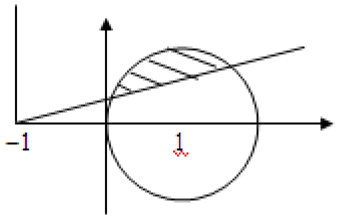
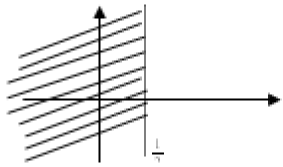
MARK SCHEME

1.

(i) (a) 	Circle One half line correct Second half line [s.c. Allow B1 for two "full" lines in correct position]	M1 A1 B1 B1 (4)
(b) Shading correct region		A1 ft (1)
(ii) (a) Rearrange $w = \frac{z-1}{z}$ to give $z = f(w)$ or $z - 1 = f(w)$		M1
$\left(z = \frac{1}{1-w}, \Rightarrow\right)$ $z - 1 = \frac{w}{1-w}$, or $\lvert z-1\rvert = \lvert z\rvert\lvert w\rvert \Rightarrow \lvert z\rvert\lvert w\rvert = 1$		A1
Completion: $(\lvert z-1\rvert = 1 \rightarrow)$ $\lvert w\rvert = \lvert 1-w\rvert = \lvert w-1\rvert$ *		A1 (3)
(b) 	Correct line shown Correct shading	M1 A1 (2)
		[10]

2.

(a) $(\cos\theta + \mathrm{i}\sin\theta)^5 = \cos 5\theta + \mathrm{i}\sin 5\theta$	M1
$(\cos\theta + \mathrm{i}\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta(\mathrm{i}\sin\theta) + 10\cos^3\theta(\mathrm{i}\sin\theta)^2$	
$+ 10\cos^2\theta(\mathrm{i}\sin\theta)^3 + 5\cos\theta(\mathrm{i}\sin\theta)^4 + (\mathrm{i}\sin\theta)^5$	M1 A1
$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$	M1
$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	A1 cso (6)
(b) $\cos 5\theta = -1$ (or 1, or 0)	M1
$5\theta = (2n \pm 1)180° \Rightarrow \theta = (2n \pm 1)36°$	A1
$x = \cos\theta = -1, -0.309, 0.809$	M1 A1 (4)
	[10]

3.

(a)	$\frac{r^2 - (r-1)^2}{r^2(r-1)^2} = \frac{2r-1}{r^2(r-1)^2}$	M1, A1 (2)
(b)	$\sum_{r=2}^{n} \frac{2r-1}{r^2(r-1)^2} = \sum_{r=2}^{n} \frac{1}{(r-1)^2} - \frac{1}{r^2}$	M1
	$= \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \ldots + \frac{1}{(n-1)^2} - \frac{1}{n^2}$	M1
	$= 1 - \frac{1}{n^2}$ (*)	A1 cso (3)
		(5 marks)

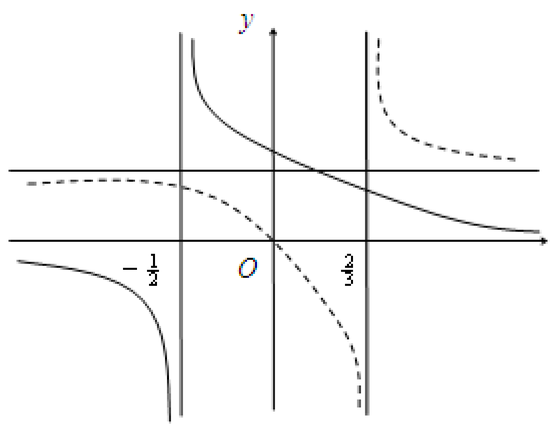
4.

