AQA Maths Further Pure 2
Mark Scheme Pack
2006-2015

Version 1.0: 0106

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2006 examination - January series

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

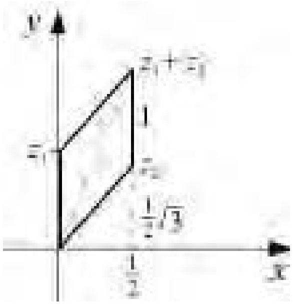
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	$\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{(r+1)^2 - r^2}{r^2(r+1)^2}$	M1		
	$= \frac{2r+1}{r^2(r+1)^2}$	A1	2	AG
(b)	$\frac{3}{1^2 \times 2^2} = \frac{1}{1^2} - \frac{1}{2^2}$			
	$\frac{5}{2^2 \times 3^2} = \frac{1}{2^2} - \frac{1}{3^2}$			
	$\frac{7}{3^2 \times 4^2} = \frac{1}{3^2} - \frac{1}{4^2}$	M1A1		A1 for at least 3 lines
	$\frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$			
	Clear cancellation	M1		
	$1 - \frac{1}{(n+1)^2}$	A1F	4	
	Total		**6**	
2(a)	$p = -4$	B1		
	$(\alpha+\beta+\gamma)^2 = \sum\alpha^2 + 2\sum\alpha\beta$	M1		
	$16 = 20 + 2\sum\alpha\beta$	A1		
	$\sum\alpha\beta = -2$	A1F		
	$q = -2$	A1F	5	
(b)	$3 - \mathrm{i}$ is a root	B1		
	Third root is -2	B1F		
	$\alpha\beta\gamma = (3+\mathrm{i})(3-\mathrm{i})(-2)$	M1		
	$= -20$	A1F		Real $\alpha\beta\gamma$
	$r = +20$	A1F	5	Real r
	Alternative to (b)			
	Substitute 3 + i into equation	M1		
	$(3+\mathrm{i})^2 = 8 + 6\mathrm{i}$	B1		
	$(3+\mathrm{i})^3 = 18 + 26\mathrm{i}$	B1		
	$r = 20$	A2,1,0		Provided r is real
	Total		**10**	

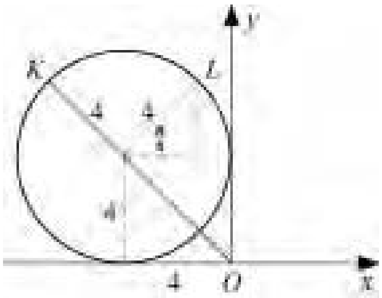
MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{1+\mathrm{i}}{1-\mathrm{i}} = \frac{(1+\mathrm{i})^2}{1-\mathrm{i}^2} = \mathrm{i}$	M1A1	2	AG
(b)	$\lvert z_2 \rvert = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1 = \lvert z_1 \rvert$	M1A1	2	
(c)	$r = 1$	B1		PI
	$\theta = \frac{1}{2}\pi,\ \frac{1}{3}\pi$	B1B1	3	Deduct 1 mark if extra solutions
(d)		B2,1F	2	Positions of the 3 points relative to each other, must be approximately correct
(e)	$\operatorname{Arg}(z_1+z_2) = \frac{5}{12}\pi$	B1		**Clearly** shown
	$\tan\frac{5}{12}\pi = \frac{1+\frac{1}{2}\sqrt{3}}{\frac{1}{2}}$	M1		Allow if BO earned
	$= 2+\sqrt{3}$	A1	3	AG must earn BO for this
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Assume result true for $n = k$			
	$\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k$			
	$\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+2)2^k$	M1A1		
	$= 2^k(k+k+2)$	m1		
	$= 2^k(2k+2)$			
	$= 2^{k+1}(k+1)$	A1		
	$n = 1 \quad 2\times 2^0 = 2 = 1\times 2^1$	B1		
	$P_k \Rightarrow P_{k+1}$ and P_1 is true	E1	6	Provided previous 5 marks earned
(b)	$\sum_{r=1}^{2n}(r+1)2^{r-1} - \sum_{r=1}^{n}(r+1)2^{r-1}$	M1		Sensible attempt at the difference between 2 series
	$= 2n\ 2^{2n} - n2^n$	A1		
	$= n\left(2^{n+1}-1\right)2^n$	A1	3	AG
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)		B1 B1 B1	 3	Circle Correct centre Touching both axes
(b)	$\lvert z\rvert \max = OK$	M1		Accept $\sqrt{4^2+4^2}+4$ as a method
	$=\sqrt{4^2+4^2}+4$	A1F		Follow through circle in incorrect position
	$=4\left(\sqrt{2}+1\right)$	A1F	3	AG
(c)	Correct position of z , ie L	M1		
	$a=-\left(4-4\cos\frac{1}{6}\pi\right)$			
	$=-\left(4-2\sqrt{3}\right)$	A1F		Follow through circle in incorrect position
	$b=4+4\sin\frac{1}{6}\pi=6$	A1F	3	
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$z+\frac{1}{z}=\cos\theta+\mathrm{i}\sin\theta+$ $\cos(-\theta)+\mathrm{i}\sin(-\theta)$	M1		Or $z+\frac{1}{z}=\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}$
	$=2\cos\theta$	A1	2	AG
(ii)	$z^2+\frac{1}{z^2}=\cos 2\theta+i\sin 2\theta$ $+\cos(-2\theta)+\mathrm{i}\sin(-2\theta)$	M1		
	$=2\cos 2\theta$	A1	2	OE
(iii)	$z^2-z+2-\frac{1}{z}+\frac{1}{z^2}$ $=2\cos 2\theta-2\cos\theta+2$	M1		
	Use of $\cos 2\theta=2\cos^2\theta-1$	m1		
	$=4\cos^2\theta-2\cos\theta$	A1	3	AG
(b)	$z+\frac{1}{z}=0$ $\quad z=\pm\mathrm{i}$	M1A1		**Alternative:**
	$z+\frac{1}{z}=1$ $\quad z^2-z+1=0$	M1A1		$\cos\theta=0$ $\quad \theta=\pm\frac{1}{2}\pi$ M1 $z=\pm\mathrm{i}$ A1
	$z=\frac{1\pm\mathrm{i}\sqrt{3}}{2}$ Accept solution to (b) if done otherwise	A1F	5	$\cos\theta=\frac{1}{2}$ $\quad \theta=\pm\frac{1}{3}\pi$ M1 $z=\mathrm{e}^{\pm\frac{1}{3}\pi\mathrm{i}}=\frac{1}{2}\left(1\pm\mathrm{i}\sqrt{3}\right)$ A1 A1
	Alternative If $\theta=+\frac{1}{2}\pi$ $\quad \theta=\frac{1}{3}\pi$	M1		
	$z=\mathrm{i}$ $\quad z=\frac{1+\sqrt{3}\,\mathrm{i}}{2}$	A1		
	Or any correct z values of θ	M1		
	Any 2 correct answers	A1		
	One correct answer only	B1		
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$2\left(\dfrac{e^{\theta}-e^{-\theta}}{2}\right)\left(\dfrac{e^{\theta}+e^{-\theta}}{2}\right)$			
	$=\dfrac{e^{2\theta}-e^{-2\theta}}{2}=\sinh 2\theta$	M1A1	2	AG
(ii)	$\left(\dfrac{e^{\theta}-e^{-\theta}}{2}\right)^2+\left(\dfrac{e^{\theta}+e^{-\theta}}{2}\right)^2$	M1		
	$=\dfrac{e^{2\theta}-2+e^{-2\theta}+e^{2\theta}+2+e^{-2\theta}}{4}$	A1		
	$=\cosh 2\theta$	A1	3	AG
(b)(i)	$\dot{x}=3\cosh^2\theta\sinh\theta''$	M1A1		Allow M1 for **reasonable** attempt at differentiation, but M0 for putting in terms of $e^{k\theta}$ or $\sinh 3\theta$ unless real progress made towards $\dot{x}^2+\dot{y}^2$
	$\dot{y}=3\sinh^2\theta\cosh\theta$	A1		Allow this M1 if not squared out, must be clear sum in question is $\dot{x}^2+\dot{y}^2$
	$\dot{x}^2+\dot{y}^2=9\cosh^4\theta\sinh^2\theta$ $+9\sinh^4\theta\cosh^2\theta$	M1		
	$=9\sinh^2\theta\cosh^2\theta\left(\cosh^2\theta+\sinh^2\theta\right)$	A1		AG Accept $\int_0^1\sqrt{\dfrac{9}{4}\sinh^2 2\theta\cosh 2\theta}\ \mathrm{d}\theta$ but limits must appear somewhere
	$=\dfrac{9}{4}\sinh^2 2\theta\cosh 2\theta$	A1	6	AG
(ii)	$S=\int_0^1\dfrac{3}{2}\sinh 2\theta\sqrt{\cosh 2\theta}\,\mathrm{d}\theta$	M1		
	$u=\cosh 2\theta \qquad \mathrm{d}u=2\sinh 2\theta\,\mathrm{d}\theta$	M1A1		
	$I=\int\dfrac{3}{4}u^{\frac{1}{2}}\,\mathrm{d}u=\dfrac{3}{4}\times\dfrac{2}{3}u^{\frac{3}{2}}$	A1F		
	$S=\left\{\dfrac{1}{2}(\cosh 2\theta)^{\frac{3}{2}}\right\}_0^1$	A1F		
	$=\dfrac{1}{2}\left\{(\cosh 2)^{\frac{3}{2}}-1\right\}$	A1	**6**	AG
	Total		**17**	
	TOTAL		**75**	

