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General Certificate of Education (A-level)
January 2012

Mathematics MFP2

(Specification 6360)

Further Pure 2

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

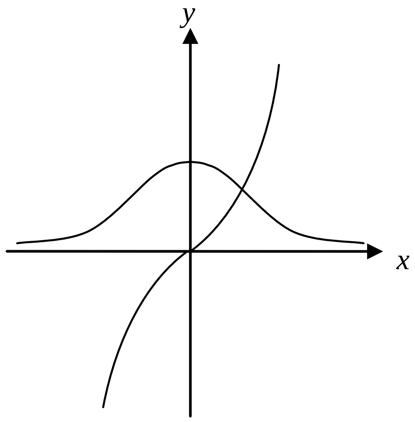
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

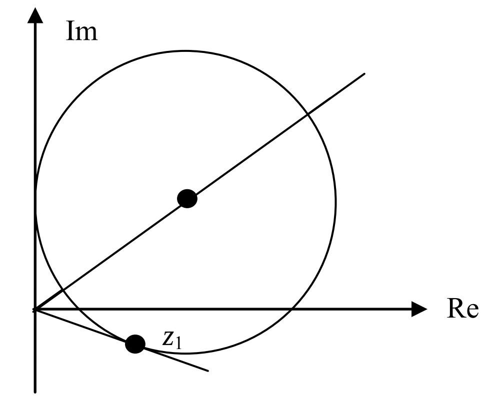
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)				
	Sketch $y = \sinh x$	B1		gradient > 0 at (0, 0); no asymptotes
	Sketch $y = \operatorname{sech} x$:			
	Symmetry about $x = 0$ with max point	B1		
	Asymptote $y = 0$	B1		must not cross x-axis
	Point (0, 1) marked or implied	B1	4	
(b)	$\sinh x = \dfrac{1}{\cosh x}$	M1		
	$\sinh 2x = 2$	M1		use of double angle formula
	Use of ln	m1		dependent on previous M2
	$x = \dfrac{1}{2}\ln\left(2+\sqrt{5}\right)$	A1	4	
	or			
	$\dfrac{1}{2}\left(e^{2x} - e^{-2x}\right) = 2$ OE	(M1)		incorrect $\sinh x$, $\cosh x$ M0 (no marks)
	$e^{4x} - 4e^{2x} - 1 = 0$	(M1)		ie multiply by e^{2x} and rewrite
	Correct use of formula	(m1)		
	Result	(A1)	(4)	
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)				
	Half-line with gradient < 1	B1	1	condone a short line, ie it stops at or inside circle
(b)(i)	Circle centre on L, x-coord 6 indicated touching $\text{Re } z = 0$ not at (0, 0)	B1 B1	 2	 not touching Re axis
(ii)	y-coord of centre is $2\sqrt{3}$ or $\dfrac{6}{\sqrt{3}}$	B1		OE; PI
	$z_0 = 6 + 2\sqrt{3}\,\mathrm{i}$, $k = 6$	B1F, B1	 3	ft error in coords of centre
(iii)	Point z_1 shown	B1		PI
	$\arg z_1 = -\dfrac{1}{6}$	B1	2	
	Total		**8**	
3(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\tanh x}$	B1		
	$\times\, \mathrm{sech}^2 x$	B1		
	$= \dfrac{1}{2\sinh x \cosh x}$	M1		for expressing in terms of $\sinh x$ and $\cosh x$
	$= \dfrac{1}{\sinh 2x}$	A1	4	AG; PI by previous line
(b)	$\sqrt{1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2} = \sqrt{1 + \dfrac{1}{\sinh^2 2x}}$	M1		use of formula; accept √ inserted at any stage
	$= \sqrt{\dfrac{\cosh^2 2x}{\sinh^2 2x}}$	m1		relevant use of $\cosh^2 - \sinh^2 = 1$
	$= \dfrac{\cosh 2x}{\sinh 2x}$	A1		OE
	Integral is $\dfrac{1}{2}\ln \sinh 2x$	M1A1		M1 for ln sinh
	$\sinh(2\ln 4) = \dfrac{255}{32}$ $\quad\sinh(2\ln 2) = \dfrac{15}{8}$	B1B1		PI
	$s = \dfrac{1}{2}\ln\left(\dfrac{17}{4}\right)$	A1F	8	ft error in $\dfrac{1}{2}$
	Total		**12**	

