Version 1.0: 0608

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2008 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

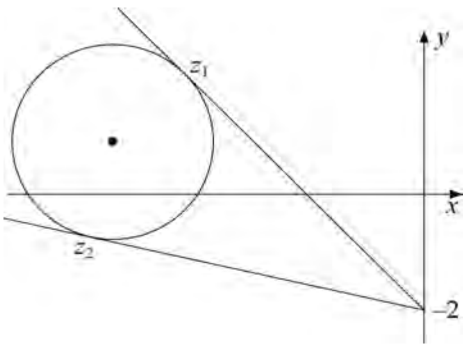
MFP2

Q	Solution	Marks	Total	Comments
1(a)	$5\left(\frac{e^x - e^{-x}}{2}\right)+\left(\frac{e^x + e^{-x}}{2}\right)$	M1		M0 if no 2s in denominator
	$= 3e^x - 2e^{-x}$	A1	2	
(b)	$3e^x - 2e^{-x} + 5 = 0$			
	$3e^{2x} + 5e^x - 2 = 0$	M1		ft if 2s missing in (a)
	$(3e^x - 1)(e^x + 2) = 0$	A1F		
	$e^x \neq -2$	E1		any **indication** of rejection
	$e^x = \frac{1}{3}$ $\quad x = \ln\frac{1}{3}$	A1F	4	provided quadratic factorises into real factors
	Total		**6**	
2(a)	$1 = A(r+2) + Br$	M1		
	$2A = 1, \quad A = \frac{1}{2}$	A1		
	$A + B = 0, \quad B = -\frac{1}{2}$	A1	3	
(b)	$r = 10 \quad \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{11.12}\right)$			if (a) is incorrect but $A = \frac{1}{2}$ and $B = -\frac{1}{2}$ used, allow full marks for (b)
	$r = 11 \quad \frac{1}{2}\left(\frac{1}{11.12} - \frac{1}{12.13}\right)$			
	. . . .			
	$r = 98 \quad \frac{1}{2}\left(\frac{1}{98.99} - \frac{1}{99.100}\right)$	M1A1		3 relevant rows seen
	$S = \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{99.100}\right)$	m1		if split into $\frac{1}{2r} - \frac{1}{r+1} + \frac{1}{2(r+2)}$, follow mark scheme, in which case $\frac{1}{2.10} - \frac{1}{2.11} + \frac{1}{2.100} - \frac{1}{2.99}$ scores m1
	$= \frac{89}{19800}$	A1	4	
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\alpha\beta\gamma = -18 + 12\mathrm{i}$	B1	1	accept $-(18 - 12\mathrm{i})$
(ii)	$\alpha + \beta + \gamma = 0$	B1	1	
(b)(i)	$\alpha = -2$	B1F	1	
(ii)	$\beta\gamma = \dfrac{\alpha\beta\gamma}{\alpha} = 9 - 6\mathrm{i}$	M1 A1F	2	ft sign errors in (a) or (b)(i) or slips such as miscopy
(iii)	$q = \sum \alpha\beta = \alpha(\beta + \gamma) + \beta\gamma$ $= -2 \times 2 + 9 - 6\mathrm{i}$ $= 5 - 6\mathrm{i}$	M1 A1F A1F	 3	 ft incorrect $\beta\gamma$ or α
(c)	$\beta = k\mathrm{i}, \quad \gamma = 2 - k\mathrm{i}$ $k\mathrm{i}(2 - ki) = 9 - 6\mathrm{i}$ $2k = -6 \quad (k^2 = 9) \quad k = -3$ $\beta = -3\mathrm{i}, \quad \gamma = 2 + 3\mathrm{i}$	B1 M1 m1 A1	 4	 imaginary parts
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	radius $\sqrt{2}$ centre $-5+\mathrm{i}$	B1,B1	2	condone $(-5, 1)$ for centre do not accept $(-5, \mathrm{i})$
(b)	$\arg(z_1 + 2\mathrm{i}) = \arg(-4+4\mathrm{i})$	M1		
	$= \frac{3\pi}{4}$	A1	2	clearly shown eg $\tan^{-1}\left(-\frac{1}{1}\right)$
(c)(i)	$\lvert z_1 + 5 - \mathrm{i}\rvert = \lvert 1+\mathrm{i}\rvert = \sqrt{2}$	B1	1	
(ii)	Gradient of line from $(-5, 1)$ to $(-4, 2)$ is 1 $\left(\frac{\pi}{4}\right)$	M1A1		M1 for a complete method
	radius $\perp$ line $\therefore$ tangent	E1	3	
(iii)				
	Circle correct	B1F		ft incorrect centre or radius
	Half line correct	B1	2	line must touch *C* generally above the circle
(d)	z_2 in correct place	B1		B0 if z_2 is directly below the centre of *C*
	with tangent shown	B1	2	
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\left(e^x + e^{-x}\right)^2$ expanded correctly	B1		$e^{2x} + 2e^0 + e^{-2x}$ is acceptable
	Result	B1	2	AG