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1(a)	$r^2+r-1=A\left(r^2+r\right)+B$	M1		Any correct method
	$A=1,\ B=-1$	A1		
		A1F	3	ft B if incorrect A and vice versa
				Or $\dfrac{r^2+r-1}{r^2+r}=1-\dfrac{1}{r(r+1)}$ B1
				$=1-\left(\dfrac{1}{r}-\dfrac{1}{r+1}\right)$ M1A1
(b)	$r=1 \quad 1-\frac{1}{1}+\frac{1}{2}$			
	$r=2 \quad 1-\frac{1}{2}+\frac{1}{3}$	M1		Do not allow M1 if merely $\sum\frac{1}{r}-\sum\frac{1}{r+1}$ is summed
	$r=99 \quad 1-\frac{1}{99}+\frac{1}{100}$	A1F		A1 for suitable (3 at least) number of rows
	Sum = $98+\dfrac{1}{100}$	m1		Must have 98 or 99
	$=98.01$	A1F	4	OE Allow correct answer with no working 4 marks
	Total		**7**	
2(a)	$\dot{x}=1-t^2,\ \dot{y}=2t$	B1		
	$\dot{x}^2+\dot{y}^2=\left(1-t^2\right)^2+4t^2$	M1		
	$=\left(1+t^2\right)^2$	A1	3	AG; must be intermediate line
(b)	$S=2\pi\int_1^2\left(1+t^2\right)t^2\,\mathrm{d}t$	M1A1		Must be correct substitutions for M1
	$=2\pi\left[\dfrac{t^3}{3}+\dfrac{t^5}{5}\right]_1^2$	m1		Allow if one term integrated correctly
	$=2\pi\left[\dfrac{8}{3}+\dfrac{32}{5}-\dfrac{1}{3}-\dfrac{1}{5}\right]$	A1F		Any form
	$=\dfrac{256\pi}{15}$	A1F	5	
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\dfrac{e^k + e^{-k}}{2} - \dfrac{3\left(e^k - e^{-k}\right)}{2} = -1$	M1		Allow if 2's are missing or if cosh*x* and sinh*x* interchanged
	$-2e^k + 4e^{-k} = -2$	A1		
	$e^{2k} - e^k - 2 = 0$	A1	3	AG Condone *x* instead of *k*
(ii)	$\left(e^k + 1\right)\left(e^k - 2\right) = 0$	M1		
	$e^k \neq -1$	E1		Must state something to earn E1. Do not accept ignoring or crossing out.
	$e^k = 2$	A1		
	$k = \ln 2$	A1F	4	
(b)(i)	$\cosh x = 3\sinh x$ or in terms of e^x	M1		
	$\tanh x = \dfrac{1}{3}$ or $2e^x = 4e^{-x}$	A1		
	$x = \dfrac{1}{2}\ln\left(\dfrac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)$ or $e^{2x} = 2$	A1F		
	$x = \dfrac{1}{2}\ln 2$	A1	4	CAO
(ii)	$\dfrac{dy}{dx} = \sinh x - 3\cosh x$ or $-e^x - 2e^{-x}$	M1		
	$= 0$ when $\tanh x = 3$ or $e^{2x} = -2$	A1		
	Correct reason	E1	3	Must give a reason
(iii)	$\dfrac{d^2y}{dx^2} = y = 0$ at $\left(\frac{1}{2}\ln 2, 0\right)$ ie one point	B1F	1	
	Total		**15**	

MFP2 (cont)

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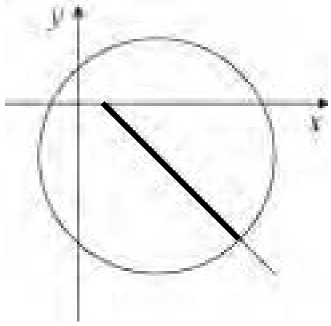
4				
(a)(i)	Circle	B1		
	Correct centre	B1		
	Enclosing the origin	B1	3	
(ii)	Half line	B1		
	Correct starting point	B1		
	Correct angle	B1	3	
(b)	Correct part of the line **indicated**	B1F	1	
	Total		**7**	
5(a)(i)	$\alpha+\beta+\gamma=4\text{i}$	B1	1	
(ii)	$\alpha\beta\gamma=4-2\text{i}$	B1	1	
(b)(i)	$\alpha+\alpha=4\text{i},\ \alpha=2\text{i}$	B1	1	AG
(ii)	$\beta\gamma=\dfrac{4-2\text{i}}{2\text{i}}\ =-2\text{i}-1$	M1		Some method must be shown, eg $\dfrac{2}{\text{i}}-1$
		A1	2	AG
(iii)	$q=\alpha\beta+\beta\gamma+\gamma\alpha$	M1		
	$=\alpha(\beta+\gamma)+\beta\gamma$	M1		Or $\alpha^2+\beta\gamma$, ie suitable grouping
	$=2\text{i}.2\text{i}-2\text{i}-1=-2\text{i}-5$	A1	3	AG
(c)	Use of $\beta+\gamma=2\text{i}$ and $\beta\gamma=-2\text{i}-1$	M1		Elimination of say γ to arrive at
	$z^2-2\text{i}z-(1+2\text{i})=0$	A1	2	$\beta^2-2\text{i}\beta-(1+2\text{i})=0$ M1A0 unless also some reference to γ being a root AG
(d)	$\text{f}(-1)=1+2\text{i}-1-2\text{i}=0$	M1		For any correct method
	$\beta=-1,\quad \gamma=1+2\text{i}$	A1A1	3	A1 for each answer
	Total		**13**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{f}(n+1)-8\mathrm{f}(n)=15^{n+1}-8^{n-1}$			
	$-8(15^{n}-8^{n-2})$	M1A1		
	$=15^{n+1}-8.15^{n}$			
	$=15^{n}(15-8)$	M1		For multiples of powers of 15 only
	$=7.15^{n}$	A1	4	For valid method ie not using 120^{n} etc
(b)	Assume $\mathrm{f}(n)$ is $\mathrm{M}(7)$			
	Then $\mathrm{f}(n+1)-8\mathrm{f}(n)=7\times15^{n}$	M1		Or considering $\mathrm{f}(n+1)-\mathrm{f}(n)$
	$\mathrm{f}(n+1)=\mathrm{M}(7)+\mathrm{M}(7)$			
	$=\mathrm{M}(7)$	A1		
	$n=2$: $\mathrm{f}(n)=15^{2}-8^{0}=224$			
	$=7\times32$	B1		$n=1$ B0
	$\mathrm{P}(n)\Rightarrow\mathrm{P}(n+1)$ and $\mathrm{P}(2)$ true	E1	4	Must score previous 3 marks to be awarded E1
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$z = e^{\frac{2k\pi i}{6}}$, $k = 0, \pm 1, \pm 2, 3$	M1 A2,1,0	3	OE M1A1 only if: (1) range for k is incorrect eg 0,1,2,3,4,5 (2) i is missing
(b)(i)	$\frac{w^2-1}{w} = w - \frac{1}{w} = 2i\sin\theta$	M1A1	2	AG
(ii)	$\frac{w}{w^2-1} = \frac{1}{2i\sin\theta}$	M1		
	$= -\frac{i}{2\sin\theta}$	A1	2	AG
(iii)	$\frac{2i}{w^2-1} = \frac{-2iw^{-1}i}{2\sin\theta}$	M1		Or for $\frac{1}{\sin\theta\, e^{i\theta}}$
	$= \frac{1}{\sin\theta}(\cos\theta - i\sin\theta)$	A1		
	$= \cot\theta - i$	A1	3	AG
(iv)	$z = \frac{2i}{w^2-1}$ Or $z + 2i = \frac{2i}{w^2-1} + 2i$	M1		ie any correct method
	$z + 2i = zw^2$	A1	2	AG
(c)(i)	No coefficient of z^6	E1	1	
(ii)	$\left(w^2\right)^6 = 1$ $w^2 = e^{\frac{k\pi i}{3}}$	B1		**Alternatively:**
	$z = \cot\frac{k\pi}{6} - i$, $k = \pm 1, \pm 2, 3$	M1 A2,1,0	4	$z + 2i = e^{\frac{k\pi i}{3}} z$ B1 $z = \frac{2i}{e^{\frac{k\pi i}{3}} - 1}$ M1 roots A2,1,0 (NB roots are $\pm\sqrt{3} - i; \pm\frac{1}{\sqrt{3}} - i; -i$)
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No Method Shown

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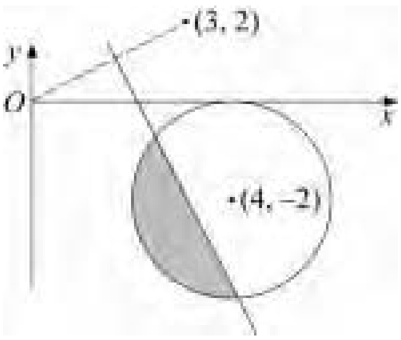
Otherwise we require evidence of a correct method for any marks to be awarded.

