AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Unit Further Pure 2

Friday 24 June 2016   Morning   Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Given that $f(r) = \frac{1}{4r-1}$, show that

$$f(r) - f(r+1) = \frac{A}{(4r-1)(4r+3)}$$

where A is an integer.

[2 marks]

(b) Use the method of differences to find the value of $\sum_{r=1}^{50} \frac{1}{(4r-1)(4r+3)}$, giving your answer as a fraction in its simplest form.

[4 marks]

QUESTION PART REFERENCE

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2 The cubic equation $3z^3 + pz^2 + 17z + q = 0$, where p and q are real, has a root $\alpha = 1 + 2\text{i}$.

(a) (i) Write down the value of another non-real root, β, of this equation.

[1 mark]

(ii) Hence find the value of $\alpha\beta$.

[1 mark]

(b) Find the value of the third root, γ, of this equation.

[3 marks]

(c) Find the values of p and q.

[3 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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3 The arc of the curve with equation $y = 4 - \ln(1 - x^2)$ from $x = 0$ to $x = \frac{3}{4}$ has length s.

(a) Show that $s = \int_0^{\frac{3}{4}} \left(\frac{1 + x^2}{1 - x^2} \right) \mathrm{d}x$.

[4 marks]

(b) Find the value of s, giving your answer in the form $p + \ln N$, where p is a rational number and N is an integer.

[6 marks]

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4 (a) Given that $y = \tan^{-1}\sqrt{(3x)}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in terms of x.

[2 marks]

(b) Hence, or otherwise, show that $\int_{\frac{1}{3}}^{1} \frac{1}{(1+3x)\sqrt{x}}\,\mathrm{d}x = \frac{\sqrt{3}\pi}{n}$, where n is an integer.

[4 marks]

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5 (a) Find the modulus of the complex number $-4\sqrt{3}+4\mathrm{i}$, giving your answer as an integer.

[2 marks]

(b) The locus of points, L, satisfies the equation $|z+4\sqrt{3}-4\mathrm{i}|=4$.

(i) Sketch the locus L on the Argand diagram below.

[3 marks]

(ii) The complex number w lies on L so that $-\pi < \arg w \leqslant \pi$.

Find the least possible value of $\arg w$, giving your answer in terms of π.

[2 marks]

(c) Solve the equation $z^3 = -4\sqrt{3}+4\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

[5 marks]

QUESTION PART REFERENCE

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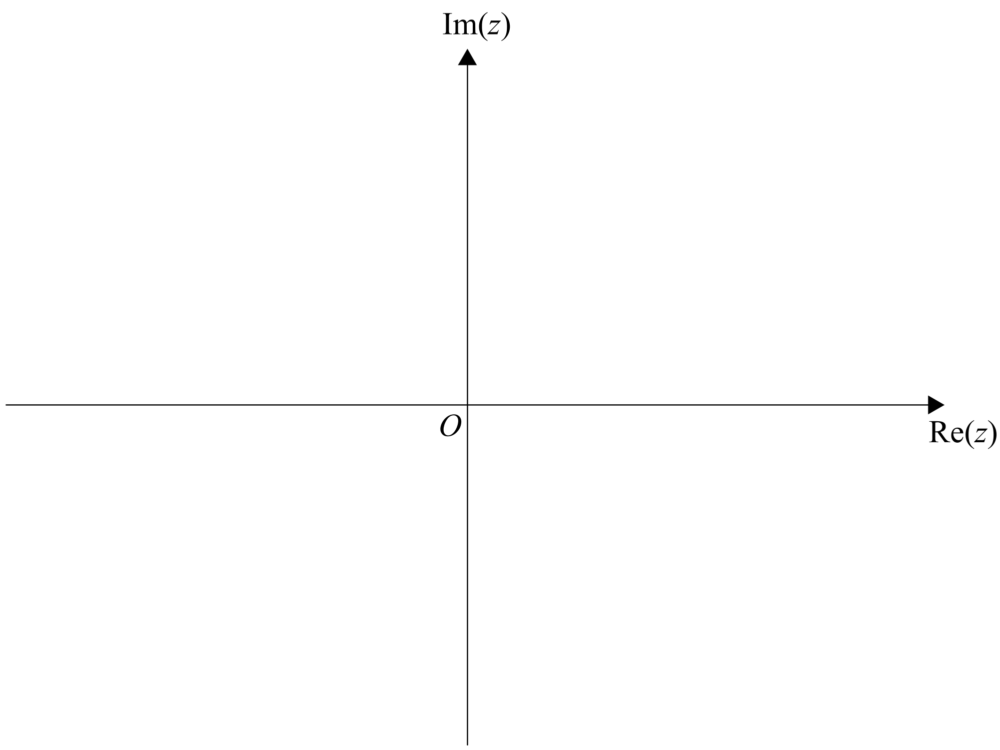
(b)(i)



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6 (a) Given that $y = \sinh x$, use the definition of $\sinh x$ in terms of e^x and e^{-x} to show that $x = \ln(y + \sqrt{y^2 + 1})$.

[4 marks]

(b) A curve has equation $y = 6\cosh^2 x + 5\sinh x$.

(i) Show that the curve has a single stationary point and find its x-coordinate, giving your answer in the form $\ln p$, where p is a rational number.

[5 marks]

(ii) The curve lies entirely above the x-axis. The region bounded by the curve, the coordinate axes and the line $x = \cosh^{-1} 2$ has area A.

Show that

$$A = a\cosh^{-1} 2 + b\sqrt{3} + c$$

where a, b and c are integers.

[5 marks]

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7 Given that $p \geqslant -1$, prove by induction that, for all integers $n \geqslant 1$,

$$(1+p)^n \geqslant 1+np$$

[6 marks]

QUESTION PART REFERENCE

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8 (a) By applying de Moivre's theorem to $(\cos\theta + \mathrm{i}\sin\theta)^4$, where $\cos\theta \neq 0$, show that

$$(1 + \mathrm{i}\tan\theta)^4 + (1 - \mathrm{i}\tan\theta)^4 = \frac{2\cos 4\theta}{\cos^4\theta}$$

[3 marks]

(b) Hence show that $z = \mathrm{i}\tan\frac{\pi}{8}$ satisfies the equation $(1+z)^4 + (1-z)^4 = 0$, and express the three other roots of this equation in the form $\mathrm{i}\tan\phi$, where $0 < \phi < \pi$.

[2 marks]

(c) Use the results from part **(b)** to find the values of:

(i) $\tan^2\frac{\pi}{8}\tan^2\frac{3\pi}{8}$;

[4 marks]

(ii) $\tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8}$.

[4 marks]

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