AQA

A-LEVEL
Mathematics

Further Pure 2 – MFP2
Mark scheme

6360
June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$r+1 = A(r+2)+B$ **or** $1 = \frac{A(r+2)}{r+1} + \frac{B}{r+1}$	**M1**		**OE** with factorials removed
	either $A=1$ or $B=-1$	**A1**		**correctly** obtained
	$\frac{1}{(r+2)r!} = \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$	**A1**	3	allow if seen in part **(b)**
(b)	$\frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \ldots$ $\frac{1}{(n+1)!} - \frac{1}{(n+2)!}$	**M1**		use of their result from part **(a)** at least twice
	Sum $= \frac{1}{2} - \frac{1}{(n+2)!}$	**A1**	2	must simplify 2! and must have scored at least **M1 A1** in part **(a)**
	Total		**5**	

(a) **Alternative Method** Substituting two values of *r* to obtain two correct equations in *A* and *B* with factorials evaluated correctly

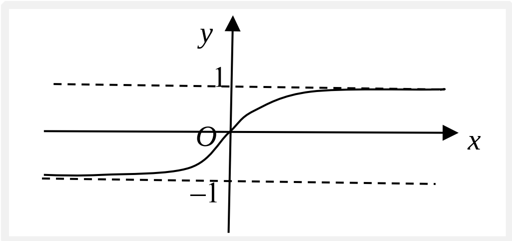
$r=0 \Rightarrow \frac{1}{2} = A + \frac{B}{2}$; $r=1 \Rightarrow \frac{1}{3} = \frac{A}{2} + \frac{B}{6}$ earns **M1** then **A1, A1** as in main scheme

NMS $\frac{1}{(r+1)!} - \frac{1}{(r+2)!}$ earns **3 marks**.

However, using an ***incorrect*** expression resulting from poor algebra such as $1 = A(r+2)! + B(r+1)!$ with candidate often fluking $A = 1$, $B = -1$ scores **M0** ie zero marks which you should denote as FIW

These candidates can then score a maximum of **M1** in part **(b).**

(b) **ISW** for incorrect simplification after correct answer seen

Q2	Solution	Mark	Total	Comment
(a)				
	Graph roughly correct through O	M1		condone infinite gradient at O for **M1**
	Correct behaviour as $x \to \pm\infty$ & grad at O	A1		
	Asymptotes have equations $y = 1$ & $y = -1$	B1	3	must state equations
(b)	$\text{sech}\, x = \dfrac{2}{e^x + e^{-x}}$ **;** $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	B1		**both** correct **ACF** or **correct** squares of these expressions seen
	($\text{sech}^2 x + \tanh^2 x =$ **)** $\dfrac{2^2 + \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$	M1		attempt to combine their squared terms with correct single denominator
	$\text{sech}^2 x + \tanh^2 x = \dfrac{e^{2x} + 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} = 1$	A1	3	**AG** valid proof convincingly shown to equal 1 **including LHS seen**
(c)	$6(1 - \tanh^2 x) = 4 + \tanh x$	B1		correct use of identity from part **(b)**
	$6\tanh^2 x + \tanh x - 2 \quad (= 0)$	M1		forming quadratic in $\tanh x$
	$\tanh x = \dfrac{1}{2}$, $\tanh x = -\dfrac{2}{3}$	A1		obtained from correct quadratic
	$\tanh x = k \Rightarrow x = \dfrac{1}{2}\ln\left(\dfrac{1+k}{1-k}\right)$	A1F		FT a value of k provided $\lvert k \rvert < 1$
	$x = \dfrac{1}{2}\ln 3$, $x = \dfrac{1}{2}\ln\dfrac{1}{5}$	A1	5	both solutions correct and no others any equivalent form involving ln
	Total		**11**	

(a) Actual asymptotes need not be shown, but if asymptotes are drawn then curve should not cross them for **A1**. Gradient should not be infinite at O for **A1.**

(b) Condone trailing equal signs up to final line provided "$\text{sech}^2 x + \tanh^2 x =$ " is seen on previous line for **A1**

Denominator may be $e^{4x} + 4e^{2x} + 6 + e^{4x} + 4e^{-2x} + e^{-4x}$ etc for **M1** and **A1**

Accept $\text{sech}^2 x + \tanh^2 x = \dfrac{\left(e^x + e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2} = 1$ for **A1**

Alternative : $\left(\dfrac{1}{\cosh^2 x} + \dfrac{\sinh^2 x}{\cosh^2 x} =\right)$ $\dfrac{1 + \left(\frac{1}{2}(e^x - e^{-x})\right)^2}{\left(\frac{1}{2}(e^x + e^{-x})\right)^2}$ scores **B1 M1**

and then **A1** for $\text{sech}^2 x + \tanh^2 x = \dfrac{\frac{1}{4}e^{2x} + \frac{1}{4}e^{-2x} + \frac{1}{2}}{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}} = 1$, (all like terms combined in any order).