Jan 07

MFP2

Q	Solution	Marks	Total	Comments
1(a)	Use of $\cosh^2 x = 1 + \sinh^2 x$	M1		Must be correct for M1
	$4\sinh^2 x - 7\sinh x + 3 = 0$	A1		
	$(4\sinh x - 3)(\sinh x - 1) = 0$	A1√		Provided quadratic factorizes
	$\sinh x = \frac{3}{4}$ or 1	A1√	4	
(b)	Use of formula for $\sinh^{-1}$	M1		
	$x = \ln 2$ or $\ln(1+\sqrt{2})$	A1√		
		A1√	3	
	Total		**7**	
2(a)				
(i)	**Circle**			
	Correct centre	B1		
	Correct radius	B1		
	Touching x-axis	B1	3	
(ii)	**Line**			
	Point $(3,2)$ indicated	B1		
	Line through $\left(1\frac{1}{2}, 1\right)$	B1√		
	Perpendicular to $(0,0) \rightarrow (3,2)$	B1	3	
(b)	Correct shaded area	B1	2	For shading inside the circle provided no other area is shaded
		B1√		Must be a circle and a straight line for second B1
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$-k^3\mathrm{i}+2(1-\mathrm{i})(-k^2)+32(1+\mathrm{i})=0$	M1		Any form
	Equate real and imaginary parts:			
	$-k^3+2k^2+32=0$	A1		
	$-2k^2+32=0$	A1		
	$k=\pm 4$	A1		
	$k=+4$	E1	5	AG
(b)	Sum of roots is $-2(1-\mathrm{i})$	M1		Or $\alpha\beta\gamma=-(32+32\mathrm{i})$ Must be correct for M1
	Third root $2-2\mathrm{i}$	A1√	2	
	Total		**7**	
4(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\cosh t}\right)=-1(\cosh t)^{-2}\sinh t$	M1A1		Or $\frac{-2\left(\mathrm{e}^t-\mathrm{e}^{-t}\right)}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}$
	$=-\operatorname{sech} t\tanh t$	A1	3	AG
(ii)	Use of $\tanh^2 t=1-\operatorname{sech}^2 t$	M1		
	Printed result	A1	2	
(b)(i)	$\dot{x}=1-\operatorname{sech}^2 t\quad(\dot{y}=-\operatorname{sech} t\tanh t)$	B1		
	$\dot{x}^2+\dot{y}^2=\left(1-\operatorname{sech}^2 t\right)^2+\operatorname{sech}^2 t-\operatorname{sech}^4 t$	M1A1		Any form
	$=1-\operatorname{sech}^2 t=\tanh^2 t$	A1	4	AG
(ii)	$s=\int_0^t \tanh t\,\mathrm{d}t$	M1		Ignore limits for M1 and first A1
	$=[\ln\cosh t]_0^t$	A1		
	$=\ln\cosh t$	A1	3	AG
(iii)	$\mathrm{e}^s=\cosh t$	M1		
	$y=\mathrm{e}^{-s}$	A1	2	AG
(c)	$S=2\pi\int_0^t \operatorname{sech} t\tanh t\,\mathrm{d}t$	M1		Ignore limits for M1 and first A1
	$=2\pi[-\operatorname{sech} t]_0^t$	A1		
	$=2\pi(1-\operatorname{sech} t)$	A1		
	$=2\pi\left(1-\mathrm{e}^{-s}\right)$	A1	4	AG
	Total		**18**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Assume true for $n = k$			
	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1}$			
	$= (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1		
	Multiply out	A1		Any form
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1		
	True for $n = 1$ shown	B1		
	$P(k) \Rightarrow P(k+1)$ and $P(1)$ true	E1	5	Allow E1 only if previous 4 marks earned
(b)	$\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6 = \cos\frac{6\pi}{6} + \mathrm{i}\sin\frac{6\pi}{6}$	M1		
	$= -1$	A1	2	
(c)	$(\cos\theta + \mathrm{i}\sin\theta)(1 + \cos\theta - \mathrm{i}\sin\theta)$	M1		
	$= \cos\theta + \cos^2\theta - \mathrm{i}\sin\theta\cos\theta$ $+ \mathrm{i}\sin\theta + \mathrm{i}\sin\theta\cos\theta + \sin^2\theta$	A1		(Accept $-\mathrm{i}^2\sin^2\theta$) Or $\mathrm{e}^{\mathrm{i}\theta}\left(1 + \mathrm{e}^{-\mathrm{i}\theta}\right)$
	$= 1 + \cos\theta + \mathrm{i}\sin\theta$	A1	3	AG
(d)	$\theta = \frac{\pi}{6}$ used	M1		In the context of part (c)
	Part (c) raised to power 6	M1		
	Use of result in part (b)	A1		
	$\left(1 + \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6 +$ $\left(1 + \cos\frac{\pi}{6} - \mathrm{i}\sin\frac{\pi}{6}\right)^6 = 0$	A1	4	AG
	Total		**14**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$1,\ e^{\pm\frac{2\pi i}{3}}$	M1A1	2	M1 for any method which would lead to the correct answers Accept e^0 or e^{0i} Also accept answers written down correctly
(b)	Any correct method Shown for one root	M1 A1	2	AG
(c)(i)	$\dfrac{\omega}{\omega+1}=\dfrac{\omega}{-\omega^2}$	M1		ie use of result in (b)
	$=-\dfrac{1}{\omega}$	A1	2	AG
(ii)	$\dfrac{\omega^2}{\omega^2+1}=-\omega$	A1	1	AG
(iii)	$\left(\dfrac{\omega}{\omega+1}\right)^k+\left(\dfrac{\omega^2}{\omega^2+1}\right)^k=\left(-\dfrac{1}{\omega}\right)^k+(-\omega)^k$	M1A1		
	Use of $\omega=e^{\frac{2\pi i}{3}}$	m1		
	$=(-1)^k\left(e^{\frac{-2k\pi i}{3}}+e^{\frac{2k\pi i}{3}}\right)$	A1		
	$=(-1)^k\,2\cos\dfrac{2k\pi}{3}$	A1	5	AG
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\tan\left((r+1)x - rx\right)$			
	$= \dfrac{\tan(r+1)x - \tan rx}{1 + \tan(r+1)x \tan rx}$	M1A1		
	Multiplying up	A1		
	Printed result	A1	4	AG
(b)	$x = \dfrac{\pi}{50}$			
	$\tan\dfrac{\pi}{50}\tan\dfrac{2\pi}{50} = \dfrac{\tan\frac{2\pi}{50}}{\tan\frac{\pi}{50}} - \dfrac{\tan\frac{\pi}{50}}{\tan\frac{\pi}{50}} - 1$			
	$\tan\dfrac{2\pi}{50}\tan\dfrac{3\pi}{50} = \dfrac{\tan\frac{3\pi}{50}}{\tan\frac{\pi}{50}} - \dfrac{\tan\frac{2\pi}{50}}{\tan\frac{\pi}{50}} - 1$	M1A1		At least three lines to be shown Accept if x's used
	………………..			
	$\tan\dfrac{19\pi}{50}\tan\dfrac{20\pi}{50} = \dfrac{\tan\frac{20\pi}{50}}{\tan\frac{\pi}{50}} - \dfrac{\tan\frac{19\pi}{50}}{\tan\frac{\pi}{50}} - 1$			
	Clear cancellation	m1		
	Sum $= \dfrac{\tan\frac{20\pi}{50}}{\tan\frac{\pi}{50}} - \dfrac{\tan\frac{\pi}{50}}{\tan\frac{\pi}{50}} - 19$	A1		
	$= \dfrac{\tan\frac{2\pi}{5}}{\tan\frac{\pi}{50}} - 20$	A1	5	AG
	Total		**9**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2007 examination - June series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

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MFP2

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{f}(r+1)-\mathrm{f}(r)=r(r+1)^2-(r-1)r^2$	M1		
	$=r\left(r^2+2r+1-r^2+r\right)$	A1		any expanded form
	$=r(3r+1)$	A1	3	AG
(b)	$r=50$ $\quad \mathrm{f}(51)-\mathrm{f}(50)$ $r=51$ $\quad \mathrm{f}(52)-\mathrm{f}(51)$ } PI $r=99$ $\quad \mathrm{f}(100)-\mathrm{f}(99)$	M1A1		OE clearly shown. Accept $\sum_1^{99}-\sum_1^{49}$
	$\sum_{r=50}^{99} r(3r+1)=\mathrm{f}(100)-\mathrm{f}(50)$	m1		clear cancellation
	$=867500$	A1F	4	cao
	Total		**7**	
2(a)	$\sum\alpha\beta=6$	B1	1	
(b)(i)	Sum of squares $<0\therefore$ not all real Coefficients real $\therefore$ conjugate pair	E1 E1	2	
(ii)	$\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum\alpha\beta$	M1A1		A1 for numerical values inserted
	$\left(\sum\alpha\right)^2=0$	A1F		
	$p=0$	A1F	4	cao
(c)(i)	$-1-3\mathrm{i}$ is a root	B1		
	Use of appropriate relationship eg $\sum\alpha=0$	M1		M0 if $\sum\alpha^2$ used unless the root 2 is checked
	Third root 2	A1F	3	incorrect $p\sqrt{}$
(ii)	$q=-(-1-3\mathrm{i})(-1+3\mathrm{i})2$	M1		allow even if sign error
	$=-20$	A1F	2	ft incorrect 3rd root
	Total		**12**	
3	$(\cos\theta+\mathrm{i}\sin\theta)^{15}=\cos 15\theta+\mathrm{i}\sin 15\theta$	M1		or $=\mathrm{e}^{15\mathrm{i}\theta}$
	$\cos 15\theta=0$ $\sin 15\theta=-1$	m1A1		or $-\mathrm{i}=\mathrm{e}^{\frac{3\pi\mathrm{i}}{2}}$
	$15\theta=\frac{3\pi}{2}$ or 270°	A1F		m1 for **both** R&I parts written down
	$\theta=\frac{\pi}{10}$ or 18°	A1F	5	ft provided the value of 15θ is a correct value
	SC $\cos 15\theta+\mathrm{i}\sin 15\theta=\mathrm{i}$ $\sin 15\theta=-1$	 (M1) (B1)		 or for $\cos 15\theta=0$
	$\theta=\frac{\pi}{10}$	(B1)	(3)	
	Total		**5**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\frac{x}{1+x^2}+\tan^{-1}x$	B1B1	2	
(b)	$\int_0^1 \tan^{-1}x\,dx = \left[x\tan^{-1}x\right]_0^1 - \int_0^1 \frac{x\,dx}{1+x^2}$	M1		either use of part (a) or integration by parts. Allow if sign error
	$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\ln\left(1+x^2\right)$	M1A1F		ft on $\int \frac{x}{1-x^2}dx$
	$I = 1\ \tan^{-1}1 - \frac{1}{2}\ln 2$	M1		
	$= \frac{\pi}{4} - \ln\sqrt{2}$	A1	5	AG
	Total		**7**	
5(a)	Explanation	E2,1,0	2	E1 for $\mathrm{i} = \mathrm{e}^{\frac{\pi \mathrm{i}}{2}}$ or $\mathrm{i}z_1 = -y_1 + \mathrm{i}x_1$
(b)(i)	Perpendicular bisector of *AB* through *O*	B1 B1	2	
(ii)	half-line from *B* parallel to *OA*	B1 B1 B1	3	If L_2 is taken to be the line *AB* give B0
(c)	$(1+\mathrm{i})z_1$	M1A1	2	ft if L_2 taken as line *AB*
	Total		**9**	
6(a)	$\left(1-\frac{1}{(k+1)^2}\right)\times\frac{k+1}{2k} = \frac{(k+1)^2-1}{(k+1)^2}\times\frac{k+1}{2k}$	M1		
	$= \frac{k^2+2k}{(k+1)^2}\times\frac{k+1}{2k}$	A1		
	$= \frac{k+2}{2(k+1)}$	A1	3	AG
(b)	Assume true for $n = k$, then			
	$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\ldots\left(1-\frac{1}{(k+1)^2}\right)$	M1		
	$= \frac{k+2}{2(k+1)}$	A1		
	True for $n = 2$ shown $1-\frac{1}{2^2} = \frac{3}{4}$	B1		
	$P_n \Rightarrow P_{n+1}$ and P_2 true	E1	4	only if the other 3 marks earned
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dy}{dx}=\frac{2}{\sqrt{x}}$	B1		accept $2x^{-\frac{1}{2}}$ etc
	$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\sqrt{1+\frac{4}{x}}$	M1A1F		ft sign error in $\frac{dy}{dx}$
	$=\sqrt{\frac{x+4}{x}}$	A1	4	AG
(b)(i)	$x=4\sinh^2\theta,\ dx=8\sinh\theta\cosh\theta\,d\theta$	M1A1		M1 for any attempt at $\frac{dx}{d\theta}$
	$I=\int\sqrt{\frac{4\sinh^2\theta+4}{4\sinh^2\theta}}\ 8\sinh\theta\cosh\theta\,d\theta$	M1		
	$=\int\frac{2\cosh\theta}{2\sinh\theta}\ 8\sinh\theta\cosh\theta\,d\theta$	m1		ie use of $\cosh^2\theta-\sinh^2\theta=1$
	$=\int 8\cosh^2\theta\,d\theta$	A1	5	AG
(ii)	Use of $2\cosh^2\theta=1+\cosh 2\theta$	M1		allow if sign error
	$I=\int 4(1+\cosh 2\theta)\,d\theta$	A1		oe
	$=4\theta+2\sinh 2\theta$	A1F		oe
	Use of $\sinh 2\theta=2\sinh\theta\cosh\theta$	m1		
	$=4\sinh^{-1}\frac{1}{2}+4\times\frac{1}{2}\sqrt{1+\frac{1}{4}}$	A1F		
	$=4\sinh^{-1}\frac{1}{2}+\sqrt{5}$	A1	6	AG
	Total		**15**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$z^3 = \frac{4 \pm \sqrt{16-32}}{2}$	M1		
	$= 2 \pm 2\mathrm{i}$	A1	2	AG
(ii)	$2+2\mathrm{i} = 2\sqrt{2}\mathrm{e}^{\frac{\pi \mathrm{i}}{4}}$, $2-2\mathrm{i} = 2\sqrt{2}\mathrm{e}^{\frac{-\pi \mathrm{i}}{4}}$	M1 A1A1		M1 for either result or for one of $r = 2\sqrt{2}$, $\theta = \pm\frac{\pi}{4}$ $\left(r = 2\sqrt{2}\ \text{A1},\ \theta = \pm\frac{\pi}{4}\ \text{A1}\right)$
	$z = \sqrt{2}\mathrm{e}^{\frac{\pi \mathrm{i}}{12}+\frac{2k\pi \mathrm{i}}{3}}$ or $\sqrt{2}\mathrm{e}^{\frac{-\pi \mathrm{i}}{12}+\frac{2k\pi \mathrm{i}}{3}}$	M1		M1 for either
	$z = \sqrt{2}\,\mathrm{e}^{\frac{\pm \pi \mathrm{i}}{12}}, \sqrt{2}\mathrm{e}^{\frac{\pm 3\pi \mathrm{i}}{4}}, \sqrt{2}\mathrm{e}^{\frac{\pm 7\pi \mathrm{i}}{12}}$	A2,1,0 F	6	allow A1 for any 3 correct ft errors in $\pm\frac{\pi}{4}$
(b)	Multiplication of brackets	M1		
	Use of $\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta} = 2\cos\theta$	A1	2	AG
(c)	$\left(z - \sqrt{2}\mathrm{e}^{\frac{\pi \mathrm{i}}{12}}\right)\left(z - \sqrt{2}\mathrm{e}^{-\frac{\pi \mathrm{i}}{12}}\right)$			
	$= z^2 - 2\sqrt{2}\cos\frac{\pi}{12}z + 2$	M1A1F		PI
	Product is $\left(z^2 - 2\sqrt{2}\cos\frac{\pi}{12}z + 2\right)$ $\left(z^2 - 2\sqrt{2}\cos\frac{7\pi}{12}z + 2\right)$ $\left(z^2 - 2\sqrt{2}\cos\frac{3\pi}{4}z + 2\right)$	A1F	3	$\left(\text{or } z^2 + 2z + 2\right)$
	Total		**13**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