(b) $\frac{dy}{dx} = x^2 - y^2 \Rightarrow \frac{d^2y}{dx^2} = 2x - 2y\frac{dy}{dx}$	M1 A1	
$\Rightarrow \frac{d^3y}{dx^3} = 2 - 2y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2$ allow at this stage	M1 A1	(4)
(c) [$y_{x=0} = 1$, $\left(\frac{dy}{dx}\right)_{x=0} = -1$,] $\left(\frac{d^2y}{dx^2}\right)_{x=0} = 0 - 2(1)(-1) = 2$	B1	
$\left(\frac{d^3y}{dx^3}\right)_{x=0} = 2 - 2(-1)^2 - 2(1)(2) = -4$	B1	
Maclaurin: $y = 1 - x + x^2 - \frac{2}{3}x^3$	M1 A1	(4)
[Alternative (c) $y = 1 + a_1x + a_2x^2 + a_3x^3$		[14]
$\Rightarrow x^2 - (1 + a_1x + a_2x^2 + a_3x^3)^2 = a_1 + 2a_2x + 3a_3x^2$ B1		
Compare coeffs $\Rightarrow a_1 = -1;\ a_2 = 1,\ a_3 = -\frac{2}{3}$. B1; M1 A1]		

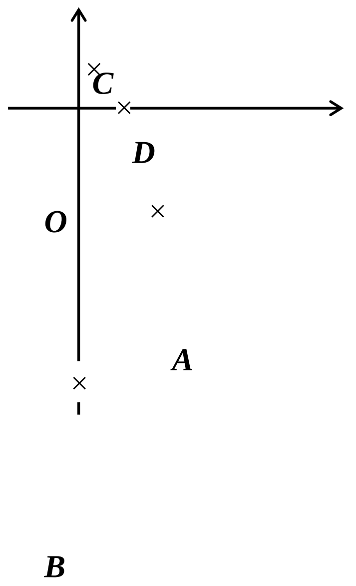
5.

Identifying as critical values $-\frac{1}{2}$, $\frac{2}{3}$	B1, B1
Establishing there are no further critical values	
Obtaining $2x^2 - 2x + 2$ or equivalent	M1
$\Delta = 4 - 16 < 0$	A1
Using exactly two critical values to obtain inequalities	M1
$-\frac{1}{2} < x < \frac{2}{3}$	A1
	(6 marks)
Identifying $x = -\frac{1}{2}$ and $x = \frac{2}{3}$ as vertical asymptotes	B1, B1
Two rectangular hyperbolae oriented correctly with respect to asymptotes in the correct half-planes.	M1
Two correctly drawn curves with no intersections	A1
As above	M1, A1

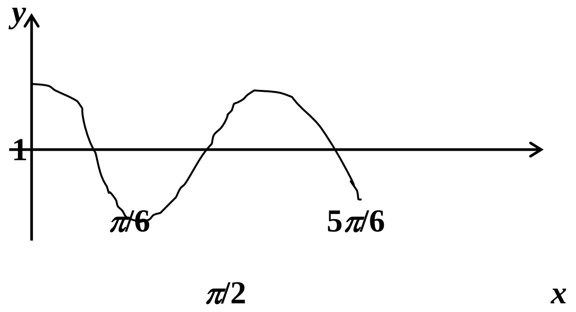
Two correctly drawn curves with no intersections	A1
As above	M1, A1



Question Number	Scheme	Marks	
12. (*a*)	$v + x\frac{dv}{dx}$,= (4 + *v*)(1 + *v*)	M1, M1	
	$x\frac{dv}{dx} = v^2 + 5v + 4 - v$	A1	
	$x\frac{dv}{dx} = (v + 2)^2$ *	A1	(4)
(*b*)	$\int \frac{1}{(v+2)^2}dv = \int \frac{1}{x}dx$	B1, M1	
	$-\frac{1}{2+v} = \ln x + c$ must have + *c*	M1 A1	
	$2 + v = -\frac{1}{\ln x + c}$	M1	
	$v = -\frac{1}{\ln x + c} - 2$	A1	(5)
(*c*)	$y = -2x - \frac{x}{\ln x + c}$	B1	(1)
			(10 marks)

