Version 1.0

General Certificate of Education
June 2010

Mathematics MFP2

Further Pure 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

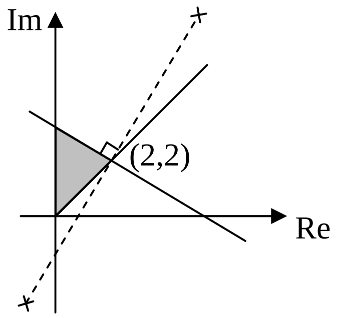
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	$\frac{9\left(e^x - e^{-x}\right)}{2} - \frac{e^x + e^{-x}}{2}$	M1		M0 if $\cosh x$ mixed up with $\sinh x$
	$= 4e^x - 5e^{-x}$	A1	2	AG
(b)	Attempt to multiply by e^x	M1		
	$4e^{2x} - 8e^x - 5 = 0$	A1		
	$\left(2e^x - 5\right)\left(2e^x + 1\right) = 0$	M1		ft provided quadratic factorises (or use of formula)
	$e^x \neq -\frac{1}{2}$	E1F		PI but not ignored
	$e^x = \frac{5}{2}$	A1F		
	$\tanh x = \frac{\frac{5}{2} - \frac{2}{5}}{\frac{5}{2} + \frac{2}{5}} = \frac{21}{29}$	M1 A1F	7	M1 PI for attempt to use $\tanh x = \frac{\sinh x}{\cosh x}$ or equivalent fraction
	Total		**9**	
2(a)	$\frac{1}{r(r+2)} = \frac{A}{r} + \frac{B}{r+2}$	M1		
	$A = \frac{1}{2}, \quad B = -\frac{1}{2}$	A1, A1F	3	ft incorrect A
(b)	$r = 1 \quad \frac{1}{1.3} = \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right)$			
	$r = 2 \quad \frac{1}{2.4} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$			
	$r = 3 \quad \frac{1}{3.5} = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$	M1		3 rows (PI) numerical values only
	$r = 48 \quad \frac{1}{48.50} = \frac{1}{2}\left(\frac{1}{48} - \frac{1}{50}\right)$	A1F		Last row – could be implied
	Cancelling appropriate pairs	M1		
	Sum $= \frac{1}{2}\left(\frac{1}{1} + \frac{1}{2} - \frac{1}{49} - \frac{1}{50}\right)$	A1F		Allow if the $\frac{1}{2}$ is missing only
	$= \frac{894}{1225}$	A1	5	CAO (or equivalent fraction)
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3				
(a)	$\lvert 2+2\text{i}+1+3\text{i}\rvert = \lvert 2+2\text{i}-5-7\text{i}\rvert$	B1		Clearly shown do not allow $\lvert 3+5\text{i}\rvert = \lvert -3-5\text{i}\rvert$ without comment
	$\arg(2+2\text{i}) = \frac{\pi}{4}$	B1	2	Clearly shown
(b)	L_1 : straight line with negative gradient	B1		
	perpendicular to line joining $(-1,-3)$ to $(5,7)$	B1		
	through $(2,2)$	B1		The point (2,2) must be shown either by (2,2) or 2+2i or with numbered axes
	L_2 : half line through O	B1		
	through $(2,2)$	B1	5	
(c)	Shading between $\frac{\pi}{4}$ and $\frac{\pi}{2}$	B1		No marks for shading if circles drawn in (b)
	Below L_1	B1	2	
	Total		**9**	
4(a)	$\alpha+\beta+\gamma=2$	B1	1	
(b)(i)	α is a root and so satisfies the equation	E1	1	
(ii)	$\sum\alpha^3 - 2\sum\alpha^2 + p\sum\alpha + 30 = 0$	M1A1		
	Substitution for $\sum\alpha^3$ and $\sum\alpha$	ml		
	$\sum\alpha^2 = p+13$	A1	4	AG
(iii)	$\left(\sum\alpha\right)^2 = \sum\alpha^2 + 2\sum\alpha\beta$ used	M1		do not allow this M mark if used in (b)(ii)
	$p = -3$	A1	2	AG
(c)(i)	$\text{f}(-2)=0$	M1		
	$\alpha = -2$	A1	2	