Q	Solution	Marks	Total	Comments
4	Assume result true for $n = k$			
	Then $u_{k+1} = \dfrac{3}{4 - \left(\dfrac{3^{k+1} - 3}{3^{k+1} - 1}\right)}$	M1		
	$= \dfrac{3(3^{k+1} - 1)}{4(3^{k+1} - 1) - (3^{k+1} - 3)}$	A1		
	$4 \times 3^{k+1} - 3^{k+1} = 3^{k+2}$	A1		clearly shown
	$u_{k+1} = \dfrac{3^{k+2} - 3}{3^{k+2} - 1}$	A1		
	$n = 1 \quad \dfrac{3^2 - 3}{3^2 - 1} = \dfrac{3}{4} = u_1$	B1		
	Induction proof set out properly	E1	6	must have earned previous 5 marks
	Total		**6**	
5	Numerator $= \mathrm{e}^{\frac{p\pi \mathrm{i}}{8}}$	B1		
	Denominator $= \mathrm{e}^{\frac{-q\pi \mathrm{i}}{12}}$	B1		
	Fraction $= \mathrm{e}^{\frac{p\pi \mathrm{i}}{8} + \frac{q\pi \mathrm{i}}{12}}$	M1		allow for attempt to subtract powers
	$= \mathrm{e}^{\frac{\pi \mathrm{i}}{24}(3p+2q)}$	A1		
	$\mathrm{i} = \mathrm{e}^{\frac{12\pi \mathrm{i}}{24}}$	m1		OE
	$3p + 2q = 12$	A1F		ft errors of sign or arithmetic slips
	$p = 2,\ q = 3$	A1	7	CAO
	Alternative 1			
	Numerator $= \cos\frac{p\pi}{8} + \mathrm{i}\sin\frac{p\pi}{8}$	(B1)		
	Denominator $= \cos\frac{-q\pi}{12} + \mathrm{i}\sin\frac{-q\pi}{12}$	(B1)		needs more than just $\cos\frac{q\pi}{12} - \sin\frac{p\pi}{12}$
	Fraction = $\left(\cos\frac{p\pi}{8} + \mathrm{i}\sin\frac{p\pi}{8}\right)\left(\cos\frac{q\pi}{12} + \mathrm{i}\sin\frac{q\pi}{12}\right)$	(M1)		
	$= \cos\frac{\pi}{24}(3p+2q) + \mathrm{i}\sin\frac{\pi}{24}(3p+2q)$	(A1)		
	$= \mathrm{i}$ if $\cos\frac{\pi}{24}(3p+2q) = 0$ or $\sin\frac{\pi}{24}(3p+2q) = 1$	(m1)		
	$3p + 2q = 12$	(A1F)		
	$p = 2,\ q = 3$	(A1)	(7)	CAO
	Alternative 2			
	LHS $\cos\frac{p\pi}{8} + \mathrm{i}\sin\frac{p\pi}{8}$	(B1)		
	RHS $\mathrm{i}\cos\frac{q\pi}{12} + \sin\frac{q\pi}{12}$	(B1)		
	$\cos\frac{p\pi}{8} = \sin\frac{q\pi}{12}$ or $\sin\frac{p\pi}{8} = \cos\frac{q\pi}{12}$	(M1)		
	Introduction of $\dfrac{\pi}{2}$	(m1)		
	$\dfrac{p\pi}{8} = \dfrac{\pi}{2} - \dfrac{q\pi}{12}$	(A1)		
	$3p + 2q = 12$	(A1F)		
	$p = 2,\ q = 3$	(A1)	(7)	CAO (correct answers, insufficient working 3/7 only)
	Total		**7**	