(b)(i)	$\frac{dy}{dx} = \sinh x$	B1		
	$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\sinh^2 x}$			
	$= \cosh x$	M1		use of $\cosh^2 x - \sinh^2 x = 1$
	$S = 2\pi\int_0^{\ln a} \cosh^2 x \, dx$	A1	3	AG (clearly derived)
(ii)	Use of $\cosh^2 x = \frac{1}{2}(1+\cosh 2x)$	M1		allow one slip in formula M0 if $\int \cosh^2 x \, dx$ is given as $\sinh^2 x$
	$S = \pi\left[x + \frac{1}{2}\sinh 2x\right]_0^{\ln a}$	A1		
	$= \pi\left[\ln a + \frac{1}{2}\left(\frac{e^{2\ln a} - e^{-2\ln a}}{2}\right)\right]$	M1		
	$= \pi\left[\ln a + \frac{1}{4}\left(a^2 - a^{-2}\right)\right]$	A1F		
	$= \pi\left[\ln a + \frac{1}{4a^2}\left(a^4 - 1\right)\right]$	A1	5	AG
	Total		**10**	
6	$u = x - 2$			
	$du = dx$ or $\frac{du}{dx} = 1$	B1		clearly seen
	$32 + 4x - x^2 = 36 - u^2$	B1		if $32 + 4x - x^2$ is written as $36 - (x-2)^2$, give B2
	$\int \frac{du}{\sqrt{36-u^2}} = \sin^{-1}\frac{u}{6}$	M1		allow if dx is used instead of du
	limits −3 and 3 or substitute back to give $\sin^{-1}\frac{x-2}{6}$	A1		
	$I = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$	A1	5	
	Total		**5**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Clear reason given	E1	1	Minimum O × E = E
(b)(i)	$(k+1)\left((k+1)^2+5\right)-k\left(k^2+5\right)$	M1		
	$=3k^2+3k+6$	A1		
	$k^2+k=k(k+1)=M(2)$	E1		**Must** be shown
	$\text{f}(k+1)-\text{f}(k)=M(6)$	E1	4	
(ii)	Assume true for $n = k$			
	$\text{f}(k+1)-\text{f}(k)=M(6)$	M1		Clear method
	$\therefore \text{f}(k+1)=M(6)+\text{f}(k)$			
	$=M(6)+M(6)$	A1		
	$=M(6)$			
	True for $n=1$	B1		
	$P(n)\to P(n+1)$ and $\text{P}(1)$ true	E1	4	Provided all other marks earned in (b)(ii)
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\left(z+\frac{1}{z}\right)\left(z-\frac{1}{z}\right)=z^2-\frac{1}{z^2}$	B1	1	
(ii)	$\left(z^2-\frac{1}{z^2}\right)^2\left(z+\frac{1}{z}\right)^2$			
	$=\left(z^4-2+\frac{1}{z^4}\right)\left(z^2+2+\frac{1}{z^2}\right)$	M1A1		Alternatives for M1A1: $\left(z^4+4z^2+6+\frac{4}{z^2}+\frac{1}{z^4}\right)\left(z^2-2+\frac{1}{z^2}\right)$ or $\left(z^3-\frac{1}{z^3}\right)^2-2\left(z^3-\frac{1}{z^3}\right)\left(z-\frac{1}{z}\right)+\left(z-\frac{1}{z}\right)^2$
	$=z^6+\frac{1}{z^6}+2\left(z^4+\frac{1}{z^4}\right)-\left(z^2+\frac{1}{z^2}\right)-4$	A1	3	CAO (not necessarily in this form)
(b)(i)	$z^n+\frac{1}{z^n}=\cos n\theta+\mathrm{i}\sin n\theta$ $+\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$	M1A1		
	$=2\cos n\theta$	A1	3	AG SC: if solution is incomplete and $(\cos\theta+\mathrm{i}\sin\theta)^{-n}$ is written as $\cos n\theta-\mathrm{i}\sin n\theta$, award M1A0A1
(ii)	$z^n-z^{-n}=2\mathrm{i}\sin n\theta$	B1	1	
(c)	RHS = $2\cos 6\theta+4\cos 4\theta-2\cos 2\theta-4$	M1 A1F		ft incorrect values in (a)(ii) provided they are cosines
	LHS = $-64\cos^4\theta\sin^2\theta$	M1		
	$\cos^4\theta\sin^2\theta$ $=-\frac{1}{32}\cos 6\theta-\frac{1}{16}\cos 4\theta+\frac{1}{32}\cos 2\theta+\frac{1}{16}$	A1	4	
(d)	$-\frac{\sin 6\theta}{192}-\frac{\sin 4\theta}{64}+\frac{\sin 2\theta}{64}+\frac{\theta}{16}\ (+k)$	M1 A1F	2	ft incorrect coefficients but not letters *A*, *B*, *C*, *D*
	Total		**14**	
	TOTAL		**75**	