Q3	Solution	Mark	Total	Comment
(a)	$\dfrac{dx}{dt}=1-\dfrac{1}{t^2}$	B1		OE eg $\dfrac{t(2t)-(t^2+1)}{t^2}$ ACF
	$\dfrac{dy}{dt}=\dfrac{2}{t}$	B1		
	$\left(\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2=\right)\quad 1-\dfrac{2}{t^2}+\dfrac{1}{t^4}+\dfrac{4}{t^2}$	M1		squaring and adding their expressions and attempting to multiply out
	$1+\dfrac{2}{t^2}+\dfrac{1}{t^4}\quad=\left(1+\dfrac{1}{t^2}\right)^2$	A1	4	**AG** be convinced
(b)	$2\pi\int_1^2(2\ln t)\left(1+\dfrac{1}{t^2}\right)dt$	B1		must have 2π , limits and dt
		M1		integration by parts - clear attempt to integrate $1+\dfrac{1}{t^2}$ and differentiate $2\ln t$
	$(2\pi)\left\{(2\ln t)\left(t-\dfrac{1}{t}\right)-\int\dfrac{2}{t}\left(t-\dfrac{1}{t}\right)(dt)\right\}$	A1		correct (may omit limits, 2π and dt)
	$2\pi\left[(2\ln t)\left(t-\dfrac{1}{t}\right)-\left(2t+\dfrac{2}{t}\right)\right]$	A1		correct including 2π (no limits required)
	$=2\pi(3\ln 2-5+4)$ $=\pi(6\ln 2-2)$	A1	5	
	Total		**9**	

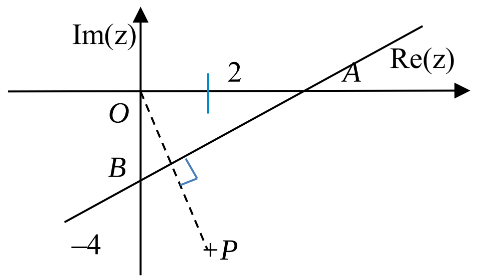
(b) May have two separate integrals and attempt both using integration by parts for **M1**

Must see $(2\pi)\left\{2t\ln t-\int 2(dt)\quad -\left(2t^{-1}\ln t-\int 2t^{-2}(dt)\right)\right\}$ (may omit limits, 2π and dt) for first **A1**

and $2\pi\left[(2t\ln t-2t)-(2t^{-1}\ln t+2t^{-1})\right]$ for second **A1**

Condone poor use of brackets if later recovered.

Q4	Solution	Mark	Total	Comment
(a)	$f(k+1) = 2^{4k+7} + 3^{3k+4}$	M1		
	convincingly showing $2^{4k+7} = 16 \times 2^{4k+3}$	E1		must see $16 = 2^4$ OE
	$f(k+1) - 16f(k)$ $= (81 - 16\times 3)\times 3^{3k}$ $= 33 \times 3^{3k}$	A1	3	
(b)	$f(1) = 209$ therefore $f(1)$ is a multiple of 11	B1		$f(1) = 209 = 11\times 19$ *or* $209 \div 11 = 19$ etc therefore true when n=1
	Assume $f(k)$ is a multiple of 11 (*) $f(k+1) = 16f(k) + 33\times 3^{3k}$	M1		attempt at $f(k+1) = ...$ using their result from part **(a)**
	$= 11M + 11N = 11(M+N)$ Therefore $f(k+1)$ is a multiple of 11	A1		where M and N are integers
	Since f(1) is multiple of 11 then f(2), f(3),… are multiples of 11 by induction (or is a multiple of 11 for all integers $n \geq 1$)	E1	4	must earn previous 3 marks and have (*) before **E1** can be awarded
	Total		**7**	
(a)	It is possible to score **M1 E0 A1**			
(b)	Withhold **E1** for conclusion such as **"**a multiple of 11 for all $n \geq 1$" or poor notation, etc			

