Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MFP2

Unit Further Pure 2

Tuesday 16 June 2015 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\frac{1}{(r+2)r!}$ in the form $\frac{A}{(r+1)!}+\frac{B}{(r+2)!}$, where A and B are integers.

[3 marks]

(b) Hence find $\sum_{r=1}^{n} \frac{1}{(r+2)r!}$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 (a) Sketch the graph of $y = \tanh x$ and state the equations of its asymptotes.

[3 marks]

(b) Use the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} to show that

$$\operatorname{sech}^2 x + \tanh^2 x = 1$$

[3 marks]

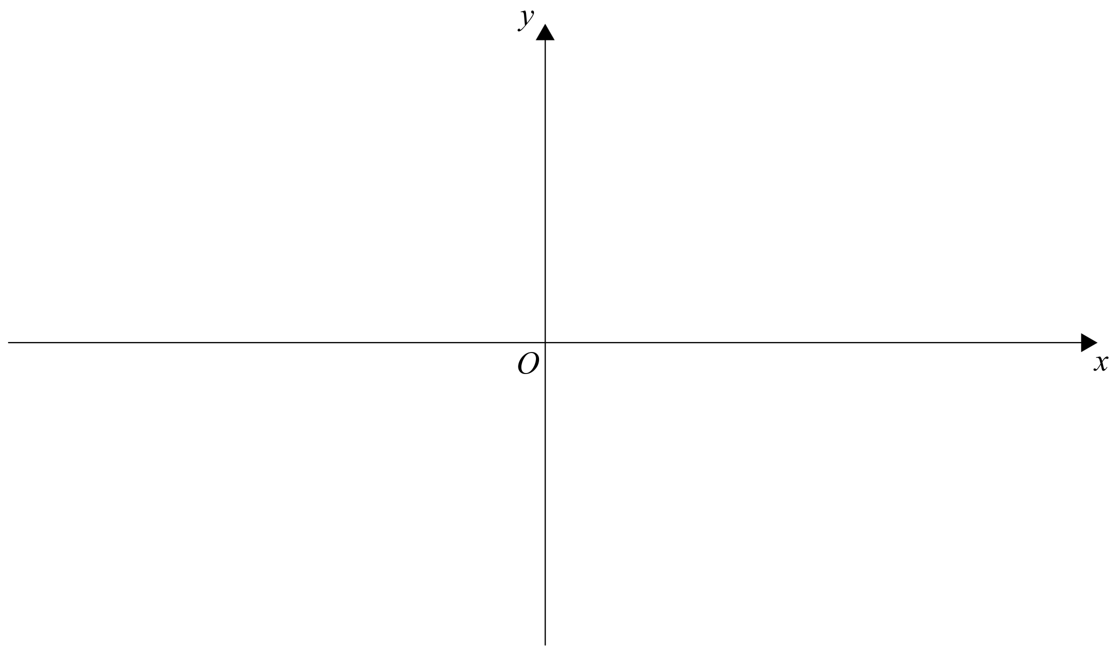
(c) Solve the equation $6 \operatorname{sech}^2 x = 4 + \tanh x$, giving your answers in terms of natural logarithms.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

(a)



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QUESTION PART REFERENCE

Answer space for question 2

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3 A curve C is defined parametrically by

$$x = \frac{t^2 + 1}{t}, \quad y = 2\ln t$$

(a) Show that $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \left(1 + \frac{1}{t^2}\right)^2$.

[4 marks]

(b) The arc of C from $t = 1$ to $t = 2$ is rotated through 2π radians about the x-axis. Find the area of the surface generated, giving your answer in the form $\pi(m\ln 2 + n)$, where m and n are integers.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 The expression $f(n)$ is given by $f(n) = 2^{4n+3} + 3^{3n+1}$.

(a) Show that $f(k+1) - 16f(k)$ can be expressed in the form $A \times 3^{3k}$, where A is an integer.

[3 marks]

(b) Prove by induction that $f(n)$ is a multiple of 11 for all integers $n \geqslant 1$.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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5 The locus of points, L, satisfies the equation

$$|z - 2 + 4\mathrm{i}| = |z|$$

(a) Sketch L on the Argand diagram below.

[3 marks]

(b) The locus L cuts the real axis at A and the imaginary axis at B.