Question Number	Scheme		Marks	
13	$z^2 = (3 - 3\text{i})(3 - 3\text{i}) = -18\text{i}$		M1 A1	(2)
(*b*)	$\dfrac{1}{z} = \dfrac{(3+3\text{i})}{(3-3\text{i})(3+3\text{i})} = \dfrac{3+3\text{i}}{18} = \dfrac{1+\text{i}}{6}$		M1 A1	(2)
(*c*)	$\lvert z \rvert = \sqrt{(9+9)} = \sqrt{18} = 3\sqrt{2}$			
	$\lvert z^2 \rvert = 18$	two correct	M1	
	$\left\lvert \dfrac{1}{z} \right\rvert = \sqrt{\dfrac{1}{18}} = \dfrac{1}{3\sqrt{2}} = \dfrac{\sqrt{2}}{6}$	all three correct	A1	(2)
(*d*)		two correct	B1	
		four correct	B1	(2)
(*e*)	$\dfrac{OB}{OD} = 18, \quad \dfrac{OA}{OC} = \dfrac{3\sqrt{2}}{\sqrt{2}/6} = 18$		M1 A1	
	$\angle AOB = \angle COD = 45 \quad \therefore$ similar		B1	(3)
				(11 marks)

Question Number	Scheme	Marks
`14. (*a*)	$y = \lambda x \cos 3x$	
	$\frac{dy}{dx} = \lambda \cos 3x - 3\lambda x \sin 3x$	M1 A1
	$\frac{d^2y}{dx^2} = -3\lambda \sin 3x - 3\lambda \sin 3x - 9\lambda x \cos 3x$	A1
	$\therefore\ -6\lambda \sin 3x - 9\lambda x \cos 3x + 9\lambda x \cos 3x = -12 \sin 3x$	
	$\lambda = 2$ cso	A1 (4)
(*b*)	$\lambda^2 - 9 = 0$	M1
	$\lambda = (\pm)3\text{i}$	A1
	$\therefore\ y = A \sin 3x + B \cos 3x$ form	M1
	$\therefore\ y = A \sin 3x + B \cos 3x + 2x \cos 3x$	A1 ft on λ's (4)
(*c*)	$y = 1, x = 0 \Rightarrow B = 1$	B1
	$\frac{dy}{dx} = 3A \cos 3x - 3B \sin 3x + 2 \cos 3x - 6x \sin 3x$	M1 A1ft on λ's
	$2 = 3A + 2 \Rightarrow A = 0$	
	$\therefore\ y = \cos 3x + 2x \cos 3x$	A1 (4)

(d)		
	axes	B1
	shape	B1 (2)
		(14 marks)

Question Number	Scheme	Marks
15. (*a*)	$\frac{1}{2}a^2\int 1+\cos^2\theta+2\cos\theta \ \ \mathrm{d}\theta$	**M1 A1correct with limits**
	$= \frac{1}{2}a^2\int 1+\frac{\cos 2\theta+1}{2}+2\cos\theta \ \mathrm{d}\theta$	**M1 A1**
	$= 2 \times \frac{1}{2}a^2\left[\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}+2\sin\theta\right]_0^{\pi}$	**A1**
	$= a^2\left[\frac{3\pi}{2}\right] = \frac{3\pi a^2}{2}$	**A1** **(6)**
(*b*)	$x = a\cos\theta + a\cos^2\theta$ $r\cos\theta$	**M1**
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -a\sin\theta - 2a\cos\theta\sin\theta$	**A1**

	$\frac{dx}{d\theta} = 0 \Rightarrow \cos\theta = -\frac{1}{2}$ **finding** θ	**M1**	
	$\theta = \frac{2\pi}{3}$ **or** $\theta = \frac{4\pi}{3}$		
	$r = \frac{a}{2}$ **or** $r = \frac{a}{2}$ **finding** r	**M1**	
	A: $r = \frac{a}{2}$, $\theta = \frac{2\pi}{3}$		
	B: $r = \frac{a}{2}$, $\theta = \frac{-2\pi}{3}$ **both** A **and** B	**A1**	**(5)**
(*c*)	$x = -\frac{1}{4}a \quad \therefore WX = 2a + \frac{1}{4}a = 2\frac{1}{4}a$	**M1 A1**	
(*d*)	$WXYZ = \frac{27\sqrt{3}a^2}{8}$	**B1 ft**	**(1)**
(*e*)	**Area** $= \frac{27\sqrt{3}}{8} \times 100 - \frac{3\pi \times 100}{2} = 113.3 \text{ cm}^2$	**M1 A1**	**(2)**
			(16 marks)