Edexcel Maths FP2

Mark Scheme Pack

2009-2013

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Mark Scheme (Results)

Summer 2009

GCE

GCE Mathematics (6668/01)

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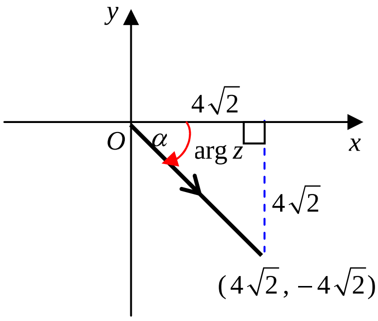
June 2009
6668 Further Pure Mathematics FP2 (new)
Mark Scheme

Question Number	Scheme		Marks
Q1 (a)	$\dfrac{1}{r(r+2)} = \dfrac{1}{2r} - \dfrac{1}{2(r+2)}$	$\dfrac{1}{2r} - \dfrac{1}{2(r+2)}$	B1 **aef**
			(1)
(b)	$\sum_{r=1}^{n} \dfrac{4}{r(r+2)} = \sum_{r=1}^{n}\left(\dfrac{2}{r} - \dfrac{2}{r+2}\right)$		
	$= \left(\dfrac{2}{1} - \dfrac{2}{3}\right) + \left(\dfrac{2}{2} - \dfrac{2}{4}\right) + \ \ldots\ldots$ $\ldots\ldots\ldots\ldots + \left(\dfrac{2}{n-1} - \dfrac{2}{n+1}\right) + \left(\dfrac{2}{n} - \dfrac{2}{n+2}\right)$	List the first two terms and the last two terms	M1
	$= \dfrac{2}{1} + \dfrac{2}{2}; - \dfrac{2}{n+1} - \dfrac{2}{n+2}$	Includes the first two underlined terms and includes the final two underlined terms.	M1
		$\dfrac{2}{1} + \dfrac{2}{2} - \dfrac{2}{n+1} - \dfrac{2}{n+2}$	A1
	$= 3 - \dfrac{2}{n+1} - \dfrac{2}{n+2}$		
	$= \dfrac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{(n+1)(n+2)}$ $= \dfrac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{(n+1)(n+2)}$	Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.	M1
	$= \dfrac{3n^2 + 5n}{(n+1)(n+2)}$		
	$= \dfrac{n(3n+5)}{(n+1)(n+2)}$	Correct Result	A1 **cso AG**
			(5)
			[6]

Question Number	Scheme		Marks
Q2 (a)	$z^3 = 4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}\ ,\ -\pi < \theta \leqslant \pi$		
			
	$r = \sqrt{(4\sqrt{2})^2 + (-4\sqrt{2})^2} = \sqrt{32+32} = \sqrt{64} = 8$ $\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$	A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}\,\mathrm{i}$.	M1
	$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$		
	So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\frac{\pi}{4}}{3}\right) + \mathrm{i}\sin\left(\frac{-\frac{\pi}{4}}{3}\right)\right)$	Taking the cube root of the modulus and dividing the argument by 3.	M1
	$\Rightarrow z = 2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	$2\left(\cos\left(-\frac{\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{\pi}{12}\right)\right)$	A1
	Also, $z^3 = 8\left(\cos\left(\frac{7\pi}{4}\right) + \mathrm{i}\sin\left(\frac{7\pi}{4}\right)\right)$ or $z^3 = 8\left(\cos\left(-\frac{9\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{9\pi}{4}\right)\right)$	Adding or subtracting 2π to the argument for z^3 in order to find other roots.	M1
	$\Rightarrow z = 2\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$	Any one of the final two roots	A1
	and $z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{-3\pi}{4}\right)\right)$	Both of the final two roots.	A1
			[6]
	Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-7\pi}{12}\right)\right)$.		
	Special Case 2: If r is incorrect (and not equal to 8) and candidate states the brackets ( ) correctly then give the first accuracy mark ONLY where this is applicable.		

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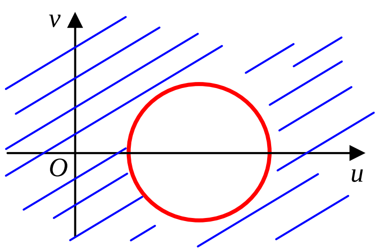
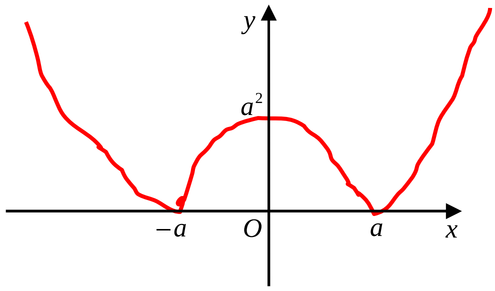
Question Number	Scheme		Marks
Q3	$\sin x\dfrac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = \sin 2x\sin x$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y\cos x}{\sin x} = \dfrac{\sin 2x\sin x}{\sin x}$	An attempt to divide every term in the differential equation by $\sin x$. Can be implied.	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y\cos x}{\sin x} = \sin 2x$		
	Integrating factor $= \mathrm{e}^{\int -\frac{\cos x}{\sin x}\,\mathrm{d}x} = \mathrm{e}^{-\ln\sin x}$	$\mathrm{e}^{\int \pm\frac{\cos x}{\sin x}(\mathrm{d}x)}$ or $\mathrm{e}^{\int \pm \text{their } \mathrm{P}(x)\,(\mathrm{d}x)}$ $\mathrm{e}^{-\ln\sin x}$ or $\mathrm{e}^{\ln\operatorname{cosec} x}$	dM1 A1 aef
	$= \dfrac{1}{\sin x}$	$\dfrac{1}{\sin x}$ or $(\sin x)^{-1}$ or $\operatorname{cosec} x$	A1 aef
	$\left(\dfrac{1}{\sin x}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{y\cos x}{\sin^2 x} = \dfrac{\sin 2x}{\sin x}$		
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sin x}\right) = \sin 2x \times \dfrac{1}{\sin x}$	$\dfrac{\mathrm{d}}{\mathrm{d}x}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F}$	M1
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sin x}\right) = 2\cos x$	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sin x}\right) = 2\cos x$ or $\dfrac{y}{\sin x} = \int 2\cos x\,(\mathrm{d}x)$	A1
	$\dfrac{y}{\sin x} = \int 2\cos x\,\mathrm{d}x$		
	$\dfrac{y}{\sin x} = 2\sin x + K$	A credible attempt to integrate the RHS with/without $+K$	dddM1
	$y = 2\sin^2 x + K\sin x$	$y = 2\sin^2 x + K\sin x$	A1 **cao**
			[8]

Question Number	Scheme		Marks
Q4	$A = \frac{1}{2}\int_0^{2\pi}(a+3\cos\theta)^2\,\mathrm{d}\theta$	Applies $\frac{1}{2}\int_0^{2\pi} r^2(\mathrm{d}\theta)$ with correct limits. Ignore $\mathrm{d}\theta$.	B1
	$(a+3\cos\theta)^2 = a^2 + 6a\cos\theta + 9\cos^2\theta$		
	$= \underline{a^2 + 6a\cos\theta + 9\left(\frac{1+\cos 2\theta}{2}\right)}$	$\cos^2\theta = \frac{\pm 1 \pm \cos 2\theta}{2}$ Correct underlined expression.	M1 A1
	$A = \frac{1}{2}\int_0^{2\pi}\left(a^2 + 6a\cos\theta + \frac{9}{2} + \frac{9}{2}\cos 2\theta\right)\mathrm{d}\theta$		
	$= \left(\frac{1}{2}\right)\left[a^2\theta + 6a\sin\theta + \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]_0^{2\pi}$	Integrated expression with at least 3 out of 4 terms of the form $\pm A\theta \pm B\sin\theta \pm C\theta \pm D\sin 2\theta$. Ignore the $\frac{1}{2}$. Ignore limits. $a^2\theta + 6a\sin\theta +$ correct ft integration. Ignore the $\frac{1}{2}$. Ignore limits.	M1* A1 ft
	$= \frac{1}{2}\left[\left(2\pi a^2 + 0 + 9\pi + 0\right) - (0)\right]$		
	$= \pi a^2 + \frac{9\pi}{2}$	$\pi a^2 + \frac{9\pi}{2}$	A1
	Hence, $\pi a^2 + \frac{9\pi}{2} = \frac{107}{2}\pi$	Integrated expression equal to $\frac{107}{2}\pi$.	dM1*
	$a^2 + \frac{9}{2} = \frac{107}{2}$		
	$a^2 = 49$		
	As $a > 0$, $a = 7$	$a = 7$	A1 cso **[8]**
	Some candidates may achieve $a = 7$ from incorrect working. Such candidates will not get full marks		

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>Q5</td><td>$y = \sec^2 x = (\sec x)^2$</td><td></td><td></td></tr>
<tr><td>(a)</td><td>$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(\sec x)^1(\sec x \tan x) = 2\sec^2 x \tan x$</td><td>Either $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$</td><td>B1 aef</td></tr>
<tr><td></td><td>Apply product rule:
$\left\{\begin{array}{ll} u = 2\sec^2 x & v = \tan x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 4\sec^2 x \tan x & \frac{\mathrm{d}v}{\mathrm{d}x} = \sec^2 x \end{array}\right\}$</td><td></td><td></td></tr>
<tr><td></td><td>$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\sec^2 x \tan^2 x + 2\sec^4 x$</td><td>Two terms added with one of either $A\sec^2 x \tan^2 x$ or $B\sec^4 x$ in the correct form.
Correct differentiation</td><td>M1
A1</td></tr>
<tr><td></td><td>$= 4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$</td><td></td><td></td></tr>
<tr><td></td><td>Hence, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6\sec^4 x - 4\sec^2 x$</td><td>Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result.</td><td>A1 **AG**
(4)</td></tr>
<tr><td>(b)</td><td>$y_{\frac{\pi}{4}} = \left(\sqrt{2}\right)^2 = \underline{2}$, $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{\frac{\pi}{4}} = 2\left(\sqrt{2}\right)^2(1) = \underline{4}$</td><td>Both $y_{\frac{\pi}{4}} = \underline{2}$ and $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{\frac{\pi}{4}} = \underline{4}$</td><td>B1</td></tr>
<tr><td></td><td>$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{\frac{\pi}{4}} = 6\left(\sqrt{2}\right)^4 - 4\left(\sqrt{2}\right)^2 = 24 - 8 = 16$</td><td>Attempts to substitute $x = \frac{\pi}{4}$ into both terms in the expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.</td><td>M1</td></tr>
<tr><td></td><td>$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 24\sec^3 x(\sec x \tan x) - 8\sec x(\sec x \tan x)$</td><td>Two terms differentiated with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct</td><td>M1</td></tr>
<tr><td></td><td>$= 24\sec^4 x \tan x - 8\sec^2 x \tan x$</td><td></td><td></td></tr>
<tr><td></td><td>$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{\frac{\pi}{4}} = 24\left(\sqrt{2}\right)^4(1) - 8\left(\sqrt{2}\right)^2(1) = 96 - 16 = 80$</td><td>$\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_{\frac{\pi}{4}} = \underline{80}$</td><td>B1</td></tr>
<tr><td></td><td>$\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + \frac{16}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{80}{6}\left(x - \frac{\pi}{4}\right)^3 + \ldots$</td><td>Applies a Taylor expansion with at least 3 out of 4 terms ft correctly.
Correct Taylor series expansion.</td><td>M1
A1
(6)</td></tr>
<tr><td></td><td>$\left\{\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + 8\left(x - \frac{\pi}{4}\right)^2 + \frac{40}{3}\left(x - \frac{\pi}{4}\right)^3 + \ldots\right\}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>**[10]**</td></tr>
</table>

Question Number	Scheme		Marks
Q6	$w = \frac{z}{z + \mathrm{i}}, \quad z = -\mathrm{i}$		
(a)	$w(z + \mathrm{i}) = z \Rightarrow wz + \mathrm{i}w = z \Rightarrow \mathrm{i}w = z - wz$ $\Rightarrow \mathrm{i}w = z(1-w) \Rightarrow z = \frac{\mathrm{i}w}{(1-w)}$	Complete method of rearranging to make z the subject. $z = \frac{\mathrm{i}w}{(1-w)}$	M1 A1 **aef**
	$\lvert z \rvert = 3 \Rightarrow \left\lvert \frac{\mathrm{i}w}{1-w} \right\rvert = 3$	Putting $\lvert z$ in terms of their $w\rvert = 3$	dM1
	$\left\{ \begin{array}{l} \lvert \mathrm{i}w \rvert = 3\lvert 1-w \rvert \Rightarrow \lvert w \rvert = 3\lvert w-1 \rvert \Rightarrow \lvert w \rvert^2 = 9\lvert w-1 \rvert^2 \\ \Rightarrow \lvert u + \mathrm{i}v \rvert^2 = 9\lvert u + \mathrm{i}v - 1 \rvert^2 \end{array} \right\}$		
	$\Rightarrow u^2 + v^2 = 9\left[(u-1)^2 + v^2\right]$	Applies $w = u + \mathrm{i}v$, and uses Pythagoras correctly to get an equation in terms of u and v without any i's. Correct equation.	ddM1 A1
	$\left\{ \begin{array}{l} \Rightarrow u^2 + v^2 = 9u^2 - 18u + 9 + 9v^2 \\ \Rightarrow 0 = 8u^2 - 18u + 8v^2 + 9 \end{array} \right\}$		
	$\Rightarrow 0 = u^2 - \frac{9}{4}u + v^2 + \frac{9}{8}$	Simplifies down to $u^2 + v^2 \pm \alpha u \pm \beta v \pm \delta = 0$.	dddM1
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 - \frac{81}{64} + v^2 + \frac{9}{8} = 0$		
	$\Rightarrow \left(u - \frac{9}{8}\right)^2 + v^2 = \frac{9}{64}$		
	{Circle} centre $\left(\frac{9}{8}, 0\right)$, radius $\frac{3}{8}$	One of centre or radius correct. Both centre and radius correct.	A1 A1 (8)
(b)		Circle indicated on the Argand diagram in the correct position in follow through quadrants. Ignore plotted coordinates. Region outside a circle indicated only.	B1ft B1 (2)
			[10]