(i) Show that the complex number represented by C, the midpoint of AB, is

$$\frac{5}{2} - \frac{5}{4}\mathrm{i}$$

[4 marks]

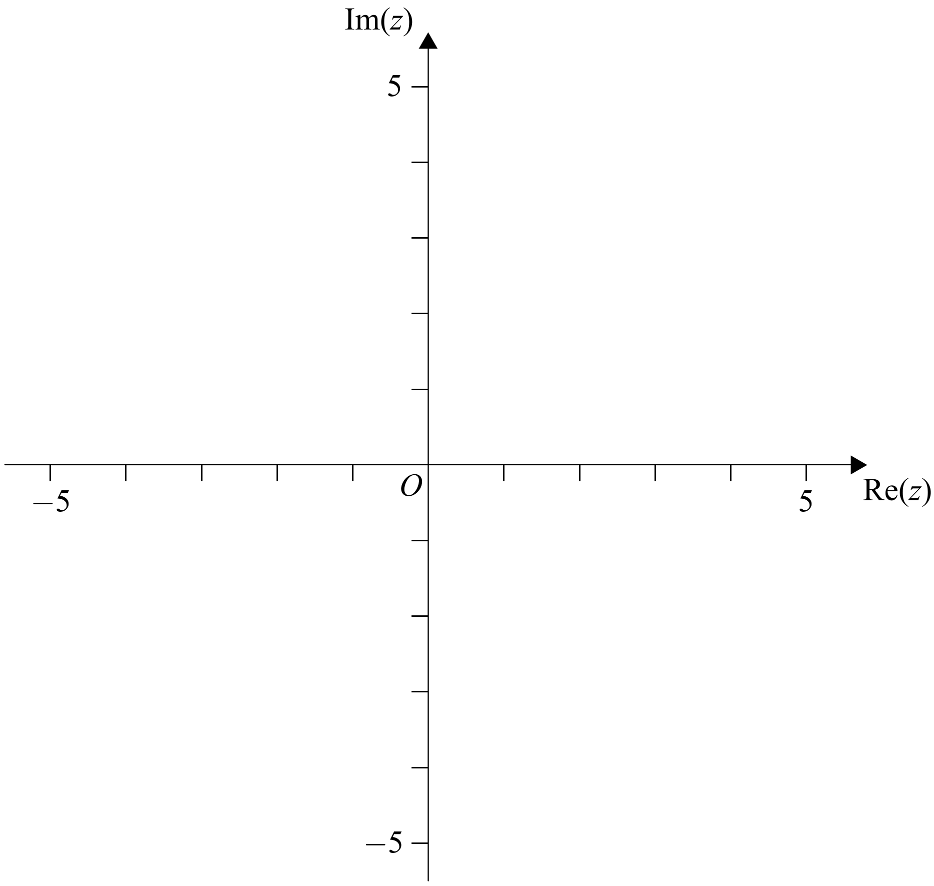
(ii) The point O is the origin of the Argand diagram. Find the equation of the circle that passes through the points O, A and B, giving your answer in the form $|z - \alpha| = k$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

(a)



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QUESTION PART REFERENCE

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6 (a) Given that $y = (x-2)\sqrt{5+4x-x^2} + 9\sin^{-1}\left(\frac{x-2}{3}\right)$, show that

$$\frac{dy}{dx} = k\sqrt{5+4x-x^2}$$

where k is an integer.

[5 marks]

(b) Hence show that

$$\int_2^{\frac{7}{2}} \sqrt{5+4x-x^2}\ dx = p\sqrt{3} + q\pi$$

where p and q are rational numbers.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

7 The cubic equation $27z^3 + kz^2 + 4 = 0$ has roots α, β and γ.

(a) Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$.

[2 marks]

(b) (i) In the case where $\beta = \gamma$, find the roots of the equation.

[5 marks]

(ii) Find the value of k in this case.

[1 mark]

(c) (i) In the case where $\alpha = 1 - \mathrm{i}$, find α^2 and α^3.

[2 marks]

(ii) Hence find the value of k in this case.

[2 marks]

(d) In the case where $k = -12$, find a cubic equation with integer coefficients which has roots $\dfrac{1}{\alpha} + 1$, $\dfrac{1}{\beta} + 1$ and $\dfrac{1}{\gamma} + 1$.

[5 marks]

QUESTION PART REFERENCE

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8 The complex number ω is given by $\omega = \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}$.

(a) (i) Verify that ω is a root of the equation $z^5 = 1$.

[1 mark]

(ii) Write down the three other non-real roots of $z^5 = 1$, in terms of ω.

[1 mark]

(b) (i) Show that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$.

[1 mark]

(ii) Hence show that $\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$.

[2 marks]

(c) Hence show that $\cos\frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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END OF QUESTIONS