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2008 examination - January series

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MFP2

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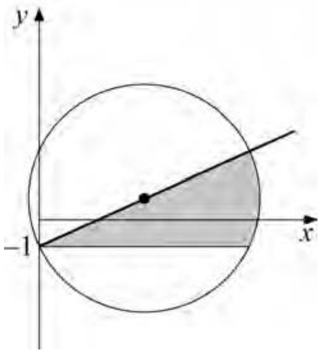
1(a)	Any method for finding r or θ	M1		
	$r = 4\sqrt{2},\ \theta = \dfrac{\pi}{4}$	A1A1	3	
(b)	$z^5 = 4\sqrt{2}\,e^{\frac{\pi i}{4}}$			M1 needs some reference to $a + 2k\pi i$
	$z = \sqrt{2}e^{\frac{\pi i}{20} + \frac{2k\pi i}{5}}$	M1 A1F A1F		A1 for r, A1 for θ] incorrect $r,\ \theta$ part (a)
	$z = \sqrt{2}\,e^{\frac{\pi i}{20}},\ \sqrt{2}\,e^{\frac{9\pi i}{20}},\ \sqrt{2}\,e^{\frac{17\pi i}{20}},$ $\sqrt{2}\,e^{\frac{-7\pi i}{20}},\ \sqrt{2}\,e^{\frac{-15\pi i}{20}}$	A2,1,0 F	5	Accept r in any form eg $32^{\frac{1}{10}}$ Correct but some answers outside range allow A1 ft incorrect r, θ in part (a)
	Total		**8**	
2(a)	Attempt to expand $(2r+1)^3 - (2r-1)^3$	M1		
	$(2r+1)^3$ or $(2r-1)^3$ expanded	A1		
	$24r^2 + 2$	A1	3	AG
(b)	$r = 1 \quad 3^3 - 1^3 = 24 \times 1^2 + 2$ $r = 2 \quad 5^3 - 3^3 = 24 \times 2^2 + 2$	M1A1		3 rows seen Do not allow M1 for $(2n+1)^3 - 1$ not equal to anything
	$r = n \quad (2n+1)^3 - (2n-1)^3 = 24 \times n^2 + 2$			
	$(2n+1)^3 - 1 = 24\sum_{r=1}^{n} r^2 + 2n$	A1		
	$8n^3 + 12n^2 + 6n + 1 - 1 - 2n = 24\sum_{r=1}^{n} r^2$	M1		M1 for multiplication of bracket or taking $(2n+1)$ out as a factor
	$8n^3 + 12n^2 + 4n = 24\sum_{r=1}^{n} r^2$	A1		CAO
	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1	6	AG
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$z=-\mathrm{i}$ $\quad\left\|-2\sqrt{3}-2\mathrm{i}\right\|=\sqrt{12+4}=4$	M1 A1	 2	$\left\|-2\sqrt{3}-2\mathrm{i}\right\|$ 4
(ii)	Centre of circle is $2\sqrt{3}+\mathrm{i}$ Substitute into line arg $\left(2\sqrt{3}+2\mathrm{i}\right)=\dfrac{\pi}{6}$ shown	B1 M1 A1	 3	Do not accept $\left(2\sqrt{3},1\right)$ unless attempt to solve using trig
(b)	 Circle: centre correct through $(0,-1)$ Half line: through $(0,-1)$ through centre of circle	 B1 B1 B1 B1	 4	
(c)	Shading inside circle and below line Bounded by $y=-1$	B1F B1	 2	
	Total		**11**	
4(a)(i)	$\sum\alpha=-\mathrm{i}$	B1	1	
(ii)	$\sum\alpha\beta=3$	B1	1	
(iii)	$\alpha\beta\gamma=1+\mathrm{i}$	B1	1	
(b)(i)	$\sum\alpha^2=\left(\sum\alpha\right)^2-2\sum\alpha\beta$ used $=(-\mathrm{i})^2-2\times3$ $=-7$	M1 A1F A1F	 3	Allow if sign error or 2 missing ft errors in (a)
(ii)	$\sum\alpha^2\beta^2=\left(\sum\alpha\beta\right)^2-2\sum\alpha\beta\cdot\beta\gamma$ $=\left(\sum\alpha\beta\right)^2-2\alpha\beta\gamma\sum\alpha$ $=9-2(1+\mathrm{i})(-\mathrm{i})$ $=7+2\mathrm{i}$	M1 A1 A1F A1F	 4	Allow if sign error in 2 missing ft errors in (a) ft errors in (a)
(iii)	$\alpha^2\beta^2\gamma^2=(1+\mathrm{i})^2=2\mathrm{i}$	M1 A1F	 2	 ft sign error in $\alpha\beta\gamma$
(c)	$z^3+7z^2+(7+2\mathrm{i})z-2\mathrm{i}=0$	B1F B1F	 2	Correct numbers in correct places Correct signs
	Total		**14**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5	Assume result true for $n=k$			
	Then $\sum_{r=1}^{k+1}(r^2+1)r!$			
	$=\left((k+1)^2+1\right)(k+1)!+k(k+1)!$	M1A1		
	Taking out $(k+1)!$ as factor	m1		
	$=(k+1)!(k^2+2k+1+1+k)$	A1		
	$=(k+1)(k+2)!$	A1		
	$k=1$ shown $(1^2+1)1!=2$, $1\times 2!=2$	B1		
	$P_k \Rightarrow P_{k+1}$ and P_1 true	E1	7	If all 6 marks earned
	Total		**7**	
6(a)(i)	$\cos 3\theta+\mathrm{i}\sin 3\theta=(\cos\theta+\mathrm{i}\sin\theta)^3$	M1		
	$=\cos^3\theta+3\mathrm{i}\cos^2\theta\sin\theta+3\mathrm{i}^2\cos\theta\sin^2\theta$ $+\mathrm{i}^3\sin^3\theta$	A1		
	Real parts: $\cos 3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$	A1	3	AG
(ii)	Imaginary parts: $\sin 3\theta=3\cos^2\theta\sin\theta-\sin^3\theta$	A1F	1	
(iii)	$\tan 3\theta=\dfrac{\sin 3\theta}{\cos 3\theta}$	M1		Used
	$=\dfrac{3\cos^2\theta\sin\theta-\sin^3\theta}{\cos^3\theta-3\sin^2\theta\cos\theta}$	A1F		Error in $\sin 3\theta$
	$=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$			
	$=\dfrac{\tan^3\theta-3\tan\theta}{3\tan^2\theta-1}$	A1	3	AG
(b)(i)	$\tan\dfrac{3\pi}{12}=1$	B1		Used (possibly implied)
	$\tan\dfrac{\pi}{12}$ is a root of $1=\dfrac{x^3-3x}{3x^2-1}$	M1		Must be hence
	$x^3-3x^2-3x+1=0$	A1	3	
(ii)	Other roots are $\tan\dfrac{5\pi}{12}$, $\tan\dfrac{9\pi}{12}$	B1B1	2	
(c)	$\tan\dfrac{\pi}{12}+\tan\dfrac{5\pi}{12}+\tan\dfrac{9\pi}{12}=3$	M1		Must be hence
	$\tan\dfrac{\pi}{12}+\tan\dfrac{5\pi}{12}=4$	A1	2	
	Total		**14**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dy}{dx} = \frac{1}{\tanh\frac{x}{2}} \ldots$	B1		
	$\operatorname{sech}^2\frac{x}{2}\ldots$	B1		
	$\frac{1}{2}$	B1		
	$= \frac{1}{2\frac{\sinh\frac{x}{2}}{\cosh\frac{x}{2}}\cosh^2\frac{x}{2}}$	M1		OE ie expressing in $\sinh\frac{x}{2}$ and $\cosh\frac{x}{2}$
	$= \frac{1}{2\sinh\frac{x}{2}\cosh\frac{x}{2}}$			
	$= \frac{1}{\sinh x}$	M1		ie use of $\sinh 2A = 2\sinh A\cosh A$
	$= \operatorname{cosech} x$	A1	6	AG
	Alternative			
	$\ln\sinh\frac{x}{2} - \ln\cosh\frac{x}{2}$	(B1)		
	$\frac{1}{2}\frac{\cosh\frac{x}{2}}{\sinh\frac{x}{2}} - \frac{1}{2}\frac{\sinh\frac{x}{2}}{\cosh\frac{x}{2}}$	(B1B1)		
	$\frac{\cosh^2\frac{x}{2} - \sinh^2\frac{x}{2}}{2\sinh\frac{x}{2}\cosh\frac{x}{2}}$	(M1)		
	Use of $\sinh 2A = 2\sinh A\cosh A$	(M1)		
	result	(A1)		
(b)(i)	$s = \int_1^2 \sqrt{1+\operatorname{cosech}^2 x}\, dx$	M1		
	$= \int_1^2 \coth x\, dx$	A1	2	AG
(ii)	$s = [\ln\sinh x]_1^2$	M1		needs to be correct
	$= \ln\sinh 2 - \ln\sinh 1$	A1		
	$= \ln\frac{2\sinh 1\cosh 1}{\sinh 1}$	A1F		must be seen
	$= \ln(2\cosh 1)$	A1	4	AG
	Total		**12**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2008 examination – June series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