Q5	Solution	Mark	Total	Comment
(a)				Ignore the line *OP* drawn in full or circles drawn as part of construction for locus *L*.
	Straight line Through midpoint of *OP*, *P* correct Perpendicular to *OP*, *P* correct	**M1** **A1** **A1**	3	*P* represents 2 – 4i
(b)(i)	$(x-2)^2+(y+4)^2=x^2+y^2$	**M1**		
	$2y-x+5=0$	**A1**		
	$A(5,0)$ & $B(0,-2.5)$	**A1**		may have 5 + 0i and 0 – 2.5i
	$C\left(\frac{5}{2},-\frac{5}{4}\right)\Rightarrow$ complex num $=\frac{5}{2}-\frac{5}{4}\mathrm{i}$	**A1**	4	
(ii)	***either*** $\alpha=\frac{5}{2}-\frac{5}{4}\mathrm{i}$ ***or*** $k=\frac{5\sqrt{5}}{4}$	**M1**		allow statement with correct value for centre or radius of circle
	$\left\lvert z-\frac{5}{2}+\frac{5}{4}\mathrm{i}\right\rvert=\frac{5\sqrt{5}}{4}$	**A1**	2	must have exact surd form
	Total		**9**	
(a)	Withhold the final **A1** (if 3 marks earned) if the straight line does not go beyond the Re(z) axis and negative Im(z) axis. The two **A1** marks can be awarded independently.			
(b)(i)	**Alternative 1:** grad $OP=-2\Rightarrow$ grad $L=0.5$ **M1 ;** $y+2=\frac{1}{2}(x-1)$ OE **A1** then **A1, A1** as per scheme **Alternative 2:** substituting $z=x$ (or a) then $z=\mathrm{i}y$ (or $\mathrm{i}b$) into given locus equation Both $(x-2)^2+4^2=x^2$ **and** $2^2+(y+4)^2=y^2$ **M1;** $4-4x+16=0$ **and** $4+8y+16=0$ **OE** for **A1** then **A1, A1** as per scheme.			

Q6	Solution	Mark	Total	Comment
(a)	$\sqrt{5+4x-x^2}+\dfrac{(x-2)\frac{1}{2}(4-2x)}{\sqrt{5+4x-x^2}}$	M1		product rule (condone one error)
		A1		correct unsimplified
	(+) $\dfrac{9\times\frac{1}{3}}{\sqrt{1-\left(\frac{x-2}{3}\right)^2}}$	B1		or $\dfrac{9}{\sqrt{3^2-(x-2)^2}}$ correct unsimplified
	$\dfrac{5+4x-x^2}{\sqrt{5+4x-x^2}}$	A1		last two terms above combined correctly
	$\left(\dfrac{dy}{dx}=\right)2\sqrt{5+4x-x^2}$	A1cso	5	$k=2$
(b)	$\dfrac{1}{k}\left\{(x-2)\sqrt{5+4x-x^2}+9\sin^{-1}\left(\dfrac{x-2}{3}\right)\right\}$	M1		ft "their" k
	$\dfrac{1}{\text{"their" } k}\left[\dfrac{3}{2}\sqrt{\dfrac{27}{4}}+9\sin^{-1}\dfrac{1}{2}\right]$	m1		correct sub of limits (simplified at least this far)
	$=\dfrac{9}{8}\sqrt{3}+\dfrac{3}{4}\pi$	A1 cso	3	must have earned **5 marks** in part **(a)** to be awarded this mark
	Total		**8**	
(a)	Second **A1** ; may combine all three terms correctly and obtain $\dfrac{10+8x-2x^2}{\sqrt{5+4x-x^2}}$			

