General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MFP2

Unit Further Pure 2

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) Show that

$$12\cosh x - 4\sinh x = 4e^{x} + 8e^{-x}$$ *(2 marks)*

(b) Solve the equation

$$12\cosh x - 4\sinh x = 33$$

giving your answers in the form $k\ln 2$. *(5 marks)*

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2 Two loci, L_1 and L_2, in an Argand diagram are given by

$$L_1 : |z + 6 - 5i| = 4\sqrt{2}$$

$$L_2 : \quad \arg(z + i) = \frac{3\pi}{4}$$

The point P represents the complex number $-2 + i$.

(a) Verify that the point P is a point of intersection of L_1 and L_2. *(2 marks)*

(b) Sketch L_1 and L_2 on one Argand diagram. *(6 marks)*

(c) The point Q is also a point of intersection of L_1 and L_2. Find the complex number that is represented by Q. *(2 marks)*

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3 (a) Show that $\dfrac{1}{5r-2} - \dfrac{1}{5r+3} = \dfrac{A}{(5r-2)(5r+3)}$, stating the value of the constant A. *(2 marks)*

(b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)}$$ *(4 marks)*

(c) Find the value of

$$\sum_{r=1}^{\infty} \frac{1}{(5r-2)(5r+3)}$$ *(1 mark)*

4 The roots of the equation

$$z^3 - 5z^2 + kz - 4 = 0$$

are α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta\gamma$. *(2 marks)*

(ii) Hence find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. *(2 marks)*

(b) The value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ is -4.

(i) Explain why α, β and γ cannot all be real. *(1 mark)*

(ii) By considering $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, find the possible values of k. *(4 marks)*

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5 (a) Using the definition $\tanh y = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$, show that, for $|x| < 1$,

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$ *(3 marks)*

(b) Hence, or otherwise, show that $\dfrac{d}{dx}(\tanh^{-1} x) = \dfrac{1}{1-x^2}$. *(3 marks)*

(c) Use integration by parts to show that

$$\int_0^{\frac{1}{2}} 4\tanh^{-1} x \, dx = \ln\left(\frac{3^m}{2^n}\right)$$

where m and n are positive integers. *(5 marks)*

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6 A curve is defined parametrically by

$$x = t^3 + 5, \qquad y = 6t^2 - 1$$

The arc length between the points where $t = 0$ and $t = 3$ on the curve is s.

(a) Show that $s = \displaystyle\int_0^3 3t\sqrt{t^2 + A}\, dt$, stating the value of the constant A. *(4 marks)*

(b) Hence show that $s = 61$. *(4 marks)*

Turn over ▶

7 The polynomial $\mathrm{p}(n)$ is given by $\mathrm{p}(n) = (n-1)^3 + n^3 + (n+1)^3$.

(a) (i) Show that $\mathrm{p}(k+1) - \mathrm{p}(k)$, where k is a positive integer, is a multiple of 9. *(3 marks)*

(ii) Prove by induction that $\mathrm{p}(n)$ is a multiple of 9 for all integers $n \geqslant 1$. *(4 marks)*

(b) Using the result from part **(a)(ii)**, show that $n(n^2 + 2)$ is a multiple of 3 for any positive integer n. *(2 marks)*

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8 (a) Express $-4 + 4\sqrt{3}\,\mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(3 marks)*

(b) (i) Solve the equation $z^3 = -4 + 4\sqrt{3}\,\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(4 marks)*

(ii) The roots of the equation $z^3 = -4 + 4\sqrt{3}\,\mathrm{i}$ are represented by the points P, Q and R on an Argand diagram.

Find the area of the triangle PQR, giving your answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(c) By considering the roots of the equation $z^3 = -4 + 4\sqrt{3}\,\mathrm{i}$, show that

$$\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$$

(4 marks)