Question Number	Scheme		Marks
Q7	$y = \lvert x^2 - a^2 \rvert,\ a > 1$		
(a)		Correct Shape. Ignore cusps. Correct coordinates.	B1 B1
			(2)
(b)	$\lvert x^2 - a^2 \rvert = a^2 - x\ ,\ a > 1$		
	$\{\lvert x \rvert > a\},\quad x^2 - a^2 = a^2 - x$	$x^2 - a^2 = a^2 - x$	M1 aef
	$\Rightarrow x^2 + x - 2a^2 = 0$		
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$	Applies the quadratic formula or completes the square in order to find the roots.	M1
	$\Rightarrow x = \dfrac{-1 \pm \sqrt{1 + 8a^2}}{2}$	Both correct "simplified down" solutions.	A1
	$\{\lvert x \rvert < a\},\quad -x^2 + a^2 = a^2 - x$	$-x^2 + a^2 = a^2 - x$ or $x^2 - a^2 = x - a^2$	M1 aef
	$\{\Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0\}$		
	$\Rightarrow x = 0\ ,1$	$x = 0$ $x = 1$	B1 A1
			(6)
(c)	$\lvert x^2 - a^2 \rvert > a^2 - x\ ,\ a > 1$		
	$x < \dfrac{-1 - \sqrt{1 + 8a^2}}{2}\quad \{\text{or}\}\quad x > \dfrac{-1 + \sqrt{1 + 8a^2}}{2}$	x is less than their least value x is greater than their maximum value	B1 ft B1 ft
	$\{\text{or}\}\quad 0 < x < 1$	For $\{\lvert x \rvert < a\}$, Lowest $< x <$ Highest $0 < x < 1$	M1 A1
			(4)
			[12]

Question Number	Scheme		Marks
Q8	$\dfrac{d^2x}{dt^2}+5\dfrac{dx}{dt}+6x = 2e^{-t}$, $x=0, \dfrac{dx}{dt}=2$ at $t=0$.		
(a)	AE, $m^2+5m+6=0 \Rightarrow (m+3)(m+2)=0$ $\Rightarrow m=-3,-2$.		
	So, $x_{CF} = Ae^{-3t}+Be^{-2t}$	$Ae^{m_1 t}+Be^{m_2 t}$, where $m_1 \neq m_2$. $Ae^{-3t}+Be^{-2t}$	M1 A1
	$\left\{ x = ke^{-t} \Rightarrow \dfrac{dx}{dt} = -ke^{-t} \Rightarrow \dfrac{d^2x}{dt^2}=ke^{-t} \right\}$		
	$\Rightarrow ke^{-t}+5(-ke^{-t})+6ke^{-t}=2e^{-t} \Rightarrow 2ke^{-t}=2e^{-t}$ $\Rightarrow k=1$	Substitutes ke^{-t} into the differential equation given in the question. Finds $k=1$.	M1 A1
	$\{$So, $x_{PI}=e^{-t}\}$		
	So, $x = Ae^{-3t}+Be^{-2t}+e^{-t}$	their x_{CF} + their x_{PI}	M1*
	$\dfrac{dx}{dt} = = -3Ae^{-3t}-2Be^{-2t}-e^{-t}$	Finds $\dfrac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}	dM1*
	$t=0, x=0 \Rightarrow \quad 0 = A+B+1$ $t=0, \dfrac{dx}{dt}=2 \Rightarrow \quad 2=-3A-2B-1$	Applies $t=0$, $x=0$ to x and $t=0, \dfrac{dx}{dt}=2$ to $\dfrac{dx}{dt}$ to form simultaneous equations.	ddM1*
	$\left\{\begin{array}{l} 2A+2B=-2 \\ -3A-2B=3 \end{array}\right\}$		
	$\Rightarrow A=-1, B=0$		
	So, $x=-e^{-3t}+e^{-t}$	$x=-e^{-3t}+e^{-t}$	A1 **cao** (8)

Question Number	Scheme		Marks
	$x = -e^{-3t} + e^{-t}$		
(b)	$\frac{dx}{dt} = 3e^{-3t} - e^{-t} = 0$	Differentiates their x to give $\frac{dx}{dt}$ and puts $\frac{dx}{dt}$ equal to 0.	M1
	$3 - e^{2t} = 0$	A credible attempt to solve.	dM1*
	$\Rightarrow t = \frac{1}{2}\ln 3$	$t = \frac{1}{2}\ln 3$ or $t = \ln\sqrt{3}$ or awrt 0.55	A1
	So, $x = -e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3} = -e^{\ln 3^{-\frac{3}{2}}} + e^{\ln 3^{-\frac{1}{2}}}$ $x = -3^{-\frac{3}{2}} + 3^{-\frac{1}{2}}$	Substitutes their t back into x and an attempt to eliminate out the ln's.	ddM1
	$= -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \underline{\frac{2\sqrt{3}}{9}}$	uses exact values to give $\frac{2\sqrt{3}}{9}$	A1 **AG**
	$\frac{d^2x}{dt^2} = -9e^{-3t} + e^{-t}$ At $t = \frac{1}{2}\ln 3$, $\frac{d^2x}{dt^2} = -9e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3}$	Finds $\frac{d^2x}{dt^2}$ and substitutes their t into $\frac{d^2x}{dt^2}$	dM1*
	$= -9(3)^{-\frac{3}{2}} + 3^{-\frac{1}{2}} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = -\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}}$		
	As $\frac{d^2x}{dt^2} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \left\{-\frac{2}{\sqrt{3}}\right\} < 0$ then x is maximum.	$-\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} < 0$ and maximum conclusion.	A1
			(7)
			[15]

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Further Pure Mathematics FP2 6668
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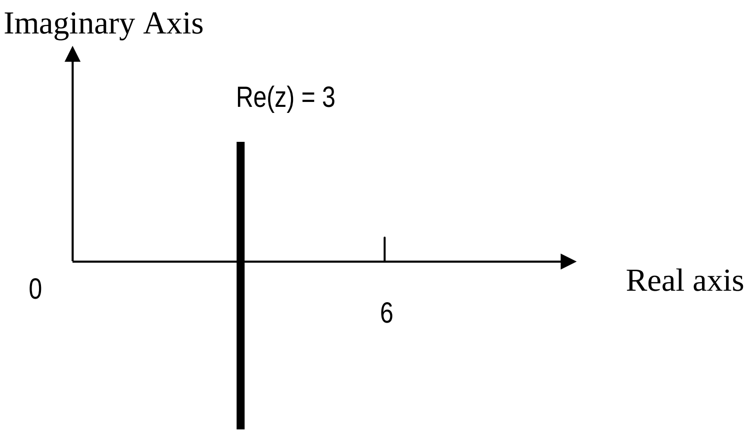
Question Number	Scheme	Marks
1(a)	$\dfrac{1}{3r-1}-\dfrac{1}{3r+2}$	M1 A1 (2)
(b)	$\displaystyle\sum_{r=1}^{n}\frac{3}{(3r-1)(3r+2)}=\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+\ldots\frac{1}{3n-1}-\frac{1}{3n+2}$ $=\dfrac{1}{2}-\dfrac{1}{3n+2}=\dfrac{3n}{2(3n+2)}$ *	M1 A1ft A1 (3)
(c)	Sum = f(1000) – f(99) $\dfrac{3000}{6004}-\dfrac{297}{598}=0.00301$ or 3.01×10^{-3}	M1 A1 (2) **7**

Question Number	Scheme	Marks
2	$\mathrm{f}''(t) = -x - \cos x, \qquad \mathrm{f}''(0) = -1$	B1
	$\mathrm{f}'''(t) = (-1 + \sin x)\dfrac{\mathrm{d}x}{\mathrm{d}t}, \qquad \mathrm{f}'''(0) = -0.5$	M1A1
	$\mathrm{f}(t) = \mathrm{f}(0) + t\mathrm{f}'(0) + \dfrac{t^2}{2}\mathrm{f}''(0) + \dfrac{t^3}{3!}\mathrm{f}'''(0) + \ldots$	
	$\quad = 0.5t - 0.5t^2 - \frac{1}{12}t^3 + \ldots$	M1 A1 **5**

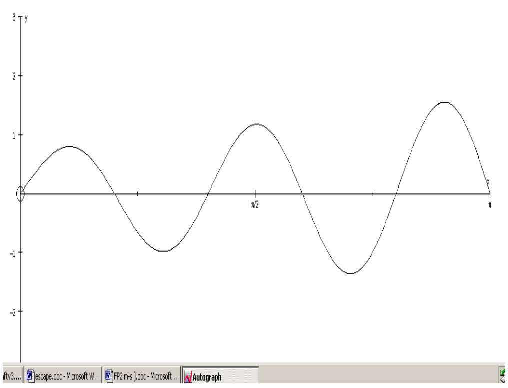
Question Number	Scheme	Marks
3(a)	$(x+4)(x+3)^2 - 2(x+3) = 0$, $(x+3)(x^2+7x+10) = 0$ so $(x+2)(x+3)(x+5) = 0$ or alternative method including calculator	M1
	Finds critical values –2 and -5	A1 A1
	Establishes $x > -2$	A1ft
	Finds and uses critical value –3 to give $-5 < x < -3$	M1A1 (6)
(b)	$x > -2$	B1ft (1) **7**

Question Number	Scheme	Marks
4(a)	Modulus = 16	B1
	Argument = $\arctan(-\sqrt{3}) = \dfrac{2\pi}{3}$	M1A1 (3)
(b)	$z^3 = 16^3(\cos(\frac{2\pi}{3}) + \mathrm{i}\sin(\frac{2\pi}{3}))^3 = 16^3(\cos 2\pi + \mathrm{i}\sin 2\pi)$ =4096 or 16^3	M1 A1 (2)
(c)	$w = 16^{\frac{1}{4}}(\cos(\frac{2\pi}{3}) + \mathrm{i}\sin(\frac{2\pi}{3}))^{\frac{1}{4}} = 2(\cos\left(\frac{\pi}{6}\right) + \mathrm{i}\sin\left(\frac{\pi}{6}\right))\ \left(= \sqrt{3} + \mathrm{i}\right)$	M1 A1ft
	OR $-1+\sqrt{3}\mathrm{i}$ OR $-\sqrt{3}-\mathrm{i}$ OR $1-\sqrt{3}\mathrm{i}$	M1A2(1,0) (5)
		10

Question Number	Scheme	Marks
5(a)	$1.5+\sin 3\theta = 2 \quad \rightarrow \quad \sin 3\theta = 0.5 \quad \therefore 3\theta = \frac{\pi}{6}\ \left(\text{or } \frac{5\pi}{6}\right),$	M1 A1,
	and $\therefore \theta = \frac{\pi}{18}$ or $\frac{5\pi}{18}$	A1 (3)
(b)	Area = $\frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5+\sin 3\theta)^2 \mathrm{d}\theta\right]$, $-\frac{1}{9}\pi \times 2^2$	M1, M1
	$= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25+3\sin 3\theta+\frac{1}{2}(1-\cos 6\theta))\mathrm{d}\theta\right] - \frac{1}{9}\pi \times 2^2$	M1
	$= \frac{1}{2}\left[(2.25\theta-\cos 3\theta+\frac{1}{2}(\theta-\frac{1}{6}\sin 6\theta))\right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \frac{1}{9}\pi \times 2^2$	M1 A1
	$= \frac{13\sqrt{3}}{24} - \frac{5\pi}{36}$	M1 A1 (7)
		10

Question Number	Scheme	Marks
6(a)	 Vertical Straight line Through 3 on real axis	B1 B1 (2)
(b)	These are points where line x = 3 meets the circle centre (3, 4) with radius 5. The complex numbers are 3 + 9i and 3 – i.	M1 A1 A1 (3)
(c)	$\lvert z-6\rvert = \lvert z\rvert \Rightarrow \lvert \frac{30}{w}-6\rvert = \lvert \frac{30}{w}\rvert$ $\therefore \lvert 30-6w\rvert = \lvert 30\rvert \quad \Rightarrow \quad \therefore \lvert 5-w\rvert = \lvert 5\rvert$ This is a circle with Cartesian equation $(u-5)^2+v^2=25$	M1 M1 A1 M1 A1 (5) **10**

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = \frac{dy}{dz} . \frac{dz}{dx}$ and $\frac{dy}{dz} = 2z$ so $\frac{dy}{dx} = 2z.\frac{dz}{dx}$	M1 M1 A1
	Substituting to get $2z.\frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ *	M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2\tan x dx} = e^{2\ln\cos x} = \cos^2 x$	M1 A1
	$\therefore \frac{d}{dx}\left(z\cos^2 x\right) = \cos^2 x \;\; \therefore z\cos^2 x = \int \cos^2 x dx$	M1
	$\therefore z\cos^2 x = \int \frac{1}{2}(\cos 2x + 1)\,dx = \frac{1}{4}\sin 2x + \frac{1}{2}x + c$	M1 A1
	$\therefore z = \frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x$	A1 (6)
(c)	$\therefore y = (\frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x)^2$	B1ft (1)
		12

Question Number	Scheme	Marks
8(a)	Differentiate twice and obtaining $\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \sin 5x + 5\lambda x \cos 5x$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$	M1 A1
	Substitute to give $\lambda = \frac{3}{10}$	M1 A1 (4)
(b)	Complementary function is $y = A\cos 5x + B\sin 5x$ or $P\mathrm{e}^{5ix} + Q\mathrm{e}^{-5ix}$	M1 A1
	So general solution is $y = A\cos 5x + B\sin 5x + \frac{3}{10}x\sin 5x$ or in exponential form	A1ft (3)
(c)	y= 0 when x = 0 means A = 0	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5B\cos 5x + \frac{3}{10}\sin 5x + \frac{3}{2}x\cos 5x$ and at x = 0 $\frac{\mathrm{d}y}{\mathrm{d}x} = 5$ and so 5 = 5A	M1 M1
	So B = 1	A1
	So $y = \sin 5x + \frac{3}{10}x\sin 5x$	A1 (5)
(d)	 "Sinusoidal" through O amplitude becoming larger	B1
	Crosses x axis at $\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$	B1 (2)
		14

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Mark Scheme (Results)

June 2011

GCE Further Pure FP2 (6668) Paper 1

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General Instructions for Marking

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2. The Edexcel Mathematics mark schemes use the following types of marks:

   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

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   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper
   - $\sqsubseteq$ The second mark is dependent on gaining the first mark

June 2011
Further Pure Mathematics FP 26668
Mark Scheme

Question Number	Scheme	Marks
1.	$3x = (x-4)(x+3) \qquad x^2 - 4x - 12 = 0$ $x = -2,\ x = 6$ both Other critical values are $x = -3,\ x = 0$ $-3 < x < -2, \qquad 0 < x < 6$	M1 A1 B1, B1 M1 A1 A1 (7) **7**
	1st M1 for $\pm(x^2 - 4x - 12)$ – '=0' not required. B marks can be awarded for values appearing in solution e.g. on sketch of graph or in final answer. 2nd M1 for attempt at method using graph sketch or +/- If cvs correct but correct inequalities are not strict award A1A0.	

Question Number	Scheme	Marks
2.		
(a)	$\frac{d^3y}{dx^3} = e^x\left(2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx}\right) + e^x\left(2y\frac{dy}{dx} + y^2 + 1\right)$	M1 A1
	$\frac{d^3y}{dx^3} = e^x\left(2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 4y\frac{dy}{dx} + y^2 + 1\right)$ $(k = 4)$	A1
		(3)
(b)	$\left(\frac{d^2y}{dx^2}\right)_0 = e^0(4+1+1) = 6$	B1
	$\left(\frac{d^3y}{dx^3}\right)_0 = e^0(12+8+8+1+1) = 30$	B1
	$y = 1 + 2x + \frac{6x^2}{2} + \frac{30x^3}{6} = 1 + 2x + 3x^2 + 5x^3$	M1 A1ft
		(4) **7**
(a)	1st M1 for evidence of Product Rule 1st A1 for completely correct expression or equivalent 2nd A1 for correct expression or $k = 4$ stated	
(b)	2nd M1 require four terms and denominators of 2 and 6 (might be implied) A1 follow through from their values in the final answer.	