$\surd$ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

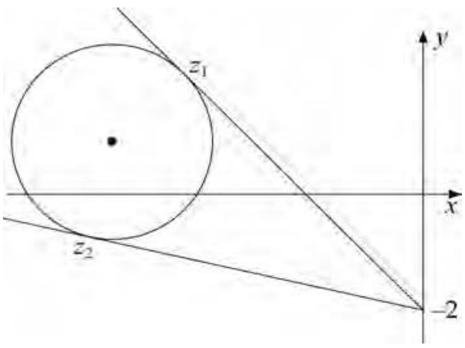
MFP2

Q	Solution	Marks	Total	Comments
1(a)	$5\left(\frac{e^x - e^{-x}}{2}\right)+\left(\frac{e^x + e^{-x}}{2}\right)$	M1		M0 if no 2s in denominator
	$= 3e^x - 2e^{-x}$	A1	2	
(b)	$3e^x - 2e^{-x} + 5 = 0$			
	$3e^{2x} + 5e^x - 2 = 0$	M1		ft if 2s missing in (a)
	$(3e^x - 1)(e^x + 2) = 0$	A1F		
	$e^x \neq -2$	E1		any **indication** of rejection
	$e^x = \frac{1}{3} \qquad x = \ln\frac{1}{3}$	A1F	4	provided quadratic factorises into real factors
	Total		**6**	
2(a)	$1 = A(r+2) + Br$	M1		
	$2A = 1, \quad A = \frac{1}{2}$	A1		
	$A + B = 0, \quad B = -\frac{1}{2}$	A1	3	
(b)	$r = 10 \qquad \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{11.12}\right)$			if (a) is incorrect but $A = \frac{1}{2}$ and $B = -\frac{1}{2}$ used, allow full marks for (b)
	$r = 11 \qquad \frac{1}{2}\left(\frac{1}{11.12} - \frac{1}{12.13}\right)$			
	....			
	$r = 98 \qquad \frac{1}{2}\left(\frac{1}{98.99} - \frac{1}{99.100}\right)$	M1A1		3 relevant rows seen
	$S = \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{99.100}\right)$	m1		if split into $\frac{1}{2r} - \frac{1}{r+1} + \frac{1}{2(r+2)}$, follow mark scheme, in which case $\frac{1}{2.10} - \frac{1}{2.11} + \frac{1}{2.100} - \frac{1}{2.99}$ scores m1
	$= \frac{89}{19800}$	A1	4	
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\alpha\beta\gamma = -18 + 12\text{i}$	B1	1	accept $-(18 - 12\text{i})$
(ii)	$\alpha + \beta + \gamma = 0$	B1	1	
(b)(i)	$\alpha = -2$	B1F	1	
(ii)	$\beta\gamma = \dfrac{\alpha\beta\gamma}{\alpha} = 9 - 6\text{i}$	M1 A1F	2	ft sign errors in (a) or (b)(i) or slips such as miscopy
(iii)	$q = \sum \alpha\beta = \alpha(\beta + \gamma) + \beta\gamma$ $= -2 \times 2 + 9 - 6\text{i}$ $= 5 - 6\text{i}$	M1 A1F A1F	3	 ft incorrect $\beta\gamma$ or α
(c)	$\beta = k\text{i}, \quad \gamma = 2 - k\text{i}$ $k\text{i}(2 - ki) = 9 - 6\text{i}$ $2k = -6 \quad (k^2 = 9) \quad k = -3$ $\beta = -3\text{i}, \quad \gamma = 2 + 3\text{i}$	B1 M1 m1 A1	4	 imaginary parts
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	radius $\sqrt{2}$ centre $-5+\text{i}$	B1,B1	2	condone $(-5, 1)$ for centre do not accept $(-5, \text{i})$
(b)	$\arg(z_1+2\text{i}) = \arg(-4+4\text{i})$	M1		
	$= \frac{3\pi}{4}$	A1	2	clearly shown eg $\tan^{-1}\left(-\frac{1}{1}\right)$
(c)(i)	$\lvert z_1+5-\text{i}\rvert = \lvert 1+\text{i}\rvert = \sqrt{2}$	B1	1	
(ii)	Gradient of line from $(-5, 1)$ to $(-4, 2)$ is 1 $\left(\frac{\pi}{4}\right)$	M1A1		M1 for a complete method
	radius $\perp$ line $\therefore$ tangent	E1	3	
(iii)				
	Circle correct	B1F		ft incorrect centre or radius
	Half line correct	B1	2	line must touch C generally above the circle
(d)	z_2 in correct place	B1		B0 if z_2 is directly below the centre of C
	with tangent shown	B1	2	
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\left(e^x + e^{-x}\right)^2$ expanded correctly	B1		$e^{2x} + 2e^0 + e^{-2x}$ is acceptable
	Result	B1	2	AG
(b)(i)	$\dfrac{dy}{dx} = \sinh x$	B1		
	$\sqrt{1+\left(\dfrac{dy}{dx}\right)^2} = \sqrt{1+\sinh^2 x}$			
	$= \cosh x$	M1		use of $\cosh^2 x - \sinh^2 x = 1$
	$S = 2\pi\int_0^{\ln a} \cosh^2 x \, dx$	A1	3	AG (clearly derived)
(ii)	Use of $\cosh^2 x = \frac{1}{2}(1+\cosh 2x)$	M1		allow one slip in formula M0 if $\int \cosh^2 x \, dx$ is given as $\sinh^2 x$
	$S = \pi\left[x + \frac{1}{2}\sinh 2x\right]_0^{\ln a}$	A1		
	$= \pi\left[\ln a + \frac{1}{2}\left(\frac{e^{2\ln a} - e^{-2\ln a}}{2}\right)\right]$	M1		
	$= \pi\left[\ln a + \frac{1}{4}\left(a^2 - a^{-2}\right)\right]$	A1F		
	$= \pi\left[\ln a + \frac{1}{4a^2}\left(a^4 - 1\right)\right]$	A1	5	AG
	Total		**10**	
6	$u = x - 2$			
	$du = dx$ or $\dfrac{du}{dx} = 1$	B1		clearly seen
	$32 + 4x - x^2 = 36 - u^2$	B1		if $32 + 4x - x^2$ is written as $36 - (x-2)^2$, give B2
	$\int \dfrac{du}{\sqrt{36-u^2}} = \sin^{-1}\dfrac{u}{6}$	M1		allow if dx is used instead of du
	limits −3 and 3 or substitute back to give $\sin^{-1}\dfrac{x-2}{6}$	A1		
	$I = \dfrac{\pi}{6} + \dfrac{\pi}{6} = \dfrac{\pi}{3}$	A1	5	
	Total		**5**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Clear reason given	E1	1	Minimum O × E = E
(b)(i)	$(k+1)\left((k+1)^2+5\right)-k\left(k^2+5\right)$	M1		
	$=3k^2+3k+6$	A1		
	$k^2+k=k(k+1)=M(2)$	E1		**Must** be shown
	$\text{f}(k+1)-\text{f}(k)=M(6)$	E1	4	
(ii)	Assume true for $n = k$			
	$\text{f}(k+1)-\text{f}(k)=M(6)$	M1		Clear method
	$\therefore \text{f}(k+1)=M(6)+\text{f}(k)$			
	$=M(6)+M(6)$	A1		
	$=M(6)$			
	True for $n=1$	B1		
	$P(n)\rightarrow P(n+1)$ and $\text{P}(1)$ true	E1	4	Provided all other marks earned in (b)(ii)
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\left(z+\frac{1}{z}\right)\left(z-\frac{1}{z}\right)=z^2-\frac{1}{z^2}$	B1	1	
(ii)	$\left(z^2-\frac{1}{z^2}\right)^2\left(z+\frac{1}{z}\right)^2$			
	$=\left(z^4-2+\frac{1}{z^4}\right)\left(z^2+2+\frac{1}{z^2}\right)$	M1A1		Alternatives for M1A1: $\left(z^4+4z^2+6+\frac{4}{z^2}+\frac{1}{z^4}\right)\left(z^2-2+\frac{1}{z^2}\right)$ or $\left(z^3-\frac{1}{z^3}\right)^2-2\left(z^3-\frac{1}{z^3}\right)\left(z-\frac{1}{z}\right)+\left(z-\frac{1}{z}\right)^2$
	$=z^6+\frac{1}{z^6}+2\left(z^4+\frac{1}{z^4}\right)-\left(z^2+\frac{1}{z^2}\right)-4$	A1	3	CAO (not necessarily in this form)
(b)(i)	$z^n+\frac{1}{z^n}=\cos n\theta+\mathrm{i}\sin n\theta$ $+\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$	M1A1		
	$=2\cos n\theta$	A1	3	AG SC: if solution is incomplete and $(\cos\theta+\mathrm{i}\sin\theta)^{-n}$ is written as $\cos n\theta-\mathrm{i}\sin n\theta$, award M1A0A1
(ii)	$z^n-z^{-n}=2\mathrm{i}\sin n\theta$	B1	1	
(c)	RHS = $2\cos 6\theta+4\cos 4\theta-2\cos 2\theta-4$	M1 A1F		ft incorrect values in (a)(ii) provided they are cosines
	LHS = $-64\cos^4\theta\sin^2\theta$	M1		
	$\cos^4\theta\sin^2\theta$ $=-\frac{1}{32}\cos 6\theta-\frac{1}{16}\cos 4\theta+\frac{1}{32}\cos 2\theta+\frac{1}{16}$	A1	4	
(d)	$-\frac{\sin 6\theta}{192}-\frac{\sin 4\theta}{64}+\frac{\sin 2\theta}{64}+\frac{\theta}{16}\ (+k)$	M1 A1F	2	ft incorrect coefficients but not letters *A*, *B*, *C*, *D*
	Total		**14**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

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2009 examination - January series

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Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