Q7	Solution	Mark	Total	Comment
(a)	$\alpha\beta+\beta\gamma+\gamma\alpha=0$	B1		
	$\alpha\beta\gamma=-\frac{4}{27}$	B1	2	
(b)(i)	$\alpha\beta+\alpha\beta+\beta^2=0$; $\alpha\beta^2=-\frac{4}{27}$	B1		May use γ instead of β throughout **(b)(i)**
		M1		Clear attempt to eliminate either α or β from "their" equations
	$\alpha^3=-\frac{1}{27}$ **or** $\beta^3=\frac{8}{27}$	A1		correct
	either $\alpha=-\frac{1}{3}$ or $\beta=\frac{2}{3}$	A1		
	$\alpha=-\frac{1}{3}$, $\beta=\frac{2}{3}$, $\gamma=\frac{2}{3}$	A1	5	all 3 roots clearly stated
(ii)	$\left(\sum\alpha=1=-\frac{k}{27}\ \Rightarrow\right)$ $k=-27$	B1	1	or substituting correct root into equation
(c)(i)	$\alpha^2=-2\text{i}$	B1		
	$\alpha^3=-2-2\text{i}$	B1	2	
(ii)	$27(-2-2\text{i})-2\text{i}k+4=0$	M1		correctly substituting "their" $\alpha^2=-2\text{i}$ and "their" $\alpha^3=-2-2\text{i}$
	$k=-27+25\text{i}$	A1	2	
(d)	$y=\frac{1}{z}+1\Rightarrow z=\frac{1}{y-1}$	B1		may use any letter instead of y
	$\frac{27}{(y-1)^3}-\frac{12}{(y-1)^2}+4=0$	M1		sub their z into cubic equation
	$27-12(y-1)+4(y-1)^3=0$	A1		removing denominators correctly
	$27-12y+12+4(y^3-3y^2+3y-1)=0$	A1		correct and $(y-1)^3$ expanded correctly
	$4y^3-12y^2+35=0$	A1	5	
	Alternative: $\sum\alpha'=3+\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}=3$	(B1)		sum of new roots =3
	$\sum\alpha'\beta'=3+\frac{2(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha+\beta+\gamma}{\alpha\beta\gamma}$	(M1)		**M1** for either of the other two formulae correct in terms of $\alpha\beta\gamma, \alpha\beta+\beta\gamma+\gamma\alpha$ and $\alpha+\beta+\gamma$
	$=0$	(A1)		
	$\prod=1+\frac{\alpha\beta+\beta\gamma+\gamma\alpha+1+\alpha+\beta+\gamma}{\alpha\beta\gamma}$			
	$=\frac{-35}{4}$	(A1)		
	$4y^3-12y^2+35=0$	(A1)	(5)	may use any letter instead of y
	Total		**17**	

Q8	Solution	Mark	Total	Comment
(a)(i)	$(\omega^5 =)\cos 2\pi + \mathrm{i}\sin 2\pi = 1$ So ω is a root of $z^5 = 1$	B1	1	must have conclusion plus verification that $\omega^5 = 1$
(ii)	$\omega^2, \omega^3, \omega^4$.	B1	1	**OE** powers mod 5 (must not include 1)
(b)(i)	$1+\omega+\omega^2+\omega^3+\omega^4 = \dfrac{1-\omega^5}{1-\omega} = 0$	B1	1	or clear statement that sum of roots (of $z^5 - 1 = 0$) is zero
(ii)	$\left(\omega+\dfrac{1}{\omega}\right)^2 + \left(\omega+\dfrac{1}{\omega}\right) - 1$ $= \omega^2 + 2 + \dfrac{1}{\omega^2} + \omega + \dfrac{1}{\omega} - 1$ $= \dfrac{1+\omega+\omega^2+\omega^3+\omega^4}{\omega^2} = 0$	M1 A1	2	correct expansion **AG** correctly shown to = 0 do not allow simply multiplying by ω^2
(c)	$\dfrac{1}{\omega} = \cos\dfrac{2\pi}{5} - \mathrm{i}\sin\dfrac{2\pi}{5}$ $\Rightarrow \omega + \dfrac{1}{\omega} = 2\cos\dfrac{2\pi}{5}$ Solving quadratic $\left(\omega+\dfrac{1}{\omega} =\right) \dfrac{-1\pm\sqrt{5}}{2}$ Rejecting negative root since $\cos\dfrac{2\pi}{5} > 0$ Hence $\cos\dfrac{2\pi}{5} = \dfrac{\sqrt{5}-1}{4}$	M1 A1 M1 A1	4	 **SC1** if result merely stated must see both values must see this line for final **A1** It is possible to score **SC1 M1 A1**
	Total		9	
(b)(ii)	May replace $\dfrac{1}{\omega^2}$ by ω^3 and $\dfrac{1}{\omega}$ by ω^4 and/or 1 by ω^5 in valid proof. **Alternative:** $1+\omega+\omega^2+\omega^3+\omega^4 = 0 \Rightarrow \dfrac{1}{\omega^2}+\dfrac{1}{\omega}+1+\omega+\omega^2 = 0$ **M1** $\left(\omega+\dfrac{1}{\omega}\right)^2 - 2 + \left(\omega+\dfrac{1}{\omega}\right) + 1 = 0 \Rightarrow \left(\omega+\dfrac{1}{\omega}\right)^2 + \left(\omega+\dfrac{1}{\omega}\right) - 1 = 0$ **A1**			