Question Number	Scheme	Marks
3.	$\frac{dy}{dx}+5\frac{y}{x}=\frac{\ln x}{x^2}$ Integrating factor $e^{\int \frac{5}{x}}$	M1
	$e^{\int \frac{5}{x}}=e^{5\ln x}=x^5$	A1
	$\int x^3 \ln x dx=\frac{x^4\ln x}{4}-\int \frac{x^3}{4}dx$	M1 M1 A1
	$=\frac{x^4\ln x}{4}-\frac{x^4}{16}\ (+C)$	A1
	$x^5 y=\frac{x^4\ln x}{4}-\frac{x^4}{16}+C$ $y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{C}{x^5}$	M1 A1
		(8) **8**
	1st M1 for attempt at correct Integrating Factor 1st A1 for simplified IF 2nd M1 for $\frac{\ln x}{x^2}$ times their IF to give their '$x^3 \ln x$' 3rd M1 for attempt at correct Integration by Parts 2nd A1 for both terms correct 3rd A1 constant not required 4th M1 $x^5 y=$ their answer + C	

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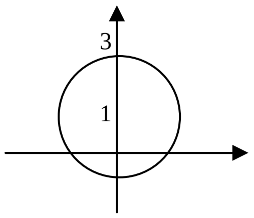
Question Number	Scheme	Marks
4. **(a)**	$(2r+1)^3=(2r)^3+3(2r)^2+3(2r)+1$	M1
	$A=8,\ B=12,\ C=6$	A1 (2)
(b)	$(2r-1)^3=(2r)^3-3(2r)^2+3(2r)-1$	M1
	$(2r+1)^3-(2r-1)^3=24r^2+2$ (*)	A1cso (2)
(c)	$r=1:\quad 3^3-1^3 = 24\times1^2+2$	
	$r=2:\quad 5^3-3^3 = 24\times2^2+2$	
	$\vdots \qquad\qquad \vdots$	M1 A1
	$r=n:\quad (2n+1)^3-(2n-1)^3 = 24\times n^2+2$	
	Summing: $(2n+1)^3-1=24\sum r^2+\left(\sum\ \right)2$	M1
	$\left(\sum 2\right)=2n$	B1
	Proceeding to $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$	A1cso
		(5) **9**
(a) **(b)** **(c)**	1st M1 require coefficients of 1,3,3,1 or equivalent 1st M1 require 1,-3,3,-1 or equivalent 1st M1 for attempt with at least 1,2 and *n* if summing expression incorrect. RHS of display not required at this stage. 1st A1 for 1,2 and n correct. 2nd M1 require cancelling and use of $24r^2+2$ Award B1 for correct *kn* for their approach 2nd A1 is for correct solution only	

Question Number	Scheme	Marks
5. **(a)**	$x^2+(y-1)^2=4$	M1 A1 (2)
(b)	 M1: Sketch of circle A1: Evidence of correct centre and radius	M1 A1 (2)
(c)	$w=\dfrac{(x+\mathrm{i}y)+\mathrm{i}}{3+\mathrm{i}(x+\mathrm{i}y)}=\dfrac{x+\mathrm{i}(y+1)}{(3-y)+\mathrm{i}x}$ $=\dfrac{\left[x+\mathrm{i}(y+1)\right]\left[(3-y)-\mathrm{i}x\right]}{\left[(3-y)+\mathrm{i}x\right]\left[(3-y)-\mathrm{i}x\right]}$ On x-axis, so imaginary part = 0: $(y+1)(3-y)-x^2=0$ $(y+1)(3-y)-x^2=0 \Rightarrow x^2+(y-1)^2=4$, so Q is on C	M1 M1 M1 A1 A1cso (5) **9**
Alt. (c)	Let $w=u+\mathrm{i}v$: $u=\dfrac{z+\mathrm{i}}{3+\mathrm{i}z}$ (since $v=0$) $z=\dfrac{3u-\mathrm{i}}{1-u\mathrm{i}}$ $z-\mathrm{i}=\dfrac{3u-\mathrm{i}-\mathrm{i}-u}{1-u\mathrm{i}}=\dfrac{2(u-\mathrm{i})}{1-u\mathrm{i}}$ $\lvert z-\mathrm{i}\rvert=\dfrac{2\sqrt{u^2+1}}{\sqrt{u^2+1}}=2$, so Q is on C	M1 dM1 M1 A1 A1cso
(a) **(b)** **(c)**	M1 Use of $z=x+\mathrm{i}y$ and find modulus Award A0 if circle doesn't intersect x - axis twice 1st M for subbing $z=x+\mathrm{i}y$ and collecting real and imaginary parts 2nd M for multiply numerator and denominator by their complex conjugate 3rd M for equating imaginary parts of numerator to 0 Award A1 for equation matching part (a), statement not required.	

Question Number	Scheme	Marks
6.	$2+\cos\theta=\frac{5}{2}\Rightarrow\theta=\frac{\pi}{3}$	B1
	$\frac{1}{2}\int(2+\cos\theta)^2 d\theta=\frac{1}{2}\int(4+4\cos\theta+\cos^2\theta)d\theta$	M1
	$=\frac{1}{2}\left[4\theta+4\sin\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right]$	M1 A1
	Substituting limits $\left(\frac{1}{2}\left[\frac{9\pi}{6}+4\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{8}\right]=\frac{1}{2}\left(\frac{3\pi}{2}+\frac{17\sqrt{3}}{8}\right)\right)$	M1
	Area of triangle = $\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{25}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}\left(=\frac{25\sqrt{3}}{32}\right)$	M1 A1
	Area of R = $\frac{3\pi}{4}+\frac{17\sqrt{3}}{16}-\frac{25\sqrt{3}}{32}=\frac{3\pi}{4}+\frac{9\sqrt{3}}{32}$	M1 A1
		(9) **9**
	1st M1 for use of $\frac{1}{2}\int r^2\, d\theta$ and correct attempt to expand 2nd M1 for use of double angle formula - $\sin 2\theta$ required in square brackets 3rd M1 for substituting their limits 4th M1 for use of $\frac{1}{2}$ base x height 5th M1 area of sector – area of triangle Please note there are no follow through marks on accuracy.	

Question Number	Scheme	Marks
7. **(a)**	$\sin 5\theta = \text{Im}(\cos\theta + \text{i}\sin\theta)^5$	B1
	$5\cos^4\theta(\text{i}\sin\theta) + 10\cos^2\theta(\text{i}^3\sin^3\theta) + \text{i}^5\sin^5\theta$	M1
	$= \text{i}(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$	A1
	$\left(\text{Im}(\cos\theta + \text{i}\sin\theta)^5\right) = 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$	M1
	$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta \quad (*)$	A1cso
		(5)
(b)	$16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = 5\left(3\sin\theta - 4\sin^3\theta\right)$	M1
	$16\sin^5\theta - 10\sin\theta = 0$	M1
	$\sin^4\theta = \frac{5}{8} \qquad \theta = 1.095$	A1
	Inclusion of solutions from $\sin\theta = -\sqrt[4]{\frac{5}{8}}$	M1
	Other solutions: $\theta = 2.046,\ 4.237,\ 5.188$	A1
	$\sin\theta = 0 \Rightarrow \theta = 0,\ \theta = \pi\ (3.142)$	B1
		(6) **11**
(a)	Award B if solution considers Imaginary parts and equates to $\sin 5\theta$ 1st M1 for correct attempt at expansion and collection of imaginary parts 2nd M1 for substitution powers of $\cos\theta$	
(b)	1st M for substituting correct expressions 2nd M for attempting to form equation Imply 3rd M if 4.237 or 5.188 seen. Award for their negative root. Ignore 2π but 2nd A0 if other extra solutions given.	

Question Number	Scheme	Marks
8.		
(a)	$m^2+6m+9=0 \qquad m=-3$	M1
	C.F. $\quad x=(A+Bt)e^{-3t}$	A1
	P.I. $\quad x=P\cos 3t+Q\sin 3t$	B1
	$\dot{x}=-3P\sin 3t+3Q\cos 3t$ $\ddot{x}=-9P\cos 3t-9Q\sin 3t$	M1
	$(-9P\cos 3t-9Q\sin 3t)+6(-3P\sin 3t+3Q\cos 3t)+9(P\cos 3t+Q\sin 3t)=\cos$	M1
	$-9P+18Q+9P=1$ and $-9Q-18P+9Q=0$	M1
	$P=0$ and $Q=\dfrac{1}{18}$	A1
	$x=(A+Bt)e^{-3t}+\dfrac{1}{18}\sin 3t$	A1ft
		(8)
(b)	$t=0: \quad x=A=\dfrac{1}{2}$	B1
	$\dot{x}=-3(A+Bt)e^{-3t}+Be^{-3t}+\dfrac{3}{18}\cos 3t$	M1
	$t=0: \quad \dot{x}=-3A+B+\dfrac{1}{6}=0 \qquad B=\dfrac{4}{3}$	M1 A1
	$x=\left(\dfrac{1}{2}+\dfrac{4t}{3}\right)e^{-3t}+\dfrac{1}{18}\sin 3t$	A1
		(5)
(c)	$t\approx\dfrac{59\pi}{6} \quad (\approx 30.9)$	B1
	$x\approx-\dfrac{1}{18}$	B1ft
		(2) **15**
(a)	1st M1 Form auxiliary equation and correct attempt to solve. Can be implied from correct exponential. 2nd M1 for attempt to differentiate PI twice 3rd M1 for substituting their expression into differential equation 4th M1 for substitution of both boundary values	
(b)	1st M1 for correct attempt to differentiate their answer to part (a) 2nd M1 for substituting boundary value	

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Mark Scheme (Results)

Summer 2012

GCE Further Pure FP2 (6668) Paper 1

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Summer 2012
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Summer 2012
6668 Further Pure 2
FP2 Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- $\sqcap$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use correct formula (with values for a, b and c), leading to x = …

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c,\quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

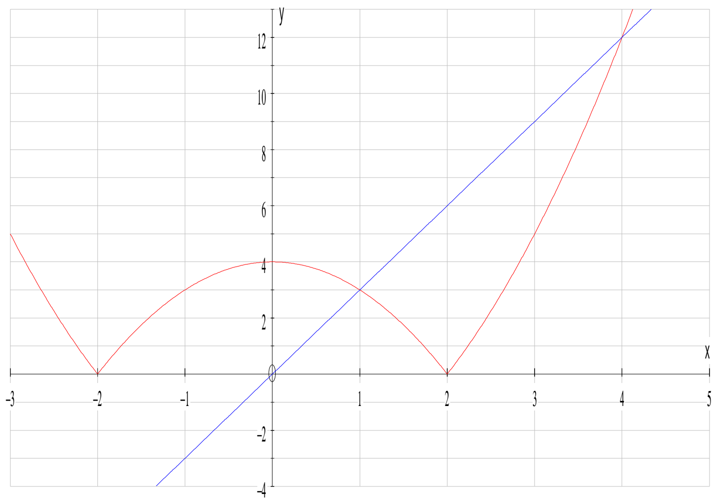
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Summer 2012
6668 Further Pure Mathematics FP2
Mark Scheme

Question Number	Scheme	Marks
1.	$x^2 - 4 = 3x$ and $x^2 - 4 = -3x$, or graphical method, or squaring both sides, leading to $x = ...$	M1
	$(x = -4,\ x = -1)$ $x = 1,\ x = 4$ **seen anywhere**	B1 B1
	Using only 2 critical values to find an inequality	dM1
	$x < 1$ $x > 4$ **both strict, ignore 'and'**	A1
		(5)
		5
	Notes	
		
	1[st] M1 accept $\pm(x^2-4) > 3x$ or $\pm(x^2-4) = 3x$ Require **modulus** of parabola and straight line with positive gradient through origin for graphical method. 1[st] B1 for x=1, 2[nd] B1 for x=4 2[nd] M1 dependent upon first M1 A0 for error in solution of quadratic leading to correct answer.	

Question Number	Scheme	Marks
2.	$y = r\sin\theta = \sin\theta + 2\sin\theta\cos\theta$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta + 2\cos 2\theta$	M1
	$4\cos^2\theta + \cos\theta - 2 = 0$	A1oe
	$\cos\theta = \dfrac{-1 \pm \sqrt{1+32}}{8}$	M1 A1
	$OP = r = 1 + \dfrac{-1+\sqrt{1+32}}{4} = \dfrac{3+\sqrt{33}}{4}$	M1 A1
		(7)
		7
	Notes B1 for $sin\theta + 2\sin\theta cos\theta$ or $sin\theta(1 + 2cos\theta)$ 1st M1 for use of Product Rule or Chain Rule (require 2 or condone ½) 1st A1 equation required 2nd M1 Valid attempt at solving 3 term quadratic (usual rules) to give $cos\theta = \cdots$ 2nd A1 for exact or 3 dp or better (-0.843.....and 0.593....) 3rd M1 for 1+2x 'their $cos\theta$' 3rd A1 for any form A0 if negative solution not discounted.	

