General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MFP2

Unit Further Pure 2

Wednesday 19 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 (a) Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 4 + 3\mathrm{i}| = 5$$ *(3 marks)*

(b) (i) Indicate on your diagram the point P representing z_1, where both

$$|z_1 - 4 + 3\mathrm{i}| = 5 \quad \text{and} \quad \arg z_1 = 0$$ *(1 mark)*

(ii) Find the value of $|z_1|$. *(1 mark)*

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2 (a) Given that

$$u_r = \frac{1}{6}r(r+1)(4r+11)$$

show that

$$u_r - u_{r-1} = r(2r+3)$$ *(3 marks)*

(b) Hence find the sum of the first hundred terms of the series

$$1 \times 5 + 2 \times 7 + 3 \times 9 + \ldots + r(2r+3) + \ldots$$ *(3 marks)*

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3 (a) Show that $(1+\mathrm{i})^3 = 2\mathrm{i} - 2$. *(2 marks)*

(b) The cubic equation

$$z^3 - (5+\mathrm{i})z^2 + (9+4\mathrm{i})z + k(1+\mathrm{i}) = 0$$

where k is a real constant, has roots α, β and γ.

It is given that $\alpha = 1 + \mathrm{i}$.

(i) Find the value of k. *(3 marks)*

(ii) Show that $\beta + \gamma = 4$. *(1 mark)*

(iii) Find the values of β and γ. *(5 marks)*

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4 (a) Prove that the curve

$$y = 12\cosh x - 8\sinh x - x$$

has exactly one stationary point. *(7 marks)*

(b) Given that the coordinates of this stationary point are (a, b), show that $a + b = 9$. *(4 marks)*

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5 (a) Given that $u = \sqrt{1 - x^2}$, find $\dfrac{\mathrm{d}u}{\mathrm{d}x}$. *(2 marks)*

(b) Use integration by parts to show that

$$\int_0^{\frac{\sqrt{3}}{2}} \sin^{-1} x \, \mathrm{d}x = a\sqrt{3}\,\pi + b$$

where a and b are rational numbers. *(6 marks)*

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6 (a) Given that

$$x = \ln(\sec t + \tan t) - \sin t$$

show that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sin t \tan t$$ *(4 marks)*

(b) A curve is given parametrically by the equations

$$x = \ln(\sec t + \tan t) - \sin t, \quad y = \cos t$$

The length of the arc of the curve between the points where $t = 0$ and $t = \dfrac{\pi}{3}$ is denoted by s.

Show that $s = \ln p$, where p is an integer. *(6 marks)*

Turn over ▶

7 (a) Given that

$$f(k) = 12^k + 2 \times 5^{k-1}$$

show that

$$f(k+1) - 5f(k) = a \times 12^k$$

where a is an integer. *(3 marks)*

(b) Prove by induction that $12^n + 2 \times 5^{n-1}$ is divisible by 7 for all integers $n \geqslant 1$. *(4 marks)*

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8 (a) Express in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$:

(i) $4(1 + i\sqrt{3})$;

(ii) $4(1 - i\sqrt{3})$. *(3 marks)*

(b) The complex number z satisfies the equation

$$(z^3 - 4)^2 = -48$$

Show that $z^3 = 4 \pm 4\sqrt{3}\,i$. *(2 marks)*

(c) (i) Solve the equation

$$(z^3 - 4)^2 = -48$$

giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(5 marks)*

(ii) Illustrate the roots on an Argand diagram. *(3 marks)*

(d) (i) Explain why the sum of the roots of the equation

$$(z^3 - 4)^2 = -48$$

is zero. *(1 mark)*

(ii) Deduce that $\cos\frac{\pi}{9} + \cos\frac{3\pi}{9} + \cos\frac{5\pi}{9} + \cos\frac{7\pi}{9} = \frac{1}{2}$. *(3 marks)*