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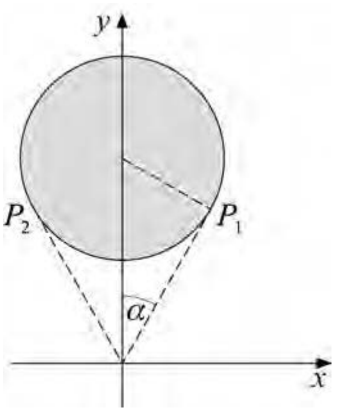
1(a)	LHS = $1 + \frac{1}{2}\left(e^{2\theta} - 2 + e^{-2\theta}\right)$	M1		Expansion of $\frac{1}{2}(e^{\theta} - e^{-\theta})^2$ correctly
		A1		Any form
	$= \frac{1}{2}\left(e^{2\theta} + e^{-2\theta}\right) = \cosh 2\theta$	A1	3	AG
(b)	$3 + 6\sinh^2\theta = 2\sinh\theta + 11$	M1		
	$3\sinh^2\theta - \sinh\theta - 4 = 0$	A1		OE
	$(3\sinh\theta - 4)(\sinh\theta + 1) = 0$	M1		Attempt to factorise or formula
	$\sinh\theta = \frac{4}{3}$ or -1	A1F		ft if factorises or real roots found
	$\theta = \ln 3$	A1F		
	$\theta = \ln\left(\sqrt{2} - 1\right)$	A1F	6	
	Total		**9**	
2(a)		B1		Circle
		B1		Correct centre
		B1		Correct radius
		B1F	4	Inside shading
(b)	Correct points P_1 and P_2 indicated	B1F		Possibly by tangents drawn ft mirror image of circle in *x*-axis
	$\sin\alpha = \frac{2}{4}$	M1		
	$\alpha = \frac{\pi}{6}$	A1		
	Range is $\frac{\pi}{3} \leqslant \arg z \leqslant \frac{2\pi}{3}$	A1	4	Deduct 1 for angles in degrees
	Total		**8**	

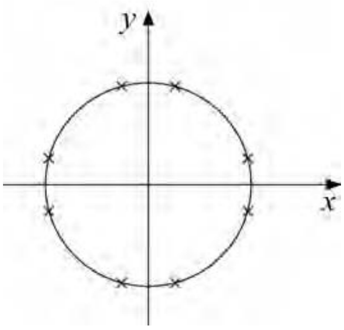
MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathrm{f}(r)-\mathrm{f}(r-1)$			
	$=\frac{1}{4}r^2(r+1)^2-\frac{1}{4}(r-1)^2r^2$	M1		
	$=\frac{1}{4}r^2(r^2+2r+1-r^2+2r-1)$	A1		Correct expansions of $(r+1)^2$ and $(r-1)^2$
	$=r^3$	A1	3	AG
(b)	$r=n$: $n^3=\frac{1}{4}n^2(n+1)^2-\frac{1}{4}(n-1)^2n^2$	M1 A1		For either $r = n$ or $r = 2n$. PI
	$r=2n$:			
	$(2n)^3=\frac{1}{4}(2n)^2(2n+1)^2-\frac{1}{4}(2n-1)^2(2n)^2$	A1		
	$\sum_{r=n}^{2n}r^3=\frac{1}{4}.4n^2(2n+1)^2-\frac{1}{4}(n-1)^2n^2$	M1		
	$=\frac{3}{4}n^2(5n+1)(n+1)$	A1	5	AG
				Alternatively $\sum_{r=1}^{2n}r^3$ and $\sum_{r=1}^{n-1}r^3$ stated M1A1A1 (M1 for either) Difference M1 Answer A1
	Total		**8**	
4(a)	Use of $\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum\alpha\beta$	M1		
	$1=-5+2\sum\alpha\beta$	A1		
	$\sum\alpha\beta=3$	A1	3	AG
(b)	$1(-5-3)=-23-3\alpha\beta\gamma$	M1		For use of identity
	$\alpha\beta\gamma=-5$	A1	2	
(c)	$z^3-z^2+3z+5=0$	M1 A1F	2	For correct signs and "= 0"
(d)	$\alpha^2+\beta^2+\gamma^2<0\Rightarrow$ non real roots	B1		
	Coefficients real $\therefore$ conjugate pair	B1	2	
(e)	$\mathrm{f}(-1)=0\Rightarrow z+1$ is a factor	M1A1		
	$(z+1)(z^2-2z+5)=0$	A1		
	$z=-1,\ 1\pm 2\mathrm{i}$	A1	4	
	Total		**13**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{du}{dx} = 2\cosh x \sinh x$	M1		Any correct method
	$= \sinh 2x$	A1	2	AG
(b)	$I = \int_{x=0}^{x=1} \frac{du}{1+u^2}$	M1A1		Ignore limits here
	$= \left[\tan^{-1} u\right]_{x=0}^{x=1}$	A1		
	$= \left[\tan^{-1}\left(\cosh^2 x\right)\right]_0^1$	A1		Or A1 for change of limits
	$= \tan^{-1}\left(\cosh^2 1\right) - \tan^{-1}\left(\cosh^2 0\right)$			
	$= \tan^{-1}\left(\cosh^2 1\right) - \frac{\pi}{4}$	A1	5	AG
	Total		**7**	
6	Assume result true for $n = k$			
	Then $\sum_{r=1}^{k+1} \frac{2^r \times r}{(r+1)(r+2)}$			**SC** If no series at all indicated on LHS, deduct 1 and give E0 at end
	$= \frac{2^{k+1}}{k+2} + \frac{2^{k+1}(k+1)}{(k+2)(k+3)} - 1$	M1A1		
	$= \frac{2^{k+1}(k+3+k+1)}{(k+2)(k+3)} - 1$	M1		Putting over common denominator (not including the –1, unless separated later)
	$= \frac{2^{k+1}2(k+2)}{(k+2)(k+3)} - 1$	A1		
	$= \frac{2^{k+2}}{k+3} - 1$	A1		
	$k = 1$: LHS = $\frac{1}{3}$, RHS = $\frac{2^2}{3} - 1$	B1		
	$P_k \Rightarrow P_{k+1}$ and P_1 true	E1	7	Must be completely correct
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{d}{dx}\left(\cosh^{-1}\frac{1}{x}\right)=\frac{1}{\sqrt{\frac{1}{x^2}-1}}\left(-\frac{1}{x^2}\right)$	M1A1		M0 if $\frac{dy}{dx}$ = f(y) and no attempt to substitute back to x
	$=\frac{-1}{x\sqrt{1-x^2}}$	A1	3	AG
(b)(i)	$\frac{d}{dx}\left(\sqrt{1-x^2}\right)=\frac{-2x}{2\sqrt{1-x^2}}$	B1 B1		For numerator For denominator (not $(1-x^2)^{\frac{1}{2}}$)
	$\frac{dy}{dx}=\frac{-x}{\sqrt{1-x^2}}+\frac{1}{x\sqrt{1-x^2}}$			
	$=\frac{1-x^2}{x\sqrt{1-x^2}}=\frac{\sqrt{1-x^2}}{x}$	M1 A1	4	For attempt to put over a common denominator AG
(ii)	$s=\int_{\frac{1}{4}}^{\frac{3}{4}}\sqrt{1+\frac{1-x^2}{x^2}}\,dx=\int_{\frac{1}{4}}^{\frac{3}{4}}\frac{1}{x}\,dx$	M1 A1A1		For use of $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$
	$=[\ln x]_{\frac{1}{4}}^{\frac{3}{4}}$	M1		
	$=\ln\frac{3}{4}-\ln\frac{1}{4}=\ln 3$	A1	5	AG
	Total		**12**	
8(a)	Correct multiplication of brackets $e^{i\theta}+e^{-i\theta}=2\cos\theta$	M1 A1	2	Clearly shown
(b)	$2\cos\theta=1$	M1		**SC** If 'hence' not used and, say, $z^8-z^4+1=0$ is solved by formula, lose M1A1, but then continue M1m1 etc if $\frac{\pi}{3}$ is obtained
	$\theta=\frac{\pi}{3}$	A1		
	$z^4=e^{\frac{\pi i}{3}}$ or $e^{\frac{-\pi i}{3}}$	M1		
	$z=e^{\frac{\pi i}{12}+\frac{2k\pi i}{4}}$ or $e^{\frac{-\pi i}{12}+\frac{2k\pi i}{4}}$	m1		
	$e^{\pm\frac{\pi i}{12}}$, $e^{\pm\frac{7\pi i}{12}}$, $e^{\pm\frac{5\pi i}{12}}$, $e^{\pm\frac{11\pi i}{12}}$	A2, 1, 0F	6	A1 if 3 roots correct
(c)		B2,1,0		B1 for 4 roots indicated correctly on a circle. CAO
	Indication that $r=1$	B1	3	
	Total		**11**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2009 examination - June series

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Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

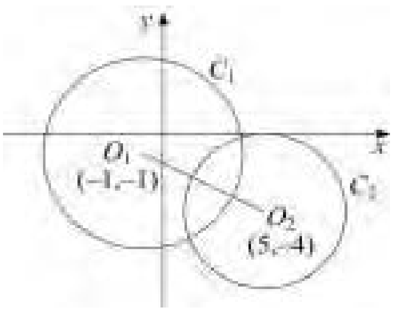
Q	Solution	Marks	Total	Comments
1(a)	$z^4 = 16e^{\frac{4\pi i}{12}}$	M1		Allow M1 if $z^4 = 2e^{\frac{4\pi i}{12}}$
	$= 16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$	A1		OE could be $2ae^{\frac{\pi i}{3}}$ or $2a\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$
	$= 8 + 8\sqrt{3}i$; $a = 8$	A1F	3	ft errors in 2^4
(b)	For other roots, $r = 2$	B1		for realising roots are of form $2 \times e^{i\theta}$
	$\theta = \frac{\pi}{12} + \frac{2k\pi}{4}$	M1A1		M1 for strictly correct θ i.e must be $\left(\text{their } \frac{\pi}{3} + 2k\pi\right) \times \frac{1}{4}$
	Roots are $2e^{\frac{7\pi i}{12}}$, $2e^{\frac{-5\pi i}{12}}$, $2e^{\frac{-11\pi i}{12}}$	A2,1, 0 F	5	ft error in $\frac{\pi}{12}$ or r $\left[\text{accept } 2e^{\left(\frac{\pi}{12}+\frac{2k\pi}{4}\right)i} \quad k = -1, -2, 1\right]$
	Total		**8**	
2(a)	$A = \frac{1}{2}, B = -\frac{1}{2}$	B1, B1F	2	For either A or B For the other
(b)	Method of differences clearly shown	M1		
	$\text{Sum} = \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$	A1		
	$= \frac{n}{2n+1}$	A1	3	AG
(c)	$\frac{1}{2(2n+1)} < 0.001$ or $\frac{n}{2n+1} > 0.499$	M1		Condone use of equals sign
	$1 < 0.004n + 0.002$ or $n > 0.998n + 0.499$			
	$n > \frac{0.998}{0.004}$ or $0.004n > 0.998$	A1		OE
	$n = 250$	A1F	3	ft if say 0.4999 used If method of trial and improvement used, award full marks for a completely correct solution showing working
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$2+3\text{i}$	B1	1	
(b)(i)	$\alpha\beta=13$	B1	1	
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha=25$	M1		M1A0 for -25 (no ft)
	$\gamma(\alpha+\beta)=12$	A1F		
	$\gamma=3$	A1F	3	ft error in $\alpha\beta$
(iii)	$p=-\sum\alpha=-7$	M1 A1F		M1 for a correct method for either p or q
	$q=-\alpha\beta\gamma=-39$	A1F	3	ft from previous errors p and q must be real for sign errors in p and q allow M1 but A0
	Alternative for (b)(ii) and (iii):			
(ii)	Attempt at $(z-2+3\text{i})(z-2-3\text{i})$	(M1)		
	$z^2-4z+13$	(A1)		
	cubic is $(z^2-4z+13)(z-3)\therefore\gamma=3$	(A1)	(3)	
(iii)	Multiply out or pick out coefficients	(M1)		
	$p=-7,\ q=-39$	(A1, A1)	(3)	
	Total		**8**	
4(a)	Sketch, approximately correct shape	B1		B0 if curve touches asymptotes lines of answer booklet could be used for asymptotes be strict with sketch
	Asymptotes at $y=\pm1$	B1	2	
(b)	Use of $u=\dfrac{\sinh x}{\cosh x}$	M1		
	$=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$ or $\dfrac{e^{2x}-1}{e^{2x}+1}$	A1		
	$u\left(e^x+e^{-x}\right)=e^x-e^{-x}$	M1		M1 for multiplying up
	$e^{-x}(1+u)=e^x(1-u)$	A1		A1 for factorizing out e's or M1 for attempt at $1+u$ and $1-u$ in terms of e^x
	$e^{2x}=\dfrac{1+u}{1-u}$	m1		
	$x=\dfrac{1}{2}\ln\left(\dfrac{1+u}{1-u}\right)$	A1	6	AG