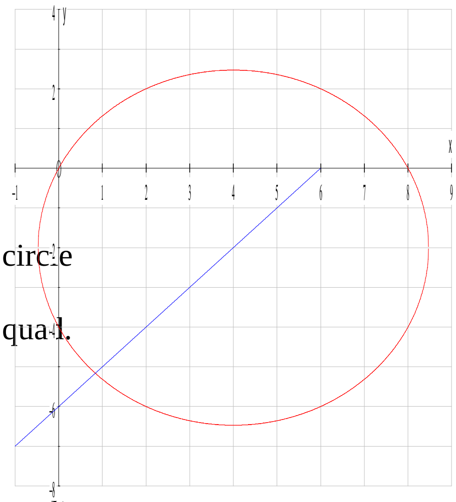
Question Number	Scheme	Marks
3.		
(a)	$r=\sqrt{(-2)^2+(2\sqrt{3})^2}=4$	B1
	$\tan\theta=-\sqrt{3}$ (Also allow M mark for $\tan\theta=\sqrt{3}$)	M1
	M mark can be implied by $\theta=\pm\dfrac{2\pi}{3}$ or $\theta=\pm\dfrac{\pi}{3}$	
	$\theta=\dfrac{2\pi}{3}$	A1
		(3)
(b)	Finding the 4th root of their r: $r=4^{1/4}$ $(=\sqrt{2})$	M1
	For one root, dividing their θ by 4: $\theta=\dfrac{2\pi}{3}\div 4=\dfrac{\pi}{6}$	M1
	For another root, add or subtract a multiple of 2π to their θ **and** divide by 4 in correct order.	M1
	$\sqrt{2}(\cos\theta+\mathrm{i}\sin\theta)$, where $\theta=-\dfrac{5\pi}{6},\ -\dfrac{\pi}{3},\ \dfrac{\pi}{6},\ \dfrac{2\pi}{3}$	A1 A1
		(5)
		8
	Notes	
(a)	M1 Accept $\pm\sqrt{3}$ *or* $\pm\dfrac{1}{\sqrt{3}}$	
	A1 Accept awrt 2.1. A0 if in degrees.	
(b)	2nd M1 for awrt 0.52	
	1st A1 for two correct values	
	2nd A1 for all correct values values in correct form and no more	

Question Number	Scheme	Marks
4.	$m^2+5m+6=0 \qquad m=-2,\ -3$	M1
	C.F. $\quad (x=)Ae^{-2t}+Be^{-3t}$	A1
	P.I. $\quad x=P\cos t+Q\sin t$	B1
	$\dot{x}=-P\,\mathbf{sin}\,t+Q\,\mathbf{cos}\,t$	
	$\ddot{x}=-P\,\mathbf{cos}\,t-Q\,\mathbf{sin}\,t$	M1
	$(-P\cos t-Q\sin t)+5(-P\sin t+Q\cos t)+6(P\cos t+Q\sin t)=2\cos t-\text{si}$	M1
	$-P+5Q+6P=2$ and $-Q-5P+6Q=-1$, and solve for P and Q	M1
	$P=\dfrac{3}{10}$ and $Q=\dfrac{1}{10}$	A1 A1
	$x=Ae^{-2t}+Be^{-3t}+\dfrac{3}{10}\cos t+\dfrac{1}{10}\sin t$	B1 ft
		(9)
		9
	Notes 1st M1 form quadratic and attempt to solve (usual rules) 1st B1 Accept negative signs for coefficients. Coefficients must be different. 2nd M1for differentiating their trig PI twice 3rd M1 for substituting x, $\dot{x}$ and $\ddot{x}$ expressions 4th M1 Form 2 equations in two unknowns and attempt to solve 1st A1 for one correct, 2nd A1 for two correct 2nd B1 for x=their CF + their PI as functions of t Condone use of the wrong variable (e.g. x instead of t) for all marks except final B1.	

Question Number	Scheme	Marks
5.		
(a)	$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=3+2y\frac{dy}{dx}$ (Using differentiation of product or quotient **and** also differentiation of implicit function)	M1
	$x\frac{d^2y}{dx^2}+(1-2y)\frac{dy}{dx}=3$ **ag**	A1 cso
		(2)
(b)	$\left(x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\right)+.....$	B1
	$.....\left[(1-2y)\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)^2\right]=0$	M1 A1
	At x = 1: $\frac{dy}{dx}=4$	B1
	$\frac{d^2y}{dx^2}=7$ $\frac{d^3y}{dx^3}=32$	B1, B1
	$(y=)\mathrm{f}(1)+\mathrm{f}'(1)(x-1)+\frac{\mathrm{f}''(1)(x-1)^2}{2}+\frac{\mathrm{f}'''(1)(x-1)^3}{6}...$	M1
	$y=1+4(x-1)+\frac{7}{2}(x-1)^2+\frac{16}{3}(x-1)^3$ (or equiv.)	A1 ft
		(8)
		10
	Notes	
(a)	Finding second derivative and substituting into given answer acceptable	
(b)	1st M1 for differentiating second term to obtain an expression involving $\frac{d^2y}{dx^2}$ and $\left(\frac{dy}{dx}\right)^2$	
	B1B1B1 for 4,7,32 seen respectively	
	2nd M1 require $\mathrm{f}(1)$ or $1,\mathrm{f}'(1)$ etc and x-1and at least first 3 terms	
	A1 for 4 terms following through their constants	
	Condone f(x)= instead of y=	

Question Number	Scheme	Marks
6.		
(a)	$\frac{1}{r(r+2)} = \frac{1}{2}\left(\frac{1}{r} - \frac{1}{r+2}\right) = \frac{1}{2r}, -\frac{1}{2r+4}$	B1,B1oe
		(2)
(b)	$r=1: \quad \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right)$	M1
	$r=2: \quad \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$	
	$r=3: \quad \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$	
	$r=n-1: \quad \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$	
	$r=n: \quad \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$	A1
	Summing: $\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$	M1 A1
	$= \frac{1}{2}\left(\frac{3(n+1)(n+2) - 2(n+1) - 2(n+2)}{2(n+1)(n+2)}\right) = \frac{n(3n+5)}{4(n+1)(n+2)}$	M1 A1cao
		(6)
(c)	$\sum_{r=1}^{2n} \frac{1}{r(r+2)} = \frac{2n(6n+5)}{4(2n+1)(2n+2)}$	B1oe
	$S_{2n} - S_n \quad = \frac{2n(6n+5)}{4(2n+1)(2n+2)} - \frac{n(3n+5)}{4(n+1)(n+2)}$	M1
	$= \frac{n(6n+5)(n+2) - n(3n+5)(2n+1)}{4(n+1)(n+2)(2n+1)}$	
	$= \frac{n(6n^2+17n+10-6n^2-13n-5)}{4(n+1)(n+2)(2n+1)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$ (*ag*)	A1 cso
		(3)
		11
(a)	1^{st} and 2^{nd} B1 Any form is acceptable	
(b)	1^{st} M1 must include at least 4 out of 5 of (r=)1,2,3 and n-1, n 1^{st} A1 require all terms that do not cancel to be accurate 2^{nd} M1 Summed expression involving all terms that do not cancel 2^{nd} A1 Correct expression 3^{rd} M1 for attempt to find single fraction	
(c)	1^{st} M1 for expression for $S_{2n} - S_n$	

Question Number	Scheme	Marks
7.		
(a)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$ seen	B1
	$3x^3v^2\left(v + x\frac{dv}{dx}\right) = x^3 + v^3x^3 \quad \Rightarrow \quad 3v^2x\frac{dv}{dx} = 1 - 2v^3$ (**ag**)	M1 A1 cso
		(3)
(b)	$\int \frac{3v^2}{1-2v^3}\,dv = \int \frac{1}{x}\,dx$	M1
	$-\frac{1}{2}\ln(1-2v^3) = \ln x \quad (+C)$	M1 A1
	$-\ln(1-2v^3) = \ln x^2 + \ln A$	
	$Ax^2 = \frac{1}{1-2v^3}$	M1
	$1 - \frac{2y^3}{x^3} = \frac{1}{Ax^2}$	
	$y = \sqrt[3]{\frac{x^3 - Bx}{2}}$ or equivalent	dM1 A1cso
		(6)
(c)	Using y = 2 at x = 1: $12\frac{dy}{dx} = 1 + 8$	M1
	At x = 1, $\frac{dy}{dx} = \frac{3}{4}$	A1
		(2)
		11
	Notes	
(a)	M1 for substituting y and $\frac{dy}{dx}$ obtaining an expression in v and x only	
(b)	1st M1 for separating variables 2nd M1 for attempting to integrate both sides 1st A1both sides required or equivalent expressions. (Modulus not required.) 3rd M1 Removing logs, dealing correctly with constant 4th M1 dep on 1st M. Substitute $v = \frac{y}{x}$ and rearranging to $y = f(x)$	
(c)	M1 for finding a numerical value for $\frac{dy}{dx}$ A1 for correct numerical answer oe.	

Question Number	Scheme	Marks
8.		
(a)	$\lvert x+\mathrm{i}y-6\mathrm{i}\rvert = 2\lvert x+\mathrm{i}y-3\rvert$	M1
	$x^2+(y-6)^2 = 4\left[(x-3)^2+y^2\right]$	M1 A1
	$x^2+y^2-12y+36 = 4x^2-24x+36+4y^2$	
	$3x^2+3y^2-24x+12y=0$	
	$(x-4)^2+(y+2)^2=20$	M1
	Centre $(4,-2)$, Radius $\sqrt{20}=2\sqrt{5}=$ awrt 4.47	A1 A1
		(6)
(b)	 Centre in correct quad for their circle	M1
	Passes through O centre in 4th quad.	A1cao
	Half line with positive gradient	B1
	Correct position, clearly through (6, 0)	B1
		(4)
(c)	Equation of line $y=x-6$	B1
	Attempting simultaneous solution of $(x-4)^2+(y+2)^2=20$ and $y=x-6$	M1
	$x=4\pm\sqrt{10}$	A1
	$\left(4-\sqrt{10}\right)+\mathrm{i}\left(-2-\sqrt{10}\right)$	A1cao
		(4)
		14
	Notes	
(a)	1st M Substituting $z=x+iy$ oe	
	2nd M implementing modulus of both sides and squaring. Require Re^2 **plus** Im^2 on both sides & no terms in i. Condone 2 instead of 4 here.	
	3rd M1 for gathering terms and attempting to find centre and / or radius	

Question Number	Scheme	Marks
	2nd A1 for centre, 3rd A1 for radius	
Alt 8(c)	**For geometric approach in this part.**	
	Centre (4,-2) on line, can be implied.	B1
	Use of Pythagoras or trigonometry to find lengths of isosceles triangle	M1
	$x = 4 - \sqrt{10}$	A1
	$\left(4-\sqrt{10}\right)+\mathrm{i}\left(-2-\sqrt{10}\right)$	A1cao

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Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 2 (6668/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c|, \text{ leading to x =}$$

$$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where } |pq|=|c| \text{ and } |mn|=|a|, \text{ leading to x =}$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	$z = x \qquad w = \dfrac{x+2\text{i}}{\text{i}x}$	M1A1
	$w = \dfrac{1}{\text{i}} + \dfrac{2\text{i}}{\text{i}x}$	
	$u + \text{i}v = -\text{i} + \dfrac{2}{x}$	
	$\left(u = \dfrac{2}{x}\right) \qquad v = -1$	M1
	$\therefore w$ is on the line $v+1=0$	A1
		4 Marks

NOTES

M1 for replacing at least one z with x to obtain (ie show an appreciation that $y = 0$)

A1 $w = \dfrac{x+2\text{i}}{\text{i}x}$

M1 for writing w as $u + \text{i}v$ and equating real or imaginary parts to obtain either u or v in terms of x or just a number

A1 for giving the equation of the line $v+1=0$ oe must be in terms of v

Question Number	Scheme	Marks
	Q1 - ALTERNATIVE 1: $w=\dfrac{x+iy+2i}{i(x+iy)}$ Replacing z with $x+iy$ $w=\dfrac{x+iy+2i}{-y+ix}\times\dfrac{-y-ix}{-y-ix}$ $w=\dfrac{(x+i(y+2))(-y-ix)}{y^2+x^2}$ $w=\dfrac{2x-i(x^2+y^2+2y)}{y^2+x^2}$ $w=\dfrac{2x-ix^2}{x^2}=\dfrac{2}{x}-i$ **Using y = 0**. This is where the first M1 may be awarded. A1 if correct even if expression is unsimplified **but denominator must be real** $v=-1$ M1, A1 as in main scheme	 M1A1 M1A1
	Q1 - ALTERNATIVE 2: $z=\dfrac{2i}{iw-1}$ Writing the transformation as a function of w $z=\dfrac{2i}{i(u+iv)-1}$ $z=\dfrac{2i}{(-v-1)+iu}\times\dfrac{(-v-1)-iu}{(-v-1)-iu}$ $z=\dfrac{2u+2i(-v-1)}{(-v-1)^2+u^2}=\dfrac{2u}{(-v-1)^2+u^2}+i\left(\dfrac{2(-v-1)}{(-v-1)^2+u^2}\right)$ $\left(\dfrac{2(-v-1)}{(v-1)^2+u^2}\right)=0$ or simply $-2(v+1)=0$ **Using y = 0** . This is where the first M1 may be awarded. A1 if correct even if expression is unsimplified **but denominator must be real** $v=-1$ M1, A1 as in main mark scheme above	 M1A1 M1A1

Question Number	Scheme	Marks
2.	**NB** Allow the first 5 marks with = instead of inequality	
	$\frac{6x}{3-x} > \frac{1}{x+1}$	
	$6x(3-x)(x+1)^2 - (3-x)^2(x+1) > 0$	M1
	$(3-x)(x+1)(6x^2+6x-3+x) > 0$	
	$(3-x)(x+1)(3x-1)(2x+3) > 0$	M1dep
	Critical values $3, -1$	B1
	and $-\frac{3}{2}, \frac{1}{3}$	A1, A1
	Use critical values to obtain both of $-\frac{3}{2} < x < -1$ $\quad \frac{1}{3} < x < 3$	M1A1cso
		7 Marks

NOTES

M1 for multiplying through by $(x+1)^2(3-x)^2$

OR: for collecting one side of the inequality and attempting to form a single fraction (see alternative in mark scheme)

M1dep for collecting on one side of the inequality and factorising the result of the above (usual rules for factorising the quadratic)
OR: for factorising the numerator - must be a three term quadratic - usual rules for factorising a quadratic (see alternative in mark scheme)
Dependent on the first M mark

B1 for the critical values 3, -1

A1 for either $-\frac{3}{2}$ or $\frac{1}{3}$

A1 for the second of these

NB: the critical values need not be shown explicitly - they may be shown on a sketch or just appear in the ranges or in the working for the ranges.

M1 using **their** 4 critical values to obtain appropriate ranges e.g. use a sketch graph of a quartic, (which must be the correct shape and cross the *x*-axis at the cvs) or a table or number line

A1cso for both of $-\frac{3}{2} < x < -1$, $\quad \frac{1}{3} < x < 3$

Notes for Question 2 Continued

Set notation acceptable i.e. $\left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{3},3\right)$ All brackets must be round; if square brackets appear anywhere then A0.

If both ranges correct, no working is needed for the last 2 marks, but any working shown must be correct.

Purely graphical methods are unacceptable as the question specifies “Use algebra…”.