(ii)	$(z+2)(z^2-4z+5)=0$	M1		For attempting to find quadratic factor
	$z = \frac{4\pm\sqrt{-4}}{2}$	m1		Use of formula or completing the square m0 if roots are not complex
	$=2\pm\text{i}$	A1	3	CAO
	Total		**13**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	Divide $\cosh^2 t - \sinh^2 t = 1$ by $\cosh^2 t$	M1		Or $\frac{\sinh^2 t}{\cosh^2 t} + \frac{1}{\cosh^2 t}$
	Rearrange	A1	2	AG If solved back to front with no conclusion ending $\cosh^2 t - \sinh^2 t = 1$ B1 only
(ii)	$\frac{d}{dt}\left(\frac{\sinh t}{\cosh t}\right) = \frac{\cosh^2 t - \sinh^2 t}{\cosh^2 t}$	M1A1		
	$= \text{sech}^2 t$	A1	3	AG
(iii)	$\frac{d}{dt}(\text{sech}\, t) = -(\cosh t)^{-2} \sinh t$	M1A1		Allow A1 if negative sign missing
	$= -\text{sech}\, t \tanh t$	A1	3	AG
(b)(i)	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \text{sech}^4 t + \text{sech}^2 t \tanh^2 t$	M1		Allow slips of sign before squaring for this M1
	Use of $\tanh^2 t + \text{sech}^2 t = 1$	m1		Correct formula only for m1
	$= \text{sech}^2 t$	A1		
	$\therefore s = \int_0^{\frac{1}{2}\ln 3} \text{sech}\, t \, dt$	A1	4	AG (including limits)
(ii)	$u = e^t \quad du = e^t\, dt$	B1		
	$\int \text{sech}\, t\, dt = \int \frac{2}{u^2 + 1}\, du$	M1A1		CAO M1 for putting integrand in terms of u (<u>no</u> sech (ln$u$))
	$\left[2\tan^{-1} u\right]$	A1		Or $2\tan^{-1} e^t$
	Change limits correctly or change back to t	m1		At some stage
	$= \frac{2\pi}{3} - \frac{2\pi}{4} = \frac{\pi}{6}$	A1	6	CAO
	Total		**18**	
6(a)	$\frac{1}{(k+2)!} = \frac{k+3}{(k+3)!}$	M1		
	Result	A1	2	
(b)	Assume true for $n = k$ For $n = k + 1$			
	$\sum_{r=1}^{k+1} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{k+1}}{(k+2)!} + \frac{(k+1)2^{k+1}}{(k+3)!}$	M1A1		If no LHS of equation, M1A0
	$= 1 - 2^{k+1}\left(\frac{1}{(k+2)!} - \frac{k+1}{(k+3)!}\right)$	m1		m1 for a suitable combination clearly shown
	$= 1 - \frac{2^{k+2}}{(k+3)!}$	A1		clearly shown or stated true for $n = k + 1$
	True for $n = 1$	B1		Shown
	Method of induction set out properly	E1	6	Provided previous 5 marks all earned
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$1+\sqrt{3}\,\mathrm{i} = 2\mathrm{e}^{\frac{\pi \mathrm{i}}{3}}$	B1		B1 both correct
	$1-\mathrm{i} = \sqrt{2}\,\mathrm{e}^{-\frac{\pi \mathrm{i}}{4}}$	B1B1	3	OE
(ii)	$2^{\frac{21}{2}}$ or equivalent single expression	B1F		No decimals; must include fractional powers
	Raising and adding powers of e	M1		
	$\frac{17\pi}{12}$ or equivalent angle	AIF	3	Denominators of angles must be different
(b)	$z = \sqrt[3]{2^{10}\sqrt{2}}\,\mathrm{e}^{\frac{17\pi \mathrm{i}}{36}+\frac{2k\pi \mathrm{i}}{3}}$	M1		
	$\sqrt[3]{2^{10}\sqrt{2}} = 8\sqrt{2}$	B1		CAO
	$\theta = \frac{17\pi}{36}, -\frac{7\pi}{36}, -\frac{31\pi}{36}$	A2,1F	4	Correct answers outside range: deduct 1 mark only
	Total		**10**	
	TOTAL		**75**	