 1|$ M1 A1

For $w = a + \text{i}b$, $|(1 + b) - a\text{i}| = |(a - 1) + \text{i}b|$ M1

$\sqrt{(1+b)^2 + a^2} = \sqrt{(a-1)^2 + b^2}$ M1

$b = -a$ Image is (line) $y = -x$ A1 6

(c)   B1 B1 2

(d) $z = \text{i}$ marked (P) on z-plane sketch. B1

$z = \text{i} \Rightarrow \dfrac{1+\text{i}}{2\text{i}} = \dfrac{\text{i}-1}{-2} = \dfrac{1}{2} - \dfrac{1}{2}\text{i}$ marked (Q) on w-plane sketch. B1 2

[14]