	Q2 – ALTERNATIVE 1:	
	$\frac{6x}{3-x}-\frac{1}{x+1}>0$	
	$\frac{6x(x+1)-(3-x)}{(3-x)(x+1)}>0$	M1
	$\frac{(3x-1)(2x+3)}{(3-x)(x+1)}>0$	M1dep
	Critical values $3, -1$	B1
	and $-\frac{3}{2}, \frac{1}{3}$	A1A1
	Use critical values to obtain both of $-\frac{3}{2}<x<-1$ $\frac{1}{3}<x<3$	M1A1cso
		7 Marks

Question Number	Scheme	Marks
3(a)	$\dfrac{2}{(r+1)(r+3)}=\dfrac{A}{r+1}+\dfrac{B}{r+3}$	
	$2=A(r+3)+B(r+1)$	
	$\dfrac{2}{(r+1)(r+3)}=\dfrac{1}{r+1}-\dfrac{1}{r+3}$	M1A1
	N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable.	**(2)**
(b)	$\sum\dfrac{2}{(r+1)(r+3)}=\sum\dfrac{1}{r+1}-\dfrac{1}{r+3}$	
	$=\left(\dfrac{1}{2}-\dfrac{1}{4}\right)+\left(\dfrac{1}{3}-\dfrac{1}{5}\right)+\left(\dfrac{1}{4}-\dfrac{1}{6}\right)+\ldots$	
	$+\left(\dfrac{1}{n-1}-\dfrac{1}{n+1}\right)+\left(\dfrac{1}{n}-\dfrac{1}{n+2}\right)+\left(\dfrac{1}{n+1}-\dfrac{1}{n+3}\right)$	M1A1ft
	$=\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{n+2}-\dfrac{1}{n+3}$	
	$=\dfrac{5(n+2)(n+3)-6(n+3)-6(n+2)}{6(n+2)(n+3)}$	M1
	$=\dfrac{5n^2+25n+30-12n-30}{6(n+2)(n+3)}$	
	$=\dfrac{n(5n+13)}{6(n+2)(n+3)}$ *	A1
		(4)
(c)	$\sum_{10}^{100}=\sum_{1}^{100}-\sum_{1}^{9}$	M1
	$=\dfrac{100(500+13)}{6\times102\times103}-\dfrac{9\times58}{6\times11\times12}=\dfrac{1425}{1751}-\dfrac{29}{44}=0.81382\ldots-0.65909\ldots$	
	= 0.1547... = 0.155	A1
		(2)
		8 Marks

Notes for Question 3

Question 3a

M1 for attempting the PFs - any valid method

A1 for correct PFs $\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$

N.B. for M mark you may see no working. Some will just use the "cover up" method to write the answer directly. This is acceptable. Award M1A1 if correct, M0A0 otherwise.

Question 3b

If all work in *r* instead of *n*, penalise last A mark only.

M1 for using **their** PFs to list at least 3 terms at the start and 2 terms at the end so the cancelling can be seen. Must start at $r = 1$ and end at $r = n$

A1ft for correct terms follow through their PFs

M1 for picking out the (4) remaining terms and attempting to form a single fraction (unsimplified numerator with at least 2 terms correct)

A1cso for $\frac{n(5n+13)}{6(n+2)(n+3)}$ * (Check all steps in the working are correct - in particular 3rd line from end in the mark scheme.)

NB: If final answer reached correctly from $\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$ (i.e. working shown from this point onwards) give 4/4 (even without individual terms listed)

Notes for Question 3 Continued
Question 3c M1 for attempting $\sum_{1}^{100} - \sum_{1}^{9}$ using the result from (b) (with numbers substituted) Use of $\sum_{1}^{100} - \sum_{1}^{10}$ scores M0 A1cso for sum = 0.155

Question Number	Scheme	Marks
4(a)	$y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5y=0$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+y\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+5\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	M1M1
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{-5\dfrac{\mathrm{d}y}{\mathrm{d}x}-3\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}}{y}$	A2,1,0 **(4)**
	Q4a – ALTERNATIVE 1:	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{-5y-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{y}=-5-\dfrac{1}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{1}{y^2}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\dfrac{2}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$	M1M1 A2,1,0
(b)	When $x=0$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2$ and $y=2$	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{1}{2}(-10-4)=-7$	M1A1
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{-10-3\times2\times-7}{2}=16$	A1
	$y=2+2x-\dfrac{7}{2(!)}x^2+\dfrac{16}{3!}x^3+\ldots$	M1
	$y=2+2x-\dfrac{7}{2}x^2+\dfrac{8}{3}x^3$	A1 **(5)**
		9 Marks

Question Number	Scheme	Marks
	Alternative: $y = 2 + 2x + ax^2 + bx^3$	M1
	$\left(2+2x+ax^2+bx^3\right)\left(2a+6bx\right)+\left(2+2ax+3bx^2...\right)^2$ $+5\left(2+2x+ax^2+bx^3\right)=0$	M1
	Coeffs x^0 : $4a+4+10=0 \quad a=-\frac{7}{2}$	A1
	Coeffs x: $4a+12b+8a+10=0 \Rightarrow b=\frac{8}{3}$	A1
	$y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$	A1

NOTES

Accept the dash notation in this question

Question 4a

M1 for using the product rule to differentiate $y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

M1 for differentiating $5y$ and using the product rule or chain rule to differentiate $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$

A2,1,0 for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{-5\frac{\mathrm{d}y}{\mathrm{d}x}-3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}}{y}$ Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if more than one error. If there are two sign errors and no other error then give A1A0.

Do NOT deduct if the two $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ terms are shown separately.

Alternative to Q4a

Can be re-arranged first and then differentiated.

M1M1 for differentiating, product and chain rule both needed (or quotient rule as an alternative to product rule)

A2,1,0 for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3-\frac{2}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$ Give A1A1 if fully correct, A1A0 if **one** error and A0A0 if more than one error

Notes for Question 4 Continued
Question 4b M1 for substituting $\frac{\mathrm{d}y}{\mathrm{d}x}=2$ and $y=2$ in **the equation** to obtain a numerical value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ A1 for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-7$ A1 for obtaining the correct value, 16, for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ M1 for using the series $y=\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\frac{x^3}{3!}\mathrm{f}'''(0)+\ldots$ (2! or 2, 3! or 6) (The general series may be shown explicitly or implied by their substitution) A1 for $y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$ oe Must have y = .. and be in ascending powers of **Alternative to Q4b** M1 for setting $y=2+2x+ax^2+bx^3$ M1 for $\left(2+2x+ax^2+bx^3\right)\left(2a+6bx\right)+\left(2+2ax+3bx^2\ldots\right)^2+5\left(2+2x+ax^2+bx^3\right)=0$ A1 for equating constant terms to get $a=-\frac{7}{2}$ A1 for equating coeffs of x^2 to get $b=\frac{8}{3}$ A1 for $y=2+2x-\frac{7}{2}x^2+\frac{8}{3}x^3$

Question Number	Scheme	Marks
5(a)	I.F. $= e^{\int 2\tan x\,dx} = e^{2\ln\sec x} = \sec^2 x$	M1A1
	$y\sec^2 x = \int \sec^2 x \sin 2x\,dx$	M1
	$y\sec^2 x = \int \dfrac{2\sin x\cos x}{\cos^2 x}dx = 2\int \tan x\,dx$	
	$y\sec^2 x = 2\ln\sec x\ (+c)$	M1depA1
	$y = \dfrac{2\ln\sec x + c}{\sec^2 x}$	A1ft
		(6)
(b)	$y = 2,\ \ x = \dfrac{\pi}{3}$	
	$2 = \dfrac{2\ln\sec\left(\dfrac{\pi}{3}\right) + c}{\sec^2\left(\dfrac{\pi}{3}\right)}$	
	$2 = \dfrac{2\ln(2) + c}{4}$	
	$c = 8 - 2\ln 2$	M1A1
	$x = \dfrac{\pi}{6} \quad y = \dfrac{2\ln\sec\left(\dfrac{\pi}{6}\right) + 8 - 2\ln 2}{\sec^2\left(\dfrac{\pi}{6}\right)}$	
	$y = \dfrac{2\ln\dfrac{2}{\sqrt{3}} + 8 - 2\ln 2}{\dfrac{4}{3}}$	M1
	$y = \dfrac{3}{4}\left(8 + 2\ln\dfrac{1}{\sqrt{3}}\right) = 6 + \dfrac{3}{2}\ln\dfrac{1}{\sqrt{3}} = 6 - \dfrac{3}{4}\ln 3$	A1
		(4)
		10 Marks

Question Number	Scheme	Marks
	Alternative: c may not appear explicitly:	
	$y\sec^2\frac{\pi}{6}-2\sec^2\frac{\pi}{3}=2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$	M1A1
	$\frac{4}{3}y-8=2\ln\frac{1}{\sqrt{3}}$	
	$y=\frac{3}{4}\left(8+2\ln\frac{1}{\sqrt{3}}\right)=6+\frac{3}{2}\ln\frac{1}{\sqrt{3}}=6-\frac{3}{4}\ln 3$	M1A1

NOTES

Question 5a

M1 for the $e^{\int 2\tan x\,dx}$ or $e^{\int \tan x\,dx}$ and attempting the integration - $e^{(2)\ln\sec x}$ should be seen if final result is not $\sec^2 x$

A1 for IF $=\sec^2 x$

M1 for multiplying the equation by **their** IF and attempting to integrate the lhs

M1dep for attempting the integration of the rhs $\sin 2x=2\sin x\cos x$ and $\sec x=\frac{1}{\cos x}$ needed. Dependent on the second M mark

A1cao for all integration correct ie $y\sec^2 x=2\ln\sec x\ (+c)$ constant not needed

A1ft for re-writing **their** answer in the form $y=...$ Accept any equivalent form but the constant must be present. eg $y=\frac{\ln\left(A\sec^2 x\right)}{\sec^2 x}$, $y=\cos^2 x\left[\ln\left(\sec^2 x\right)+c\right]$

Notes for Question 5 Continued
Question 5b M1 for using the given values $y = 2,\ x = \frac{\pi}{3}$ in **their** general solution to obtain a value for the constant of integration A1 for eg $c = 8 - 2\ln 2$ or $A = \frac{1}{4}e^8$ (Check the constant is correct for their correct answer for (a)). Answers to 3 significant figures acceptable here and can include $\cos\frac{\pi}{3}$ or $\sec\frac{\pi}{3}$ M1 for using **their** constant and $x = \frac{\pi}{6}$ in **their** general solution and attempting the simplification to the required form. A1cao for $y = 6 - \frac{3}{4}\ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$ **Alternative to 5b** M1 for finding the difference between $y\sec^2\frac{\pi}{6}$ and $2\sec^2\frac{\pi}{3}$ (or equivalent with their general solution) A1 for $y\sec^2\frac{\pi}{6} - 2\sec^2\frac{\pi}{3} = 2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$ M1 for re-arranging to $y = \ldots$ and attempting the simplification to the required form A1cao for $y = 6 - \frac{3}{4}\ln 3$ $\left(\frac{3}{4} \text{ or } 0.75\right)$

Question Number	Scheme	Marks
6(a)	$z^n + z^{-n} = e^{in\theta} + e^{-in\theta}$ $= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ $= 2\cos n\theta$ *	M1A1 **(2)**
(b)	$\left(z+z^{-1}\right)^5 = 32\cos^5\theta$	B1
	$\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1A1
	$32\cos^5\theta = \left(z^5+z^{-5}\right) + 5\left(z^3+z^{-3}\right) + 10\left(z+z^{-1}\right)$	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *	A1 **(5)**
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$	M1
	$16\cos^5\theta = -2\cos\theta$	A1
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0$	
	$8\cos^4\theta + 1 = 0$ no solution	B1
	$\cos\theta = 0$	
	$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1 **(4)** **11 Marks**

Notes for Question 6

Question 6a

M1 for using de Moivre's theorem to show that either $z^n = \cos n\theta + \mathrm{i}\sin n\theta$ or $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$

A1 for completing to the given result $z^n + z^n = 2\cos n\theta$ *

Question 6b

B1 for using the result in (a) to obtain $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$ Need not be shown explicitly.

M1 for attempting to expand $\left(z+z^{-1}\right)^5$ by binomial, Pascal's triangle or multiplying out the brackets. If ${}^nC_r$ is used do not award marks until changed to numbers

A1 for a correct expansion $\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$

M1 for replacing $\left(z^5+z^{-5}\right), \left(z^3+z^{-3}\right), \left(z+z^{-1}\right)$ with $2\cos 5\theta,\ 2\cos 3\theta,\ 2\cos\theta$ and equating their revised expression to their result for $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$

A1cso for $\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *

Question 6c

M1 for attempting re-arrange the equation with one side matching the bracket in the result in (b) Question states "hence", so no other method is allowed.

A1 for using the result in (b) to obtain $16\cos^5\theta = -2\cos\theta$ oe

B1 for stating that there is no solution for $8\cos^4\theta + 1 = 0$ oe eg $8\cos^4\theta + 1 \neq 0$ $8\cos^4\theta + 1 > 0$ or "ignore" but $\cos\theta = \sqrt[4]{-\frac{1}{8}}$ without comment gets B0

A1 for $\theta = \frac{\pi}{2}$ **and** $\frac{3\pi}{2}$ and no more in the range. **Must** be in radians, can be in decimals (1.57..., 4.71... 3 sf or better)

Question Number	Scheme	Marks
7(a)	$y = \lambda t^2 e^{3t}$	
	$\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$	M1A1
	$\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$	A1
	$2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$	M1dep
	$\lambda = 3$	A1cso
		(5)
	NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen.	
(b)	$m^2 - 6m + 9 = 0$	
	$(m-3)^2 = 0$	
	C.F. $(y =)\ (A + Bt)e^{3t}$	M1A1
	G.S. $y = (A + Bt)e^{3t} + 3t^2 e^{3t}$	A1ft
		(3)
(c)	$t = 0 \quad y = 5 \quad \Rightarrow A = 5$	B1
	$\frac{dy}{dt} = Be^{3t} + 3(A + Bt)e^{3t} + 6te^{3t} + 9t^2 e^{3t}$	M1
	$\frac{dy}{dt} = 4 \qquad 4 = B + 15$	M1dep
	$B = -11$	A1
	Solution: $y = (5 - 11t)e^{3t} + 3t^2 e^{3t}$	A1ft
		(5)
		13 Marks

Notes for Question 7

Question 7a

M1 for differentiating $y = \lambda t^2 e^{3t}$ wrt t. Product rule must be used.

A1 for correct differentiation ie $\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$

A1 for a correct second differential $\frac{d^2 y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$

M1dep for substituting their differentials in the equation and obtaining a numerical value for λ
Dependent on the first M mark.

A1cso for $\lambda = 3$ (no incorrect working seen)

NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen. Candidates who attempt the differentiation should be marked on that. If they then go straight to $\lambda = 3$ without showing the substitution, give M1A1 if differentiation correct and M1A0 otherwise, as the solution is incorrect. If $\lambda \neq 3$ then the M mark is only available if the substitution is shown.

Question 7b

M1 for solving the 3 term quadratic auxiliary equation to obtain a value or values for *m* (usual rules for solving a quadratic equation)

A1 for the CF $(y =)\ (A + Bt)e^{3t}$

A1ft for using **their** CF and **their numerical** value of λ in the particular integral to obtain the general solution $y = (A + Bt)e^{3t} + 3t^2 e^{3t}$ Must have *y* = ... and rhs must be a function of *t*.

Question 7c

B1 for deducing that $A = 5$

M1 for differentiating **their** GS to obtain $\frac{dy}{dt} = ...$ The product rule must be used.

M1dep for using $\frac{dy}{dt} = 4$ and **their** value for *A* in **their** $\frac{dy}{dt}$ to obtain an equation for *B* Dependent on the previous M mark (of (c))

A1cao and cso for $B = -11$

A1ft for using **their** numerical values *A* and *B* in **their** GS from (b) to obtain the particular solution. Must have *y* = ... and rhs must be a function of *t*.

