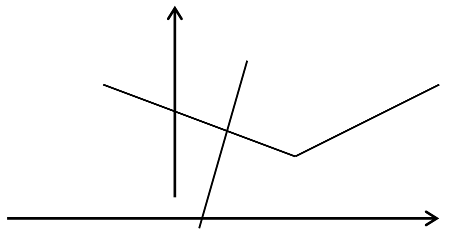
MARK SCHEME

1.



M1 A1

$1 - x = 6x - 1$ M1

$x = {}^2/_7$ A1

$x < {}^2/_7$ A1

2.(a) $I = e^{-\int \frac{1}{t} dt} = e^{-\ln t} = \frac{1}{t}$ M1, A1, A1

$\frac{d}{dt}\left(\frac{v}{t}\right) = \frac{1}{t}$ $\frac{v}{t} = \ln t + c$ M1 A1 $v = t(\ln t + c)$ A1

(b) $v = 3, t = 2, c = {}^3/_2 - \ln 2$ M1, A1 $t = 4$, ${}^v/_4 = \ln 4 + {}^3/_2 - \ln 2 = 8.77$ M1, A1

3.(a) $y' = ½x^2e^x + xe^x$ B1 $y'' = ½x^2e^x + 2xe^x + e^x$ B1

$y'' - 2y' + y = ½x^2e^x + 2xe^x + e^x - x^2e^x - 2xe^x + ½x^2e^x$ M1

$= e^x$ A1

(b) Aux. eqn $m^2 - 2m + 1 = 0$ $m = 1$ M1, A1

C.F. $e^x(A + Bx)$ A1

Gen. soln. $y = e^x(A + Bx) + ½x^2e^x$ A1ft

$x = 0, y = 1$ $A = 1$ B1

$y' = e^x(A + Bx) + Be^x + xe^x + ½x^2e^x$ M1

$y' = 2, x = 0$ $2 = A + B$, $B = 1$ M1, A1

$y = e^x(1 + x + ½x^2)$ A1 ft

4.(a) Circle B1, Diameter 3a B1

Cardiod cusp at O B1 Symmetry and 2a B1

(b) $3a\cos\theta = a(1 + \cos\theta)$, $\cos\theta = \frac{1}{2}$ M1

$\theta = \pm\,{}^{\pi}/_{3}$, $r = {}^{3a}/_{2}$ A1, A1

(c) $A_1 = \frac{1}{2}\int a^2(1+\cos\theta)^2\,\mathrm{d}\theta$ M1

$= \frac{1}{2}a^2\int\left(1 + 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right)\mathrm{d}\theta$ M1, A1

$= \frac{1}{2}a^2\left[\frac{3\theta}{2} + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]$ A1, A1

Evaluate $A_1$ using 0 and ${}^{\pi}/_{3}$ M1

$A_1 = \frac{\pi a^2}{4} + \frac{9\sqrt{3}a^2}{16}$ A1

(d) Area required = ${}^{9}/_{4}\pi a^2 - 2A_1 - 2\times$given M1 ${}^{9}/_{4}\pi a^2$ B1

$= \frac{9\pi a^2}{4} - \frac{\pi a^2}{2} - \frac{9\sqrt{3}a^2}{8} - \frac{3\pi a^2}{4} + \frac{9\sqrt{3}a^2}{8}$ M1

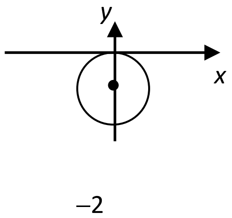
$= \pi a^2$ A1

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** | $(x > 0)$ $2x^2 - 5x > 3$ or $2x^2 - 5x = 3$ | M1 |
| | $(2x + 1)(x - 3)$, critical values $-\frac{1}{2}$ and 3 | A1, A1 |
| | $x > 3$ | A1 ft |
| | $x < 0$ $2x^2 - 5x < 3$ | M1 |
| | Using critical value 0: $-\frac{1}{2} < x < 0$ | M1, A1 ft |
| **Alt.** | $2x - 5 - \frac{3}{x} < 0$ or $(2x - 5)x^2 > 3x$ | M1 |

| | | | |
|---|---|---|---|
| | | $\frac{(2x+1)(x-3)}{x} > 0$ or $x(2x+1)(x-3) > 0$ | M1, A1 |
| | | Critical values –½ and 3, $x > 3$ | A1, A1 ft |
| | | Using critical value 0, $-\frac{1}{2} < x < 0$ | M1, A1 ft |
| | | | **(7 marks)** |
| **6.** | (*a*) | $\frac{dy}{dx} + y\left(\frac{\sin x}{\cos x}\right) = \cos^2 x$ | M1 |
| | | Int. factor $e^{\int \tan x\,dx} = e^{-\ln(\cos x)} = \sec x$ | M1, A1 |
| | | Integrate: $y\sec x = \int \cos x\,dx$ | M1 , A1 |
| | | $y\sec x = \sin x + C$ | A1 |
| | | $(y = \sin x\cos x + C\cos x)$ | **(6)** |
| | (*b*) | When $y = 0$, $\cos x(\sin x + C) = 0$ , $\cos x = 0$ | M1 |
| | | 2 solutions for this $(x = \pi/2, 3\pi/2)$ | A1 **(2)** |
| | (*c*) | $y = 0$ at $x = 0$: $C = 0$ : $y = \sin x\cos x$ | M1 |
| | | $(y = \frac{1}{2}\sin 2x)$ Shape | A1 |
| | | Scales | A1 **(3)** |
| | | | **(11 marks)** |
| **7.** | (*a*) | $2m^2 + 7m + 3 = 0$ $(2m+1)(m+3) = 0$ | |
| | | $m = -\frac{1}{2}, -3$ | |
| | | C.F. is $y = Ae^{-\frac{1}{2}t} + Be^{-3t}$ | M1, A1 |
| | | P.I. $y = at^2 + bt + c$ | B1 |
| | | $y' = 2at + b$, $y'' = 2a$ | |
| | | $2(2a) + 7(2at + b) + 3(at^2 + bt + c) \equiv 3t^2 + 11t$ | M1 |