Q	Solution	Marks	Total	Comments
6(a)	$7+4x-2x^2=9-2(x-1)^2$	M1A1	2	
(b)	Put $u=\sqrt{2}(x-1)$	M1		allow $u=k(x-1)$ any k
	$\mathrm{d}u=\sqrt{2}\,\mathrm{d}x$	A1F		
	$\mathrm{I}=\frac{1}{\sqrt{2}}\int\frac{\mathrm{d}u}{\sqrt{9-u^2}}$	A1F		ft error in (a); must have u^2 only, ie $\frac{1}{\sqrt{2}}$ outside integrand
	$=\frac{1}{\sqrt{2}}\sin^{-1}\frac{u}{3}$	A1		for $\sin^{-1}\frac{u}{p}$
	Change limits or replace u	m1		provided $\sin^{-1}$
	$=\frac{\pi}{4\sqrt{2}}$ or $\frac{\pi\sqrt{2}}{8}$	A1	6	CAO
	Alternative – if integration is attempted without substitution:			
	$\sin^{-1}$	(M1)		
	$\frac{1}{\sqrt{2}}$	(A1F)		
	$(x-1)$	(A1)		
	$\frac{\sqrt{2}}{3}$	(A1F)		
	Substitution of limits	(m1)		
	$\frac{\pi}{4\sqrt{2}}$	(A1)	(6)	CAO
	Total		**8**	
7(a)	Use of $\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum\alpha\beta$	M1		
		A1	2	AG
(b)	$p=0,\ \ q=5+6\mathrm{i}$	B1,B1	2	
(c)(i)	Substitute $3\mathrm{i}$ for z **or** use $3\mathrm{i}\beta\gamma=-r$	M1		allow for $3\mathrm{i}\beta\gamma=r$
	$-27\mathrm{i}+15\mathrm{i}-18+r=0$ **or** $\beta\gamma=5+6\mathrm{i}+\alpha^2$	A1		any form
	$r=18+12\mathrm{i}$	A1F	3	one error
(ii)	Cubic is $(z-3\mathrm{i})(z^2+3\mathrm{i}z-4+6\mathrm{i})$ or use of $\beta\gamma$ and $\beta+\gamma$	M1A1	2	clearly shown
(iii)	$\mathrm{f}(-2)=0$ or equate imaginary parts	M1		
	$\beta=-2,\ \gamma=2-3\mathrm{i}$	A1,A1F	3	correct answers no working and no check B1 only
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	$1, \mathrm{e}^{\frac{2\pi i}{5}}, \mathrm{e}^{\frac{4\pi i}{5}}, \mathrm{e}^{\frac{-2\pi i}{5}}, \mathrm{e}^{\frac{-4\pi i}{5}}$	B1	1	accept e^0
(b)	$\frac{z^5-1}{z-1} = z^4+z^3+z^2+z+1$	B1		B0 if assumed
	$=\left(z-\mathrm{e}^{\frac{2\pi i}{5}}\right)\left(z-\mathrm{e}^{\frac{4\pi i}{5}}\right)\left(z-\mathrm{e}^{\frac{-2\pi i}{5}}\right)\left(z-\mathrm{e}^{\frac{-4\pi i}{5}}\right)$	M1A1	3	accept if $\mathrm{e}^{\frac{6\pi i}{5}}, \mathrm{e}^{\frac{8\pi i}{5}}$ used here
(c)	Correct grouping of linear factors	M1		
	$\mathrm{e}^{\frac{2\pi i}{5}} + \mathrm{e}^{\frac{-2\pi i}{5}} = 2\cos\frac{2\pi}{5}$	A1		clearly shown
	$\left(z^2-2\cos\frac{2\pi}{5}z+1\right)\left(z^2-2\cos\frac{4\pi}{5}z+1\right)$	A1		
	$\div z^2$ to give answer	A1	4	AG
(d)	Substitute into LHS to give w^2+w-1	B1		
	RHS $\left(w-2\cos\frac{2\pi}{5}\right)\left(w-2\cos\frac{4\pi}{5}\right)$	B1		
	Solve $w^2+w-1=0$	M1		
	$w=\frac{-1\pm\sqrt{5}}{2}$	A1		
	$\cos\frac{2\pi}{5}=\frac{\sqrt{5}-1}{4}$	A1		
	with reasons for choice	E1	6	
	Total		**14**	
	TOTAL		**75**	