Question Number	Scheme	Marks
8 (a)	$A = (4\times)\int_0^{\frac{\pi}{4}} \frac{9}{2}\cos 2\theta \,\mathrm{d}\theta$	M1A1(limits for A mark only)
	$= 18\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$	M1
	$9\left[\sin\frac{\pi}{2} - 0\right] = 9$	A1
		(4)
(b)	$r = 3(\cos 2\theta)^{\frac{1}{2}}$	
	$r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$	M1
	$\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$	M1depA1
	At max/min: $\frac{-3\sin 2\theta\sin\theta}{(\cos 2\theta)^{\frac{1}{2}}} + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta = 0$	M1
	$\sin 2\theta\sin\theta = \cos 2\theta\cos\theta$	
	$2\sin^2\theta\cos\theta = (1-2\sin^2\theta)\cos\theta$	
	$\cos\theta(1-4\sin^2\theta) = 0$	
	$(\cos\theta = 0)\quad \sin^2\theta = \frac{1}{4}$	
	$\sin\theta = \pm\frac{1}{2} \qquad \theta = \pm\frac{\pi}{6}$	M1A1
	$r\sin\frac{\pi}{6} = 3\left(\cos\frac{\pi}{3}\right)^{\frac{1}{2}}\times\frac{1}{2} = \frac{3\sqrt{2}}{4}$	B1
	$\therefore$ length $PS = \frac{3\sqrt{2}}{2}$, (length $PQ = 6$)	

Question Number	Scheme	Marks
	Shaded area = $6\times\frac{3\sqrt{2}}{2}-9, =9\sqrt{2}-9$ oe	M1,A1 **(9)** **13 Marks**

NOTES

Question 8a

M1 for $A=\frac{1}{2}\int r^2\,\mathrm{d}\theta=\frac{1}{2}\int \alpha\cos 2\theta\,\mathrm{d}\theta$ with $\alpha=3$ or 9 4 or 2 and limits not needed for this mark - ignore any shown.

A1 for $A=(4\times)\int_0^{\frac{\pi}{4}}\frac{9}{2}\cos 2\theta\,\mathrm{d}\theta$ Correct limits $\left(0,\frac{\pi}{4}\right)$ with multiple 4 or $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ with multiple 2 needed. 4 or 2 may be omitted here, provided it is used later.

M1 for the integration $\cos 2\theta$ to become $\pm\left(\frac{1}{2}\right)\sin 2\theta$ Give M0 for $\pm 2\sin 2\theta$. Limits and 4 or 2 not needed

A1cso for using the limits and 4 or 2 as appropriate to obtain 9

ALTERNATIVES ON FOLLOWING PAGES

Notes for Question 8 Continued

Question 8b

M1 for $r\sin\theta = 3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$ or $r^2\sin^2\theta = 9\cos 2\theta\sin^2\theta$ 3 or 9 allowed

M1dep for differentiating the rhs of the above wrt θ. Product and chain rule must be used.

A1 for $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta) = \left\{-3\times\frac{1}{2}(\cos 2\theta)^{-\frac{1}{2}}\times 2\sin 2\theta\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta\right\}$ or correct differentiation of $r^2\sin^2\theta = 9\cos 2\theta\sin^2\theta$

M1 for equating their expression for $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\sin\theta)$ to 0

M1dep for solving the resulting equation to $\sin k\theta = \ldots$ or $\cos k\theta = \ldots$ incuding the use of the appropriate trig formulae (must be correct formulae)

A1 for $\sin\theta = \frac{1}{2}$ or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = (\pm)\frac{\pi}{6}$ oe ignore extra answers

B1 for the length of $\frac{1}{2}PS = \frac{3\sqrt{2}}{4}$ (1.0606...) or of PS May not be shown explicitly. Give this mark if the correct area of the rectangle is shown. Length of PQ is not needed for this mark.

M1 for attempting the shaded area by their $PS\times 6-$ **their** answer to (a). There must be evidence of PS being obtained using their θ

A1 for $9\sqrt{2}-9$ oe 3.7279....or awrt 3.73

ALTERNATIVES ON FOLLOWING PAGES

Option 1 – using $r\sin\theta$ with/without manipulation of $\cos 2\theta$ before differentiation

Use of $3(\cos 2\theta)^{\frac{1}{2}}\sin\theta$	First M mark
$3(\cos 2\theta)^{\frac{1}{2}}\cos\theta - 3\left(\frac{1}{2}\right)(\cos 2\theta)^{-\frac{1}{2}}(2)\sin 2\theta\sin\theta = 0$ $3(\cos 2\theta)^{\frac{1}{2}}\cos\theta - 3(\cos 2\theta)^{-\frac{1}{2}}\sin 2\theta\sin\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$3(\cos 2\theta)^{\frac{1}{2}} - 6(\cos 2\theta)^{-\frac{1}{2}}\sin^2\theta = 0$ $\cos 2\theta - 2\sin^2\theta = 0$	Use of $\sin 2\theta = 2\sin\theta\cos\theta$, division by $3\cos\theta$ and multiplication by $(\cos 2\theta)^{\frac{1}{2}}$ simplify the equation but do not provide specific M marks
$(1-2\sin^2\theta) - 2\sin^2\theta = 0$ $4\sin^2\theta = 1$	Use of $\cos 2\theta = 1 - 2\sin^2\theta$ gives next M mark provided a value of $\sin\theta$ or alt is reached with no errors seen
$\sin\theta = \pm\frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ reached with use of $\cos 2\theta = ...$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $3\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}\sin\theta$	First M mark Use of $\cos 2\theta = \cos^2\theta - \sin^2\theta$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(\cos^2\theta - \sin^2\theta\right)^{-\frac{1}{2}}\left(-2\cos\theta\sin\theta - 2\sin\theta\cos\theta\right)\sin\theta$ $+3\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$ $-6\left(\cos^2\theta - \sin^2\theta\right)^{-\frac{1}{2}}\cos\theta\sin^2\theta + 3\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta + 3\left(\cos^2\theta - \sin^2\theta\right) = 0$ $4\sin^2\theta = 1$	Multiplication by $\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}$, division by $3\cos\theta$ and use of $\cos^2\theta = 1 - \sin^2\theta$ simplify the equation but do not provide specific M marks
$\sin\theta = \pm\frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $3\left(2\cos^2\theta-1\right)^{\frac{1}{2}}\sin\theta$	First M mark Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(2\cos^2\theta-1\right)^{-\frac{1}{2}}\left(-4\cos\theta\sin\theta\right)\sin\theta+3\left(2\cos^2\theta-1\right)^{\frac{1}{2}}\cos\theta=0$ $-6\left(2\cos^2\theta-1\right)^{-\frac{1}{2}}\cos\theta\sin^2\theta+3\left(2\cos^2\theta-1\right)^{\frac{1}{2}}\cos\theta=0$	Second (dep) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta+3\left(2\cos^2\theta-1\right)=0$ $4\sin^2\theta=1$ or $4\cos^2\theta=3$	Multiplication by $\left(\cos^2\theta-\sin^2\theta\right)^{\frac{1}{2}}$, division by $3\cos\theta$ and use of $\sin^2\theta=1-\cos^2\theta$ or vice versa simplify the equation but do not provide specific M marks
$\sin\theta=\pm\frac{1}{2}$ or $\cos\theta=\pm\frac{\sqrt{3}}{2}$ $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ or $\cos\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark. Second accuracy mark given here.

Use of $3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\sin\theta$	First M mark Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$3\left(\frac{1}{2}\right)\left(1-2\sin^2\theta\right)^{-\frac{1}{2}}\left(-4\cos\theta\sin\theta\right)\sin\theta + 3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$ $-6\left(1-2\sin^2\theta\right)^{-\frac{1}{2}}\cos\theta\sin^2\theta + 3\left(1-2\sin^2\theta\right)^{\frac{1}{2}}\cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-6\sin^2\theta + 3\left(1-2\cos^2\theta\right) = 0$ $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$	Multiplication by $\left(\cos^2\theta - \sin^2\theta\right)^{\frac{1}{2}}$, division by $3\cos\theta$ and use of $\sin^2\theta = 1-\cos^2\theta$ or vice versa simplify the equation but do not provide specific M marks
$\sin\theta = \pm\frac{1}{2}$ or $\cos\theta = \pm\frac{\sqrt{3}}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ or $\cos\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark. Second A mark given here.

Option 2 – using $r^2 \sin^2 \theta$ with/without manipulation of $\cos 2\theta$ before differentiation

Use of $9\cos 2\theta \sin^2 \theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2 \theta$
$-9(2)\sin 2\theta \sin^2 \theta + 9(2)\cos 2\theta \sin\theta \cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-2\sin^2 \theta + \cos 2\theta = 0$ or $-\sin 2\theta \sin\theta + \cos 2\theta \cos\theta = 0$ leading to $-2\sin^2 \theta + \cos 2\theta = 0$ or $\cos 3\theta = 1$ (compound angle formula)	Division by $9\sin 2\theta$ or $18\sin\theta$ and use of $\sin 2\theta = 2\sin\theta\cos\theta$ followed by division by $\cos\theta$ will simplify the equation but not provide specific M marks
$-2\sin^2 \theta + 1 - 2\sin^2 \theta = 0$ $4\sin^2 \theta = 1$	Use of $\cos 2\theta = 1 - 2\sin^2 \theta$ gives next M mark provided a value of $\sin\theta$ or alt is reached with no errors seen
$\sin\theta = \pm\frac{1}{2}$ or $3\theta = 2\pi$ (from $\cos 3\theta = 1$) $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ or alt reached with use of $\cos 2\theta = ...$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $9\left(\cos^2\theta-\sin^2\theta\right)\sin^2\theta$ Could be expanded out to $9\cos^2\theta\sin^2\theta-9\sin^4\theta$ before differentiation in which case the derivative is immediately given by $-18\cos\theta\sin^3\theta+18\cos^3\theta\sin\theta-36\sin^3\theta\cos\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$ Use of $\cos 2\theta = 2\cos^2\theta-1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$9\left(-2\cos\theta\sin\theta-2\sin\theta\cos\theta\right)\sin^2\theta+9\left(\cos^2\theta-\sin^2\theta\right)2\sin\theta$ $-36\sin^3\theta\cos\theta+18\left(\cos^2\theta-\sin^2\theta\right)\sin\theta\cos\theta=0$ $-36\cos\theta\sin^3\theta+18\cos^3\theta\sin\theta-18\sin^3\theta\cos\theta=0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$18\cos^3\theta\sin\theta-54\sin^3\theta\cos\theta=0$ $\cos^2\theta-3\sin^2\theta=0$ $1-4\sin^2\theta=0$ or $4\cos^2\theta-3=0$	Division by $18\cos\theta\sin\theta$ and use of $\sin^2\theta=1-\cos^2\theta$ or vice versa will simplify the equation but not provide specific M marks
$\sin\theta=\pm\frac{1}{2}$ or $\cos\theta=\pm\frac{\sqrt{3}}{2}$ $\left(\theta=\frac{\pi}{6}\right)$	Value of $\sin\theta$ or $\cos\theta$ reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $9(2\cos^2\theta - 1)\sin^2\theta$ Could be expanded out to $18\cos^2\theta\sin^2\theta - 9\sin^2\theta$ before differentiation in which case the derivative is immediately given by $-36\cos\theta\sin^3\theta + 36\cos^3\theta\sin\theta - 18\sin\theta\cos\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$ Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$9(-4\cos\theta\sin\theta)\sin^2\theta + 9(2\cos^2\theta - 1)2\sin\theta\cos\theta = 0$ $-36\sin^3\theta\cos\theta + 36\cos^3\theta\sin\theta - 18\sin\theta\cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$-2\sin^2\theta + 2\cos^2\theta - 1 = 0$ $2\cos 2\theta = 1$ or $1 - 4\sin^2\theta = 0$ or $4\cos^2\theta - 3 = 0$	Division by $18\cos\theta\sin\theta$ and use of $\sin^2\theta = 1 - \cos^2\theta$ or vice versa will simplify the equation but not provide specific M marks It is also possible to use $\cos^2\theta - \sin^2\theta = \cos 2\theta$ here
$\sin\theta = \pm\frac{1}{2}$ or $\cos\theta = \pm\frac{\sqrt{3}}{2}$ or $\cos 2\theta = \frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ or alt reached with use of $\cos 2\theta = ...$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Use of $9(1-2\sin^2\theta)\sin^2\theta$ Could be expanded out to $9\sin^2\theta - 18\sin^4\theta$ before differentiation in which case the derivative is immediately given by $18\sin\theta\cos\theta - 72\cos\theta\sin^3\theta$	First M mark even if they have a slip on the 9 and use 3 but must be $\sin^2\theta$ Use of $\cos 2\theta = 2\cos^2\theta - 1$ gives 4th M mark provided a value of $\sin\theta$ or alt is reached with no errors seen after the differentiation
$9(-4\cos\theta\sin\theta)\sin^2\theta + 9(1-2\sin^2\theta)2\sin\theta\cos\theta = 0$ $-36\sin^3\theta\cos\theta - 36\sin^3\theta\cos\theta + 18\sin\theta\cos\theta = 0$	Second (dependent) M mark for differentiating using the product rule A1 awarded here for correct derivative and M1 for setting their derivative equal to 0
$1 - 4\sin^2\theta = 0$	Division by $18\cos\theta\sin\theta$ will simplify the equation but not provide specific M marks
$\sin\theta = \pm\frac{1}{2}$ or $\cos\theta = \pm\frac{\sqrt{3}}{2}$ or $\cos 2\theta = \frac{1}{2}$ $\left(\theta = \frac{\pi}{6}\right)$	Value of $\sin\theta$ or alt reached with use of $\cos 2\theta = \ldots$ and no method errors seen (arithmetic slips would be condoned) gives final M mark Second accuracy mark given here.