Q	Solution	Marks	Total	Comments
4(c)(i)	Use of $\tanh^2 x = 1 - \text{sech}^2 x$	M1		
	Printed answer	A1	2	
(ii)	$(3\tanh x - 1)(\tanh x - 2) = 0$	M1		Attempt to factorise
	$\tanh x \neq 2$	E1		Accept tanh $x\neq 2$ written down but not ignored or just crossed out
	$\tanh x = \frac{1}{3}$	A1		
	$x = \frac{1}{2}\ln 2$	M1 A1F	5	ft
	Total		**15**	
5(a)	$(\cos\theta + \text{i}\sin\theta)^{k+1} =$			
	$(\cos k\theta + \text{i}\sin k\theta)(\cos\theta + \text{i}\sin\theta)$	M1		
	Multiply out	A1		Any form
	$= \cos(k+1)\theta + \text{i}\sin(k+1)\theta$	A1		Clearly shown
	True for $n = 1$ shown	B1		
	$P(k) \Rightarrow P(k+1)$ and $P(1)$ true	E1	5	provided previous 4 marks earned
(b)	$\frac{1}{z^n} = \frac{1}{\cos n\theta + \text{i}\sin n\theta} = \cos n\theta - \text{i}\sin n\theta$	M1A1		or $z^{-n} = \cos(-n\theta) + \text{i}\sin(-n\theta)$ SC $(\cos\theta + \text{i}\sin\theta)^{-n}$ quoted as $\cos n\theta - \text{i}\sin n\theta$ earns M1A1 only
	$z^n + \frac{1}{z^n} = 2\cos n\theta$	A1	3	AG
(c)	$z + \frac{1}{z} = \sqrt{2}$			
	$2\cos\theta = \sqrt{2}$	M1		
	$\theta = \frac{\pi}{4}$	A1		
	$z^{10} + \frac{1}{z^{10}} = 2\cos\left(\frac{10\pi}{4}\right)$	M1		M0 for merely writing $z^{10} + \frac{1}{z^{10}} = 2\cos 10\theta$
	$= 0$	A1F	4	
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Centre $-1-\mathrm{i}$ or $(-1,-1)$	B1		
	Radius 5	M1 A1F		ft incorrect centre if used
	$\lvert z+1+\mathrm{i}\rvert = 5$ or $\lvert z-(-1-i)\rvert = 5$	A1F	4	ft $\lvert z+1+\mathrm{i}\rvert = 10$ earns M0B1
(b)				
	C_1 correct centre, correct radius	B1F		ft errors in (a) but fit circles need to intersect and C_1 enclose $(0,0)$
	C_2 correct centre, correct radius	B1		
	Touching x-axis	B1F	3	error in plotting centre
(c)	$O_1O_2 = 3\sqrt{5}$	M1A1		allow if circles misplaced but O_1O_2 is still $3\sqrt{5}$
	Correct length identified	m1		
	Length is $9+3\sqrt{5}$	M1 A1F	5	ft if r is taken as 10
	Total		**12**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\dfrac{ds}{dx}=\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=\sqrt{1+\left(\dfrac{s}{2}\right)^2}$	M1A1		Allow M1 for $s=\int\sqrt{1+\left(\dfrac{s}{2}\right)^2}\,dx$ then A1 for $\dfrac{dy}{dx}$
	$=\dfrac{1}{2}\sqrt{4+s^2}$	A1	3	AG
(ii)	$\displaystyle\int\frac{ds}{\sqrt{4+s^2}}=\int\frac{1}{2}\,dx$	M1		For separation of variables; allow without integral sign
	$\sinh^{-1}\dfrac{s}{2}=\dfrac{1}{2}x+C$	A1		Allow if C is missing
	$C=0$	A1		
	$s=2\sinh\dfrac{1}{2}x$	A1	4	AG if C not mentioned allow $\dfrac{3}{4}$ **SC** incomplete proof of (a)(ii), differentiating $s=2\sinh\dfrac{x}{2}$ to arrive at $\dfrac{ds}{dx}=\dfrac{1}{2}\sqrt{4+s^2}$ allow M1A1 only $\left(2/4\right)$
(iii)	$\dfrac{dy}{dx}=\sinh\dfrac{1}{2}x$	M1		
	$y=2\cosh\dfrac{1}{2}x+C$	A1		Allow if C is missing
	$C=0$	A1	3	Must be shown to be zero and CAO
(b)	$y^2=4\left(1+\sinh^2\dfrac{x}{2}\right)$	M1		Use of $\cosh^2=1+\sinh^2$
	$=4+s^2$	A1	2	AG
	Total		**12**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2010 examination - January series

