Centre Number			Candidate Number		
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General Certificate of Education Advanced Level Examination June 2010

# **Mathematics**

MFP2

**Unit Further Pure 2** 

Wednesday 9 June 2010 1.30 pm to 3.00 pm

## For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

### Time allowed

• 1 hour 30 minutes

# Instructions

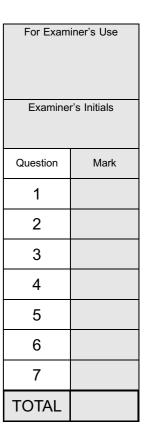
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

#### **Advice**

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.



	Answer all questions in the spaces provided.
1 (a	) Show that
	$9\sinh x - \cosh x = 4e^x - 5e^{-x} $ (2 marks)
(b	) Given that
	$9\sinh x - \cosh x = 8$
	find the exact value of $tanh x$ . (7 marks)
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2 (a)	Express $\frac{1}{r(r+2)}$ in partial fractions.	(3 marks)
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**(b)** Use the method of differences to find

$$\sum_{r=1}^{48} \frac{1}{r(r+2)}$$

giving your answer as a rational number.

(5 marks)

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**3** Two loci,  $L_1$  and  $L_2$ , in an Argand diagram are given by

$$L_1 : |z + 1 + 3i| = |z - 5 - 7i|$$

$$L_2: \arg z = \frac{\pi}{4}$$

- Verify that the point represented by the complex number 2 + 2i is a point of intersection of  $L_1$  and  $L_2$ . (2 marks)
- (b) Sketch  $L_1$  and  $L_2$  on one Argand diagram. (5 marks)
- (c) Shade on your Argand diagram the region satisfying

both 
$$|z+1+3i| \le |z-5-7i|$$

and  $\frac{\pi}{4} \leqslant \arg z \leqslant \frac{\pi}{2}$  (2 marks)

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(1 mark)

4 The roots of the cubic equation

$$z^3 - 2z^2 + pz + 10 = 0$$

are  $\alpha$ ,  $\beta$  and  $\gamma$ .

It is given that  $\alpha^3 + \beta^3 + \gamma^3 = -4$ .

- (a) Write down the value of  $\alpha + \beta + \gamma$ .
- **(b) (i)** Explain why  $\alpha^3 2\alpha^2 + p\alpha + 10 = 0$ . (1 mark)
  - (ii) Hence show that

$$\alpha^2 + \beta^2 + \gamma^2 = p + 13 \tag{4 marks}$$

- (iii) Deduce that p = -3. (2 marks)
- (c) (i) Find the real root  $\alpha$  of the cubic equation  $z^3 2z^2 3z + 10 = 0$ . (2 marks)
  - (ii) Find the values of  $\beta$  and  $\gamma$ . (3 marks)

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**5 (a)** Using the identities

$$\cosh^2 t - \sinh^2 t = 1, \quad \tanh t = \frac{\sinh t}{\cosh t} \quad \text{and} \quad \operatorname{sech} t = \frac{1}{\cosh t}$$

show that:

(i) 
$$\tanh^2 t + \operatorname{sech}^2 t = 1$$
; (2 marks)

(ii) 
$$\frac{\mathrm{d}}{\mathrm{d}t}(\tanh t) = \mathrm{sech}^2 t$$
; (3 marks)

(iii) 
$$\frac{d}{dt}(\operatorname{sech} t) = -\operatorname{sech} t \tanh t$$
. (3 marks)

(b) A curve C is given parametrically by

$$x = \operatorname{sech} t$$
,  $y = 4 - \tanh t$ 

(i) Show that the arc length, s, of C between the points where t = 0 and  $t = \frac{1}{2} \ln 3$  is given by

$$s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \, \mathrm{d}t \tag{4 marks}$$

(ii) Using the substitution  $u = e^t$ , find the exact value of s. (6 marks)

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6 (a)	Charry that	1	k+1	2	(2 m aula)
o (a)	Show that	(k+2)!	$\overline{(k+3)!}$	$=\frac{1}{(k+3)!}$ .	(2 marks)

(b) Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{r \times 2^{r}}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$$
 (6 marks)

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- 7 (a) (i) Express each of the numbers  $1 + \sqrt{3}i$  and 1 i in the form  $re^{i\theta}$ , where r > 0.
  - (ii) Hence express

$$(1+\sqrt{3}i)^8(1-i)^5$$

in the form  $re^{i\theta}$ , where r > 0.

(3 marks)

**(b)** Solve the equation

$$z^3 = (1 + \sqrt{3}i)^8 (1 - i)^5$$

giving your answers in the form  $a\sqrt{2}\,\mathrm{e}^{\mathrm{i}\theta}$ , where a is a positive integer and  $-\pi < \theta \leqslant \pi$ .

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