Using the factor formulae after differentiating $3(\cos 2\theta)^{\frac{1}{2}} \sin\theta$ **:**

M1 awarded for using $3(\cos 2\theta)^{\frac{1}{2}} \sin\theta$

$$3\left(\frac{1}{2}\right)(\cos 2\theta)^{-\frac{1}{2}}(-2\sin 2\theta)\sin\theta + 3(\cos 2\theta)^{\frac{1}{2}}\cos\theta = 0$$

M1A1 awarded for correct differentiation using product and chain rule

M1 for setting derivative equal to zero

Multiplication by $(\cos 2\theta)^{\frac{1}{2}}$ and division by 3 gives

$$\cos 2\theta \cos\theta - \sin 2\theta \sin\theta = 0$$
$$\cos 3\theta = 0$$

dM1 mark can now be awarded for using correct trigonometric formulae to reduce the equation to $\cos k\theta = \ldots$ but the A mark requires $\cos\theta = \ldots$ or $\theta = \dfrac{\pi}{6}$

$$3\theta = \frac{\pi}{2}$$
$$\theta = \frac{\pi}{6}$$

The A1 mark can now be awarded

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Summer 2013

GCE Further Pure Mathematics 2 (6668/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x =

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c,\quad q\neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1. **(a)**	$\frac{2}{(2r+1)(2r+3)} = \frac{A}{2r+1} + \frac{B}{2r+3} =, \quad \frac{1}{2r+1} - \frac{1}{2r+3}$	M1,A1 (2)
(b)	$\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + ... \frac{1}{2n+1} - \frac{1}{2n+3}$	
	$= \frac{1}{3} - \frac{1}{2n+3} = \frac{2n+3-3}{3(2n+3)}$	M1
	$\sum_{1}^{n} \frac{3}{(2r+1)(2r+3)} = \frac{3}{2} \times \frac{2n}{3(2n+3)} = \frac{n}{2n+3}$	M1depA1 (3)
		[5]

Notes for Question 1

(a)

M1 for any valid attempt to obtain the PFs

A1 for $\frac{1}{2r+1} - \frac{1}{2r+3}$

NB With no working shown award M1A1 if the correct PFs are written down, but M0A0 if either one is incorrect

(b)

M1 for using **their** PFs to split each of the terms of the sum or of $\sum \frac{2}{(2r+1)(2r+3)}$ into 2 PFs.

At least 2 terms at the start and 1 at the end needed to show the diagonal cancellation resulting in two remaining terms.

M1dep for simplifying to a single fraction and multiplying it by the appropriate constant

A1cao for $\sum = \frac{n}{2n+3}$

NB: If r is used instead of n (including for the answer), only M marks are available.

Question Number	Scheme	Marks
2 **(a)**	$z = 5\sqrt{3} - 5\text{i} = r(\cos\theta + \text{i}\sin\theta)$	
	$r = \sqrt{(5^2 \times 3 + 5^2)} = 10$	B1 (1)
(b)	$\arg z = \arctan\left(-\frac{5}{5\sqrt{3}}\right) = -\frac{\pi}{6}$ $\left(\text{or } -\frac{\pi}{6} \pm 2n\pi\right)$	M1A1 (2)
(c)	$\left\lvert\frac{w}{z}\right\rvert = \frac{2}{10} = \frac{1}{5}$ or 0.2	B1 (1)
(d)	$\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \left(-\frac{\pi}{6}\right),\ = \frac{5\pi}{12}$ $\left(\text{or } \frac{5\pi}{12} \pm 2n\pi\right)$	M1,A1 (2)
		[6]

Notes for Question 2
(a) B1 for $\lvert z\rvert = 10$ no working needed (b) M1 for $\arg z = \arctan\left(\pm\frac{5}{5\sqrt{3}}\right)$, $\tan(\arg z) = \pm\frac{5}{5\sqrt{3}}$, $\arg z = \arctan\left(\pm\frac{5\sqrt{3}}{5}\right)$ or $\tan(\arg z) = \pm\frac{5\sqrt{3}}{5}$ OR use their $\lvert z\rvert$ with sin or cos used correctly A1 for $= -\frac{\pi}{6}$ $\left(\text{or } -\frac{\pi}{6} \pm 2n\pi\right)$ (must be 4th quadrant) (c) B1 for $\left\lvert\frac{w}{z}\right\rvert = \frac{2}{10}$ or $\frac{1}{5}$ or 0.2 (d) M1 for $\arg\left(\frac{w}{z}\right) = \frac{\pi}{4} - \arg z$ using **their** arg z A1 for $\frac{5\pi}{12}$ $\left(\text{or } \frac{5\pi}{12} \pm 2n\pi\right)$ *Alternative for* (d): Find $\frac{w}{z} = \frac{(\sqrt{6}-\sqrt{2})+(\sqrt{6}+\sqrt{2})\mathrm{i}}{20}$ $\tan\left(\arg\frac{w}{z}\right) = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ M1 from their $\frac{w}{z}$ $\arg\left(\frac{w}{z}\right) = \frac{5\pi}{12}$ A1 cao Work for (c) and (d) may be seen together – give B and A marks only if modulus and argument are clearly identified ie $\frac{1}{5}\left(\cos\frac{5\pi}{12} + \mathrm{i}\sin\frac{5\pi}{12}\right)$ alone scores B0M1A0

Question Number	Scheme	Marks
3	$(x=0)\quad \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sin 0-4\times\dfrac{1}{2}=-2$	B1
	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+4\dfrac{\mathrm{d}y}{\mathrm{d}x}-\cos x\,(=0)$	M1
	$(x=0)\quad \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\cos 0-4\times\dfrac{1}{8}=\dfrac{1}{2}$	A1
	$(y=)\,y_0+x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0+\dfrac{x^2}{2!}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0+\dfrac{x^3}{3!}\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0+....$	M1 (2! or 2 and 3! or 6)
	$(y=)\,\dfrac{1}{2}+x\times\dfrac{1}{8}+\dfrac{x^2}{2}\times(-2)+\dfrac{x^3}{6}\times\dfrac{1}{2}$	
	$y=\dfrac{1}{2}+\dfrac{x}{8}-x^2+\dfrac{x^3}{12}$	A1 cao **[5]**
	Alt:	
	$y=\dfrac{1}{2}+\dfrac{x}{8}+ax^2+bx^3+...$	B1
	$y''=2a+6bx+...$	M1 Diff twice
	$2a+6bx+...=\sin x-\left(\dfrac{1}{2}+\dfrac{x}{8}+ax^2+bx^3...\right)$	A1 Correct differentiation and equation used
	$2a+2=0\quad a=-1$	M1
	$6b+\dfrac{1}{2}=1\quad b=\dfrac{1}{12}$	
	$y=\dfrac{1}{2}+\dfrac{x}{8}-x^2+\dfrac{x^3}{12}$	A1cao

Notes for Question 3

B1 for $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 = -2$ wherever seen

M1 for attempting the differentiation of the given equation. To obtain $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} \pm k\frac{\mathrm{d}y}{\mathrm{d}x} \pm \cos x\,(=0)$ oe

A1 for substituting $x=0$ to obtain $\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0 = \frac{1}{2}$

M1 for using the expansion $\left[y = \mathrm{f}(x)\right] = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2(!)}\mathrm{f}''(0) + \frac{x^3}{3!}\mathrm{f}'''(0)$ with their values for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. Factorial can be omitted in the x^2 term but must be shown explicitly in the x^3 term or implied by further working eg using 6.

A1cao for $y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12}$ (Ignore any higher powers included) Exact decimals allowed. **Must include *y* =….**

Alternative:

B1 for $y = \frac{1}{2} + \frac{x}{8} + ax^2 + bx^3 + ...$

M1 for differentiating this twice to get $y'' = 2a + 6bx + ...$ (may not be completely correct)

A1 for correct differentiation and using the given equation and the expansion of sin x to get $2a + 6bx + ... = \left(x - \frac{x^3}{3} + ...\right) - 4\left(\frac{1}{2} + \frac{x}{8} + ...\right)$

M1 for equating coefficients to obtain a value for a or b

A1 cao for $y = \frac{1}{2} + \frac{x}{8} - x^2 + \frac{x^3}{12}$ (Ignore any higher powers included)

Question Number	Scheme	Marks
4 **(a)**	Assume true for $n=k$: $z^k = r^k(\cos k\theta + \mathrm{i}\sin k\theta)$	
	$n=k+1$: $z^{k+1} = (z^k \times z =) r^k(\cos k\theta + \mathrm{i}\sin k\theta) \times r(\cos\theta + \mathrm{i}\sin\theta)$	M1
	$= r^{k+1}(\cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta))$	M1
	$= r^{k+1}(\cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta)$	M1depA1cso
	$\therefore$ <u>if true for $n=k$,</u> also <u>true for $n=k+1$</u>	
	$k=1$ <u>$z^1 = r^1(\cos\theta + \mathrm{i}\sin\theta)$</u>; <u>True for $n=1$</u> $\therefore$ <u>true for all n</u>	A1cso (5)
	Alternative: See notes for use of $r\mathrm{e}^{\mathrm{i}\theta}$ form	
(b)	$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$	
	$w^5 = 3^5\left(\cos\frac{15\pi}{4} + \mathrm{i}\sin\frac{15\pi}{4}\right)$	M1
	$w^5 = 243\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{i}\right)$ $\left[= \frac{243\sqrt{2}}{2} - \frac{243\sqrt{2}}{2}\mathrm{i} \text{ or}\right]$ oe	A1 (2)
		[7]

Notes for Question 4

(a)

NB: Allow each mark if *n*, *n* + 1 used instead of *k*, *k* + 1

M1 for using the result for $n=k$ to write $z^{k+1}\left(=z^k \times z\right)=r^k\left(\cos k\theta+\mathrm{i}\sin k\theta\right)\times r\left(\cos\theta+\mathrm{i}\sin\theta\right)$

M1 for multiplying out and collecting real and imaginary parts, using $\mathrm{i}^2=-1$
OR using sum of arguments and product of moduli to get $r^{k+1}\left(\cos\left(k\theta+\theta\right)+\mathrm{i}\sin\left(k\theta+\theta\right)\right)$

M1dep for using the addition formulae to obtain single cos and sin terms
OR factorise the argument $r^{k+1}\left(\cos\theta\left(k+1\right)+\mathrm{i}\sin\theta\left(k+1\right)\right)$
Dependent on the second M mark.

A1cso for $r^{k+1}\left(\cos\left(k+1\right)\theta+\mathrm{i}\sin\left(k+1\right)\theta\right)$ Only give this mark if all previous steps are fully correct.

A1cso All 5 underlined statements must be seen

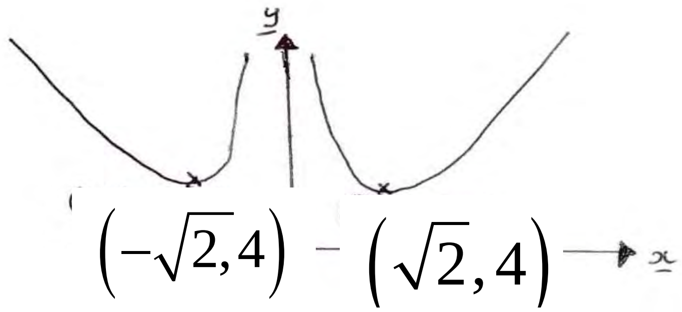
Alternative: Using Euler's form

$z=r\left(\cos\theta+i\sin\theta\right)=r\,\mathrm{e}^{\mathrm{i}\theta}$	M1 May not be seen explicitly
$z^{k+1}=z^k\times z=\left(r\,\mathrm{e}^{\mathrm{i}\theta}\right)^k\times r\,\mathrm{e}^{\mathrm{i}\theta}=r^k\,\mathrm{e}^{\mathrm{i}k\theta}\times r\,\mathrm{e}^{\mathrm{i}\theta}$	M1
$=r^{k+1}\,\mathrm{e}^{\mathrm{i}(k+1)\theta}$	M1dep on 2nd M mark
$=r^{k+1}\left(\cos\left(k+1\right)\theta+\mathrm{i}\sin\left(k+1\right)\theta\right)$	A1cso
$k=1\quad z^1=r^1\left(\cos\theta+\mathrm{i}\sin\theta\right)$	
True for $n=1$ $\therefore$ true for all *n* etc	A1 cso All 5 underlined statements must be seen

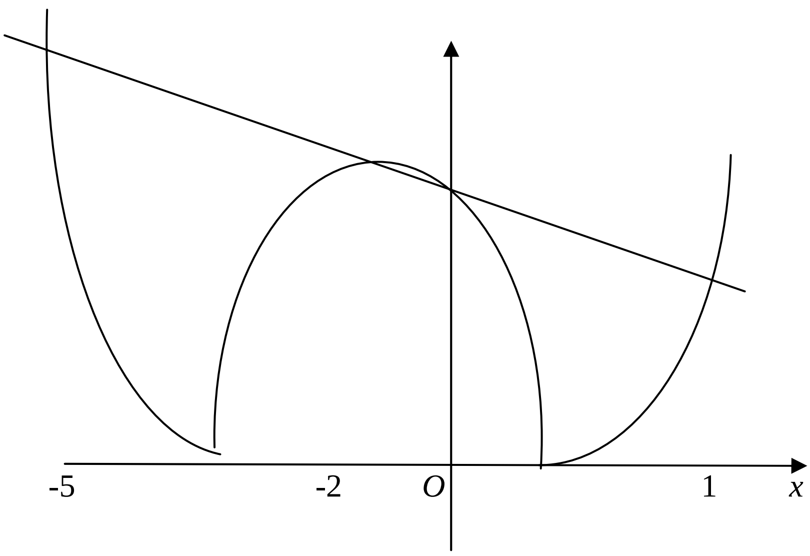
(b)

M1 for attempting to apply de Moivre to *w* or attempting to expand w^5 and collecting real and imaginary parts, but no need to simplify these.

A1cao for $243\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\mathrm{i}\right)$ $\left[=\frac{243\sqrt{2}}{2}-\frac{243\sqrt{2}}{2}\mathrm{i}\right]$ (oe eg 3^5 instead of 243)

Question Number	Scheme	Marks
5		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}+2\frac{y}{x}=4x$	M1
	I F: $\mathrm{e}^{\int\frac{2}{x}\mathrm{d}x}=\mathrm{e}^{2\ln x}=\left(x^2\right)$	M1
	$x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=4x^3$	M1dep
	$yx^2=\int 4x^3\mathrm{d}x=x^4\left(+c\right)$	M1dep
	$y=x^2+\frac{c}{x^2}$	A1cso (5)
(b)	$x=1,\ y=5\Rightarrow c=4$	M1
	$y=x^2+\frac{4}{x^2}$	A1ft (2)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x-\frac{8}{x^3}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\quad x^4=4\ ,\ x=\pm\sqrt{2}$ or $\pm\sqrt[4]{4}$	M1,A1
	$y=2+\frac{4}{2}=4$	A1cao
	Alt: Complete square on *y* = ... or use the original differential equation	M1
	$x=\pm\sqrt{2},\quad y=4$	A1,A1
		B1 shape B1 turning points shown *somewhere* (5) **[12]**

Notes for Question 5
(a) M1 for dividing the given equation by x May be implied by subsequent work. M1 for IF= $e^{\int \frac{2}{x}dx} = e^{2\ln x} = (x^2)$ $\int \frac{2}{x}dx$ must be seen together with an attempt at integrating this. ln x must be seen in the integrated function. M1dep for multiplying the equation $\frac{dy}{dx} + 2\frac{y}{x} = 4x$ by their IF dep on 2nd M mark M1dep for attempting the integration of the resulting equation - constant not needed. Dep on 2nd and 3rd M marks A1cso for $y = x^2 + \frac{c}{x^2}$ oe eg $yx^2 = x^4 + c$ *Alternative:* for first three marks: Multiply given equation by x to get straight to the third line. All 3 M marks should be given. (b) M1 for using $x = 1,\ y = 5$ in **their** expression for y to obtain a value for c A1ft for $y = x^2 + \frac{4}{x^2}$ follow through their result from (a) (c) M1 for differentiating **their** result from (b), equating to 0 and solving for x A1 for $x = \pm\sqrt{2}$ (no follow through) or $\pm\sqrt[4]{4}$ No extra real values allowed but ignore any imaginary roots shown. A1cao for using the particular solution to obtain $y = 4$. No extra values allowed. *Alternatives for these 3 marks:* M1 for making $\frac{dy}{dx} = 0$ in the given differential equation to get $y = 2x^2$ and using this with their particular solution to obtain an equation in one variable **OR** complete the square on **their** particular solution to get $y = \left(x + \frac{2}{x}\right)^2 - 4$ A1 for $x = \pm\sqrt{2}$ (no follow through) A1cao for $y = 4$ No extra values allowed B1 for the correct shape - must have two minimum points and two branches, both asymptotic to the y-axis B1 for a fully correct sketch with the coordinates of the minimum points shown somewhere on or beside the sketch. Decimals accepted here.