| | | | | |
|---|---|---|---|---|
| | | $3a = 3$, $a = 1$ $14 + 3b = 11$ , $b = -1$ | A1 | |
| | | $4 - 7 + 3c = 0$, $c = 1$ | M1, A1 | |
| | | General solution: $y = Ae^{-½t} + Be^{-3t} + (t^2 - t + 1)$ | A1 ft | **(8)** |
| | (*b*) | $y' = -½Ae^{-½t} - 3Be^{-3t} + (2t - 1)$ | M1 | |
| | | $t = 0$, $y' = 1$: $1 = -1 - ½A - 3B$ | | |
| | | $t = 0$, $y = 1$: $1 = 1 + A + B$ one of these | M1, A1 | |
| | | Solve: $A + B = 0$, $A + 6B = -4$ | | |
| | | $A = {}^4/_5$, $B = -{}^4/_5$ | M1 | |
| | | $y = (t^2 - t + 1) + \frac{4}{5}(e^{-½t} - e^{-3t})$ | A1 | **(5)** |
| | (*c*) | $t = 1$: $y = \frac{4}{5}(e^{-½} - e^{-3}) + 1$ (= 1.445...) | B1 | **(1)** |
| | | | | **(14 marks)** |

| | | | | |
|---|---|---|---|---|
| **8.** | (*a*) | $y = r\sin\theta = a(3\sin\theta + \sqrt{5}\sin\theta\cos\theta)$ | | |
| | | $\frac{dy}{d\theta} = a(3\cos\theta + \sqrt{5}\cos 2\theta)$ | M1, A1 | |
| | | $2\sqrt{5}\cos^2\theta + 3\cos\theta - \sqrt{5} = 0$ | | |
| | | $\cos\theta = \frac{-3 \pm \sqrt{9 + 40}}{4\sqrt{5}}$ , $\cos\theta = \frac{1}{\sqrt{5}}$ | M1, A1 | |
| | | $\theta = \pm 1.107...$ | A1 ft | |
| | | $r = 4a$ | A1 ft | **(6)** |
| | (*b*) | $2r\sin\theta = 20$ | M1 | |
| | | $8a\sin\theta = 20$ , $a = \frac{20}{8\sin\theta}$ = 2.795... | M1, A1 | **(3)** |

| | | | |
|---|---|---|---|
| | (*c*) | $(3+\sqrt{5}\cos\theta)^2 = 9+6\sqrt{5}\cos\theta+5\cos^2\theta$ | B1 |
| | | Integrate: $9\theta+6\sqrt{5}\sin\theta+5\left(\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right)$ | M1, A1 |
| | | Limits used: $[\ldots]_0^{2\pi} = 18\pi+5\pi$ (or upper limit: $9\pi+\frac{5\pi}{2}$) | A1 |
| | | $½\int_0^{2\pi} r^2 d\theta = a^2(23\pi)$ $\approx 282$ $m^2$ | M1, A1 **(6)** |
| | | | **(15 marks)** |
| **9.** | (*a*)(i) | $\lvert x+(y-2)i\rvert = 2\lvert x+(y+i)\rvert$ | M1 |
| | | $\therefore x^2+(y-2)^2 = 4(x^2+(y+1)^2)$ | |
| | (ii) | so $3x^2+3y^2+12y=0$ any correct from; 3 terms; isw | A1 **(2)** |
| | |  Sketch circle | B1 |
| | | Centre (0,–2) | B1 |
| | | *r* = 2 or touches axis | B1 **(3)** |
| | (*b*) | $w = 3(z-7+11i)$ | B1 |
| | | $= 3z-21+33i$ | B1 **(2)** |
| | | | **(7 marks)** |
| **10.** | (*a*) | $y\frac{d^3y}{dx^3}+\frac{dy}{dx}\frac{d^2y}{dx^2};+2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2};+\frac{dy}{dx}=0$ marks can be awarded in (b) | M1 A1; B1;B1 |
| | | $\frac{d^3y}{dx^3} = \frac{-3\frac{dy}{dx}\frac{d^2y}{dx^2}-\frac{dy}{dx}}{y}$ or sensible correct alternative | B1 **(5)** |
| | (*b*) | When $x=0$ $\frac{d^2y}{dx^2}=-2$, and $\frac{d^3y}{dx^3}=5$ | M1A1, A1 ft |
| | | $\therefore$ $y = 1+x-x^2+\frac{5}{6}x^3\ldots$ | M1, A1 ft **(5)** |

| | | | |
|---|---|---|---|
| (*c*) | Could use for $x = 0.2$ but not for $x = 50$ as | B1 | |
| | approximation is best at values close to $x = 0$ | B1 | **(2)** |
| | | | **(12 marks)** |