General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS **MFP2**
Unit Further Pure 2

Thursday 1 February 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Given that

$$4\cosh^2 x = 7\sinh x + 1$$

find the two possible values of $\sinh x$. *(4 marks)*

(b) Hence obtain the two possible values of x, giving your answers in the form $\ln p$. *(3 marks)*

2 (a) Sketch on one diagram:

(i) the locus of points satisfying $|z - 4 + 2\mathrm{i}| = 2$; *(3 marks)*

(ii) the locus of points satisfying $|z| = |z - 3 - 2\mathrm{i}|$. *(3 marks)*

(b) Shade on your sketch the region in which

both $|z - 4 + 2\mathrm{i}| \leqslant 2$

and $|z| \leqslant |z - 3 - 2\mathrm{i}|$ *(2 marks)*

3 The cubic equation

$$z^3 + 2(1 - \mathrm{i})z^2 + 32(1 + \mathrm{i}) = 0$$

has roots α, β and γ.

(a) It is given that α is of the form $k\mathrm{i}$, where k is real. By substituting $z = k\mathrm{i}$ into the equation, show that $k = 4$. *(5 marks)*

(b) Given that $\beta = -4$, find the value of γ. *(2 marks)*

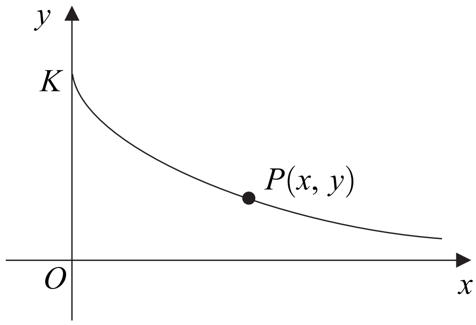
4 (a) Given that $y = \operatorname{sech} t$, show that:

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\operatorname{sech} t \tanh t$; *(3 marks)*

(ii) $\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \operatorname{sech}^2 t - \operatorname{sech}^4 t$. *(2 marks)*

(b) The diagram shows a sketch of part of the curve given parametrically by

$$x = t - \tanh t \qquad y = \operatorname{sech} t$$



The curve meets the y-axis at the point K, and $P(x, y)$ is a general point on the curve. The arc length KP is denoted by s. Show that:

(i) $\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \tanh^2 t$; *(4 marks)*

(ii) $s = \ln \cosh t$; *(3 marks)*

(iii) $y = \mathrm{e}^{-s}$. *(2 marks)*

(c) The arc KP is rotated through 2π radians about the x-axis. Show that the surface area generated is

$$2\pi(1 - \mathrm{e}^{-s})$$ *(4 marks)*

Turn over for the next question

5 (a) Prove by induction that, if n is a positive integer,

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$ *(5 marks)*

(b) Find the value of $\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6$. *(2 marks)*

(c) Show that

$$(\cos\theta + \mathrm{i}\sin\theta)(1 + \cos\theta - \mathrm{i}\sin\theta) = 1 + \cos\theta + \mathrm{i}\sin\theta$$ *(3 marks)*

(d) Hence show that

$$\left(1 + \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6 + \left(1 + \cos\frac{\pi}{6} - \mathrm{i}\sin\frac{\pi}{6}\right)^6 = 0$$ *(4 marks)*

6 (a) Find the three roots of $z^3 = 1$, giving the non-real roots in the form $\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \leqslant \pi$. *(2 marks)*

(b) Given that ω is one of the non-real roots of $z^3 = 1$, show that

$$1 + \omega + \omega^2 = 0$$ *(2 marks)*

(c) By using the result in part (b), or otherwise, show that:

(i) $\dfrac{\omega}{\omega + 1} = -\dfrac{1}{\omega}$; *(2 marks)*

(ii) $\dfrac{\omega^2}{\omega^2 + 1} = -\omega$; *(1 mark)*

(iii) $\left(\dfrac{\omega}{\omega + 1}\right)^k + \left(\dfrac{\omega^2}{\omega^2 + 1}\right)^k = (-1)^k\, 2\cos\frac{2}{3}k\pi$, where k is an integer. *(5 marks)*

7 (a) Use the identity $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ with $A = (r + 1)x$ and $B = rx$ to show that

$$\tan rx \tan(r+1)x = \frac{\tan(r+1)x}{\tan x} - \frac{\tan rx}{\tan x} - 1$$ *(4 marks)*

(b) Use the method of differences to show that

$$\tan\frac{\pi}{50}\tan\frac{2\pi}{50} + \tan\frac{2\pi}{50}\tan\frac{3\pi}{50} + \ldots + \tan\frac{19\pi}{50}\tan\frac{20\pi}{50} = \frac{\tan\frac{2\pi}{5}}{\tan\frac{\pi}{50}} - 20$$ *(5 marks)*

END OF QUESTIONS

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