Question Number	Scheme	Marks
6 **(a)**	$2x^2+6x-5=5-2x$	M1
	$2x^2+8x-10=0$	
	$x^2+4x-5=0$	
	$(x+5)(x-1)=0$ or by formula	M1
	$x=-5,\ x=1$	A1
	$-2x^2-6x+5=5-2x$	M1
	$2x^2+4x=0$	A1
	$x=0\ \ x=-2$	A1 (6)
(b)		B1 line B1 quad curve B1ft (on x-coords from (a)) (3)
(c)	$x<-5,\quad -2<x<0,\quad x>1$	B1,B1,B1 (3)
	Special case: Deduct the last B mark earned1 if $\leqslant$ or $\geqslant$ used	**[12]**

Notes for Question 6
(a)**NB: Marks for (a) can only be awarded for work shown in (a):** M1 for $2x^2+6x-5=5-2x$ M1 for obtaining a 3 term quadratic and attempting to solve by factorising, formula or completing the square A1 for $x=-5,\ x=1$ M1 for considering the part of the quadratic that needs to be reflected ie for $-2x^2-6x+5=5-2x$ oe A1 for a correct 2 term quadratic, terms in any order $2x^2+4x=0$ oe A1 for $x=0\ \ x=-2$ **NB:** The question demands that algebra is used, so solutions which do not show how the roots have been obtained will score very few if any marks, depending on what is written on the page. *Alternative:* Squaring both sides: M1 Square both sides and simplify to a quartic expression M1 Take out the common factor *x* A1 *x*, a correct linear factor and a correct quadratic factor M1 *x* and 3 linear factors A1 any two of the required values A1 all 4 values correct (b) B1 for a line drawn, with negative gradient, crossing the positive *y*-axis B1 for the quadratic curve, with part reflected and the correct shape. It should cross the *y*-axis at the same point as the line and be pointed where it meets the *x*-axis (ie not U-shaped like a turning point) B1ft for showing the *x* coordinates of the points where the line crosses the curve. They can be shown on the *x*-axis as in the MS (accept *O* for 0) or written alongside the points as long as it is clear the numbers are the *x* coordinates The line should cross the curve at all the crossing points found *and no others* for this mark to be given. (c)**NB: No follow through for these marks** B1 for any one of $x<-5$, $-2<x<0$, $x>1$ correct B1 for a second one of these correct B1 for the third one correct Special case: if $\leqslant$ or $\geqslant$ is used, deduct the last B mark earned.

Question Number	Scheme	Marks
7		
(a)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$	M1
	$\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x\frac{d^2v}{dx^2}$	M1A1
	$4x^2\left(2\frac{dv}{dx} + x\frac{d^2v}{dx^2}\right) - 8x\left(v + x\frac{dv}{dx}\right) + \left(8 + 4x^2\right)\times xv = x^4$	M1
	$4x^3\frac{d^2v}{dx^2} + 4x^3v = x^4$	M1
	$4\frac{d^2v}{dx^2} + 4v = x$ *	A1 (6)
	See end for an alternative for (a)	
(b)	$4\lambda^2 + 4 = 0$	
	$\lambda^2 = -1$ oe	M1A1
	$(v =)C\cos x + D\sin x \quad \left(\text{or } (v =)Ae^{ix} + Be^{-ix}\right)$	A1
	P.I: Try $v = kx \ (+l)$	
	$\frac{dv}{dx} = k \quad \frac{d^2v}{dx^2} = 0$	M1
	$4\times 0 + 4\left(kx\ (+l)\right) = x$	M1dep
	$k = \frac{1}{4} \quad (l = 0)$	
	$v = C\cos x + D\sin x + \frac{1}{4}x \quad \left(\text{or } v = Ae^{ix} + Be^{-ix} + \frac{1}{4}x\right)$	A1 (6)
(c)	$y = x\left(C\cos x + D\sin x + \frac{1}{4}x\right) \quad \left(\text{or } y = x\left(Ae^{ix} + Be^{-ix} + \frac{1}{4}x\right)\right)$	B1ft (1)

Question 7 continued		
	Alternative for (a):	
	$v=\frac{y}{x}$	
	$\frac{dv}{dx}=\frac{dy}{dx}\times\frac{1}{x}-y\times\frac{1}{x^2}$	M1
	$\frac{d^2v}{dx^2}=\frac{d^2y}{dx^2}\times\frac{1}{x}-\frac{dy}{dx}\times\frac{1}{x^2}-\frac{dy}{dx}\times\frac{1}{x^2}+2y\times\frac{1}{x^3}$	M1A1
	$x^3\frac{d^2v}{dx^2}=x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y$	M1
	$4x^3\frac{d^2v}{dx^2}+4x^3v=4x^2\frac{d^2y}{dx^2}-8x\frac{dy}{dx}+8y+4x^2y=x^4$	M1
	$4\frac{d^2v}{dx^2}+4v=x$ *	A1

Notes for Question 7

(a)

M1 for attempting to differentiate $y = xv$ to get $\frac{dy}{dx}$ - product rule must be used

M1 for differentiating **their** $\frac{dy}{dx}$ to obtain an expression for $\frac{d^2y}{dx^2}$ - product rule must be used

A1 for $\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x\frac{d^2v}{dx^2}$

M1 for substituting **their** $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and $y = xv$ in the original equation to obtain a differential equation in v and x

M1 for collecting the terms to have at most a 4 term equation - 4 terms only if a previous error causes $\frac{dv}{dx}$ to be included, otherwise 3 terms

A1cao and cso for $4\frac{d^2v}{dx^2} + 4v = x$ *

Alternative: (see end of mark scheme)

M1 for writing $v = \frac{y}{x}$ and attempting to differentiate by quotient or product rule to get $\frac{dv}{dx}$

M1 for differentiating **their** $\frac{dv}{dx}$ to obtain an expression for $\frac{d^2v}{dx^2}$ - product or quotient rule must be used

A1 for $\frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} - \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x^3}$

M1 for multiplying **their** $\frac{d^2v}{dx^2}$ by x^3

M1 for multiplying by 4 **and** adding $4x^2y$ to each side and equating to x^4 (as rhs is now identical to the original equation.

A1cao and cso for $4\frac{d^2v}{dx^2} + 4v = x$ *

(b)

M1 for forming the auxiliary equation and attempting to solve

A1 for $\lambda^2 = -1$ oe

A1 for the complementary function in either form. Award for a correct CF even if $\lambda = \text{i}$ only is shown.

Notes for Question 7 continued
M1 for trying one of $v = kx,\ k \neq 1$ or $v = kx + l$ and $v = mx^2 + kx + l$ as a PI and obtaining $\frac{\mathrm{d}v}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$ M1dep for substituting their differentials in the equation $4\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 4v = x$. Award M0 if the original equation is used. Dep on 2nd M mark of (b) A1cao for obtaining the correct result (either form) (c) B1ft for reversing the substitution to get $y = x\left(C\cos x + D\sin x + \frac{1}{4}x\right)$ $\left(\text{or } y = x\left(Ae^{\mathrm{i}x} + Be^{-\mathrm{i}x} + \frac{1}{4}x\right)\right)$ follow through their answer to (b)

Question Number	Scheme	Marks
8 (a)	$(y=)\,r\sin\theta = a\sin 2\theta\sin\theta$	M1
	$\left(\dfrac{dy}{d\theta}=\right)a\left(2\cos 2\theta\sin\theta+\sin 2\theta\cos\theta\right)$	M1depA1
	$\left(\dfrac{dy}{d\theta}=\right)2a\sin\theta\left(\cos 2\theta+\cos^2\theta\right)$	M1
	At P $\dfrac{dy}{d\theta}=0 \Rightarrow \sin\theta=0$ (n/a) or $2\cos^2\theta-1+\cos^2\theta=0$	M1 $\sin\theta=0$ not needed
	$3\cos^2\theta=1$	
	$\cos\theta=\dfrac{1}{\sqrt{3}}$ *	A1cso
		(6)
(b)	$r=a\sin 2\theta=2a\sin\theta\cos\theta$	
	$r=2a\sqrt{\left(1-\dfrac{1}{3}\right)}\sqrt{\dfrac{1}{3}}=2a\dfrac{\sqrt{2}}{3}$	M1A1 (2)
(c)	Area = $\int_0^{\phi}\dfrac{1}{2}r^2 d\theta=\dfrac{1}{2}a^2\int_0^{\phi}\sin^2 2\theta\, d\theta$	M1
	$=\dfrac{1}{2}a^2\int_0^{\phi}\dfrac{1}{2}(1-\cos 4\theta)\,d\theta$	M1
	$=\dfrac{1}{4}a^2\left[\theta-\dfrac{1}{4}\sin 4\theta\right]_0^{\phi}$	M1A1
	$=\dfrac{1}{4}a^2\left[\phi-\dfrac{1}{4}\left(4\sin\phi\cos\phi\left(2\cos^2\phi-1\right)\right)\right]$	M1dep on 2nd M mark
	$=\dfrac{1}{4}a^2\left[\arccos\left(\dfrac{1}{\sqrt{3}}\right)-\left(\sqrt{\dfrac{2}{3}}\times\sqrt{\dfrac{1}{3}}\times\left(\dfrac{2}{3}-1\right)\right)\right]$	M1 dep (all Ms)
	$\dfrac{1}{36}a^2\left[9\arccos\left(\dfrac{1}{\sqrt{3}}\right)+\sqrt{2}\right]$ *	A1 (7)
		[15]

Notes for Question 8

(a)

M1 for obtaining the y coordinate $y = r\sin\theta = a\sin 2\theta\sin\theta$

M1dep for attempting the differentiation to obtain $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ Product rule and/or chain rule must be used; sin to become $\pm\cos$ (cos to become $\pm\sin$). The 2 may be omitted. Dependent on the first M mark.

A1 for correct differentiation eg $\frac{\mathrm{d}y}{\mathrm{d}\theta} = a\left(2\cos 2\theta\sin\theta + \sin 2\theta\cos\theta\right)$ oe

M1 for using $\sin 2\theta = 2\sin\theta\cos\theta$ anywhere in their solution to (a)

M1 for setting $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ and getting a quadratic factor with no $\sin^2\theta$ included.

Alternative: Obtain a quadratic in $\sin\theta$ or $\tan\theta$ and complete to $\cos\theta$ = later.

A1cso for $\cos\theta = \frac{1}{\sqrt{3}}$ or $\cos\phi = \frac{1}{\sqrt{3}}$ *

Question 8 (a) Variations you may see:

y = rsinθ = asin2θsinθ

$y = a\sin 2\theta\sin\theta$	$y = 2a\sin^2\theta\cos\theta$	$y = 2a(\cos\theta - \cos^3\theta)$
$\mathrm{d}y/\mathrm{d}\theta = a(2\cos 2\theta\sin\theta + \sin 2\theta\cos\theta)$ $= a(2\cos 2\theta\sin\theta + 2\sin\theta\cos^2\theta)$ $= 2a\sin\theta(\cos 2\theta + \cos^2\theta)$ $= 2a\sin\theta(3\cos^2\theta - 1)$ or $= 2a\sin\theta(2\cos^2\theta - \sin^2\theta)$ or $= 2a\sin\theta(2 - 3\sin^2\theta)$	$\mathrm{d}y/\mathrm{d}\theta = 2a(2\sin\theta\cos^2\theta - \sin^3\theta)$ $= 2a\sin\theta(2\cos^2\theta - \sin^2\theta)$	$\mathrm{d}y/\mathrm{d}\theta = 2a(-\sin\theta + 3\sin\theta\cos^2\theta)$ $= 2a\sin\theta(3\cos^2\theta - 1)$

At P: $\mathrm{d}y/\mathrm{d}\theta = 0 \Rightarrow \sin\theta = 0$ or:		
$2\cos^2\theta - \sin^2\theta = 0$	$3\cos^2\theta - 1 = 0$	$2 - 3\sin^2\theta = 0$
$\tan^2\theta = 2$	$\cos^2\theta = 1/3$	$\sin^2\theta = 2/3$
$\tan\theta = \pm\sqrt{2} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$	$\cos\theta = \pm\frac{1}{\sqrt{3}}$	$\sin\theta = \pm\frac{\sqrt{2}}{\sqrt{3}} = \pm\frac{\sqrt{6}}{3} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$

(b)

M1 for using $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos^2\theta + \sin^2\theta = 1$ and $\cos\phi = \frac{1}{\sqrt{3}}$ in $r = a\sin 2\theta$ to obtain a numerical multiple of a for R. Need not be simplified.

A1cao for $R = 2a\frac{\sqrt{2}}{3}$

Can be done on a calculator. Completely correct answer with no working scores 2/2; incorrect answer with no working scores 0/2

Notes for Question 8 continued
(c) M1 for using the area formula $\int_0^{\phi}\frac{1}{2}r^2\mathrm{d}\theta = \frac{1}{2}a^2\int_0^{\phi}\sin^2 2\theta\,\mathrm{d}\theta$ Limits not needed M1 for preparing $\int \sin^2 2\theta\,\mathrm{d}\theta$ for integration by using $\cos 2x = 1-2\sin^2 x$ M1 for attempting the integration: $\cos 4\theta$ to become $\pm\sin 4\theta$ - the $\frac{1}{4}$ may be missing but inclusion of 4 implies differentiation - and the constant to become $k\theta$. Limits not needed. A1 for $=\frac{1}{4}a^2\left[\theta-\frac{1}{4}\sin 4\theta\right]$ Limits not needed M1dep for changing **their** integrated function to an expression in $\sin\theta$ and $\cos\theta$ and substituting limits 0 and ϕ. Dependent on the second M mark of (c) M1dep for a numerical multiple of a^2 for the area. Dependent on all previous M marks of (c) A1cso for $\frac{1}{36}a^2\left[9\arccos\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right]$ * This is a given answer, so check carefully that it can be obtained from the previous step in their working. **Also**: The final 3 marks can only be awarded if the working is **shown** ie $\sin 4\theta$ cannot be obtained by calculator.

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