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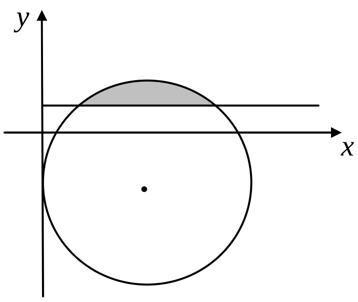
Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	LHS = $\frac{1}{4}\left(e^x + e^{-x}\right)^2 - \frac{1}{4}\left(e^x - e^{-x}\right)^2$	M1		
	Correct expansion of either square	A1		
	Shown equal to 1	A1	3	AG
(b)(i)	$8\cosh^2 x - 3$	B1	1	
(ii)	Sketch of $y = \cosh x$	B1	1	Must cross *y*-axis above *x*-axis
(iii)	$\cosh x = (\pm)1.25$	B1F		OE; ft errors in (b)(i); allow $\pm$ missing
	$x = \ln\left(1.25 + \sqrt{1.25^2 - 1}\right)$	M1		
	$= \ln 2$	A1F		
	$\ln\frac{1}{2}$ by symmetry	A1F	4	Accept $-\ln 2$ written straight down Alternatively, if solved by using $e^{2x} - 2.5e^x + 1 = 0$, allow M1 for $x = \ln\left(\frac{2.5 \pm \sqrt{2.5^2 - 4}}{2}\right)$
	Total		**9**	
2				
(a)(i)	Circle	B1		
	Correct centre	B1		*x*-coordinate $\approx -2\times$ *y*-coordinate in correct quadrant; condone $(4, -2i)$
	Touching *y*-axis	B1	3	
(ii)	Straight line	B1		
	parallel to *x*-axis	B1		
	through (0, 1)	B1	3	Assume (0, 1) if distance up *y*-axis is half distance to top of circle; no other shading outside circle
(b)	Shading: inside circle	B1F		
	above line	B1F	2	Whole question reflected in *x*-axis loses 2 marks
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\beta = 2 - 2\sqrt{3}\,\mathrm{i}$	B1	1	
(ii)	$\alpha\beta\gamma = -8$	M1		Allow for +8 but not ± 16
	$\alpha\beta = 16$	B1		
	$\gamma = -\frac{1}{2}$	A1	3	
(iii)	Either $\frac{-p}{2} = \alpha + \beta + \gamma$ or $\frac{q}{2} = \alpha\beta + \beta\gamma + \gamma\alpha$	M1		**SC** if failure to divide by 2 throughout, allow M1A1 for either *p* or *q* correct ft
	$p = -7, \quad q = 28$	A1F, A1F	3	ft incorrect γ
	Alternative to (a)(ii) and (a)(iii):			
	$(z^2 - 4z + 16)(az + b)$	(M1)		
	$\alpha\beta = 16$	(B1)		
	$a = 2,\ b = +1,\ \gamma = -\frac{1}{2}$	(A1)		
	Equating coefficients	(M1)		
	$p = -7$	(A1F)		
	$q = 28$	(A1F)		
(b)(i)	$r = 4,\ \theta = \frac{\pi}{3}$	B1,B1	2	
(ii)	$(2 + 2\sqrt{3}\,\mathrm{i})^n = \left(4\mathrm{e}^{\frac{\pi \mathrm{i}}{3}}\right)^n$	M1		
	$= 4^n\left(\cos\frac{n\pi}{3} + \mathrm{i}\sin\frac{n\pi}{3}\right)$	A1	2	AG
(iii)	$(2 - 2\sqrt{3}\,\mathrm{i})^n = 4^n\left(\cos\frac{n\pi}{3} - \mathrm{i}\sin\frac{n\pi}{3}\right)$	B1		
	$\alpha^n + \beta^n + \gamma^n = 4^n\left(\cos\frac{n\pi}{3} + \mathrm{i}\sin\frac{n\pi}{3}\right) + 4^n\left(\cos\frac{n\pi}{3} - \mathrm{i}\sin\frac{n\pi}{3}\right) + \left(-\frac{1}{2}\right)^n$	M1		
	$= 2^{2n+1}\cos\frac{n\pi}{3} + \left(-\frac{1}{2}\right)^n$	A1	3	AG
	Total		**14**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\frac{dx}{dt} = \sinh 2t$	B1		
	$\frac{dy}{dt} = 2\cosh t$	B1		
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sinh^2 2t + 4\cosh^2 t$	M1		
	Use of $\sinh 2t = 2\sinh t \cosh t$	m1		Or other correct formula for double angle
	$= 4\cosh^2 t\left(\sinh^2 t + 1\right)$	A1		For taking out factor
	$= 4\cosh^4 t$	A1F	6	ft errors of sign in $\frac{dx}{dt}$ or $\frac{dy}{dt}$
(b)(i)	$S = 2\pi\int_0^1 2\sinh t.2\cosh^2 t\,dt$	M1		Using the value obtained in (a)
	$= 8\pi\int_0^1 \sinh t.\cosh^2 t\,dt$	A1	2	AG
(ii)	$S = 8\pi\left[\frac{\cosh^3 t}{3}\right]_0^1$	M1		
	$= \frac{8\pi}{3}\left[\cosh^3 1 - 1\right]$	A1	2	OE eg $\frac{\pi}{3}\left(\left(e + \frac{1}{e}\right)^3 - 8\right)$
	Total		**10**	
5(a)(i)	$u_1 = S_1 = 1^2.2.3 = 6$	B1	1	AG
(ii)	$u_2 = S_2 - S_1 = 42$	B1	1	AG
(iii)	$u_n = S_n - S_{n-1}$	M1		
	$= n^2(n+1)(n+2) - (n-1)^2 n(n+1)$	A1		
	$= n(n+1)(4n-1)$	A1	3	AG
(b)	$\sum_{r=n+1}^{2n} u_r = S_{2n} - S_n$	M1		
	$= (2n)^2(2n+1)(2n+2) - n^2(n+1)(n+2)$	A1		
	$= 3n^2(n+1)(5n+2)$	A1	3	AG
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$t = \tan\theta \qquad \mathrm{d}t = \sec^2\theta\,\mathrm{d}\theta$	B1		OE
	$I = \int \frac{\mathrm{d}t}{\left(9\cos^2\theta + \sin^2\theta\right)\sec^2\theta}$	M1		OE
	$= \int \frac{\mathrm{d}t}{t^2 + 9}$	A1	3	AG
(b)	$I = \left[\frac{1}{3}\tan^{-1}\frac{t}{3}\right]_0^{\sqrt{3}}$	M1		M1 for $\tan^{-1}$
	$\frac{1}{3}\tan^{-1}\frac{\sqrt{3}}{3}$ or $\frac{1}{3}\tan^{-1}\frac{1}{\sqrt{3}}$	A1		
	$= \frac{\pi}{18}$	A1	3	AG
	Total		**6**	
7(a)	Assume true for $n = k$			
	$u_{k+1} = 2\left(3\times 2^{k-1} - 1\right) + 1$	M1A1		
	$= 3\times 2^k - 1$	A1		$2^{(k-1)+1}$ not necessarily seen
	True for $n = 1$ shown	B1		
	Method of induction clearly expressed	E1	5	Provided all 4 previous marks earned
(b)	$\sum_{r=1}^{n} u_r = \sum_{r=1}^{n} 3\times 2^{r-1} - n$			
	$= 3\left(2^n - 1\right) - n$	M1A1		M1 for summation, ie recognition of a GP
	$= u_{n+1} - \left(n + 2\right)$	A1	3	AG
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\left(e^{\frac{2\pi i}{7}}\right)^7 = e^{2\pi i} = 1$	B1	1	Or $z^7 = e^{2k\pi i}$ $z = e^{\frac{2k\pi i}{7}}$ $k = 1$
(ii)	Roots are $\omega^2, \omega^3, \omega^4, \omega^5, \omega^6$	M1A1	2	OE; M1A0 for incomplete set **SC** B1 for a set of correct roots in terms of $e^{i\theta}$
(b)	Sum of roots considered	M1		$\left\{ \text{or } \sum_{r=0}^{6} \omega^6 = \frac{\omega^7 - 1}{\omega - 1} = 0 \right.$
	$= 0$	A1	2	
(c)(i)	$\omega^2 + \omega^5 = e^{\frac{4\pi i}{7}} + e^{\frac{10\pi i}{7}}$	M1		
	$= e^{\frac{4\pi i}{7}} + e^{\frac{-4\pi i}{7}}$	A1		Or $\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7} + \cos\frac{4\pi}{7} - i\sin\frac{4\pi}{7}$
	$= 2\cos\frac{4\pi}{7}$	A1	3	AG
(ii)	$\omega + \omega^6 = 2\cos\frac{2\pi}{7}$; $\omega^3 + \omega^4 = 2\cos\frac{6\pi}{7}$	B1,B1		Allow these marks if seen earlier in the solution
	Using part (b)	M1		
	Result	A1	4	AG
	Total		**12**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education
June 2010

Mathematics MFP2

Further Pure 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

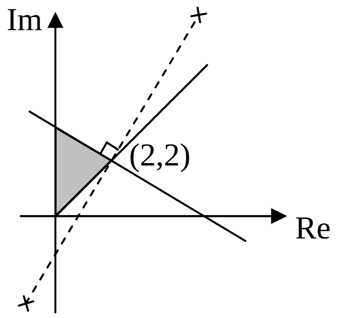
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	$\frac{9\left(e^{x}-e^{-x}\right)}{2} - \frac{e^{x}+e^{-x}}{2}$	M1		M0 if $\cosh x$ mixed up with $\sinh x$
	$= 4e^{x} - 5e^{-x}$	A1	2	AG
(b)	Attempt to multiply by e^{x}	M1		
	$4e^{2x} - 8e^{x} - 5 = 0$	A1		
	$\left(2e^{x}-5\right)\left(2e^{x}+1\right) = 0$	M1		ft provided quadratic factorises (or use of formula)
	$e^{x} \neq -\frac{1}{2}$	E1F		PI but not ignored
	$e^{x} = \frac{5}{2}$	A1F		
	$\tanh x = \frac{\frac{5}{2}-\frac{2}{5}}{\frac{5}{2}+\frac{2}{5}} = \frac{21}{29}$	M1 A1F	7	M1 PI for attempt to use $\tanh x = \frac{\sinh x}{\cosh x}$ or equivalent fraction
	Total		**9**	
2(a)	$\frac{1}{r(r+2)} = \frac{A}{r} + \frac{B}{r+2}$	M1		
	$A = \frac{1}{2}, \quad B = -\frac{1}{2}$	A1, A1F	3	ft incorrect A
(b)	$r=1 \quad \frac{1}{1.3} = \frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}\right)$			
	$r=2 \quad \frac{1}{2.4} = \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)$			
	$r=3 \quad \frac{1}{3.5} = \frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)$	M1		3 rows (PI) numerical values only
	$r=48 \quad \frac{1}{48.50} = \frac{1}{2}\left(\frac{1}{48}-\frac{1}{50}\right)$	A1F		Last row – could be implied
	Cancelling appropriate pairs	M1		
	Sum $= \frac{1}{2}\left(\frac{1}{1}+\frac{1}{2}-\frac{1}{49}-\frac{1}{50}\right)$	A1F		Allow if the $\frac{1}{2}$ is missing only
	$= \frac{894}{1225}$	A1	5	CAO (or equivalent fraction)
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3				
(a)	$\lvert 2+2i+1+3i \rvert = \lvert 2+2i-5-7i \rvert$	B1		Clearly shown do not allow $\lvert 3+5i \rvert = \lvert -3-5i \rvert$ without comment
	$\arg(2+2i) = \frac{\pi}{4}$	B1	2	Clearly shown
(b)	L_1 : straight line with negative gradient	B1		
	perpendicular to line joining $(-1,-3)$ to $(5,7)$	B1		
	through $(2,2)$	B1		The point (2,2) must be shown either by (2,2) or 2+2i or with numbered axes
	L_2 : half line through O	B1		
	through $(2,2)$	B1	5	
(c)	Shading between $\frac{\pi}{4}$ and $\frac{\pi}{2}$	B1		No marks for shading if circles drawn in (b)
	Below L_1	B1	2	
	Total		**9**	
4(a)	$\alpha+\beta+\gamma=2$	B1	1	
(b)(i)	α is a root and so satisfies the equation	E1	1	
(ii)	$\sum\alpha^3 - 2\sum\alpha^2 + p\sum\alpha + 30 = 0$	M1A1		
	Substitution for $\sum\alpha^3$ and $\sum\alpha$	ml		
	$\sum\alpha^2 = p+13$	A1	4	AG
(iii)	$\left(\sum\alpha\right)^2 = \sum\alpha^2 + 2\sum\alpha\beta$ used	M1		do not allow this M mark if used in (b)(ii)
	$p=-3$	A1	2	AG
(c)(i)	$f(-2)=0$	M1		
	$\alpha=-2$	A1	2	
(ii)	$(z+2)(z^2-4z+5)=0$	M1		For attempting to find quadratic factor
	$z=\frac{4\pm\sqrt{-4}}{2}$	m1		Use of formula or completing the square m0 if roots are not complex
	$=2\pm i$	A1	3	CAO
	Total		**13**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	Divide $\cosh^2 t - \sinh^2 t = 1$ by $\cosh^2 t$	M1		Or $\frac{\sinh^2 t}{\cosh^2 t} + \frac{1}{\cosh^2 t}$
	Rearrange	A1	2	AG If solved back to front with no conclusion ending $\cosh^2 t - \sinh^2 t = 1$ B1 only
(ii)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\sinh t}{\cosh t}\right) = \frac{\cosh^2 t - \sinh^2 t}{\cosh^2 t}$	M1A1		
	$= \operatorname{sech}^2 t$	A1	3	AG
(iii)	$\frac{\mathrm{d}}{\mathrm{d}t}(\operatorname{sech} t) = -(\cosh t)^{-2} \sinh t$	M1A1		Allow A1 if negative sign missing
	$= -\operatorname{sech} t \tanh t$	A1	3	AG
(b)(i)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \operatorname{sech}^4 t + \operatorname{sech}^2 t \tanh^2 t$	M1		Allow slips of sign before squaring for this M1
	Use of $\tanh^2 t + \operatorname{sech}^2 t = 1$	m1		Correct formula only for m1
	$= \operatorname{sech}^2 t$	A1		
	$\therefore s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \, \mathrm{d}t$	A1	4	AG (including limits)
(ii)	$u = \mathrm{e}^t \quad \mathrm{d}u = \mathrm{e}^t \, \mathrm{d}t$	B1		
	$\int \operatorname{sech} t \, \mathrm{d}t = \int \frac{2}{u^2+1} \, \mathrm{d}u$	M1A1		CAO M1 for putting integrand in terms of u (<u>no</u> sech (ln$u$))
	$\left[2\tan^{-1} u\right]$	A1		Or $2\tan^{-1}\mathrm{e}^t$
	Change limits correctly or change back to t	m1		At some stage
	$= \frac{2\pi}{3} - \frac{2\pi}{4} = \frac{\pi}{6}$	A1	6	CAO
	Total		**18**	
6(a)	$\frac{1}{(k+2)!} = \frac{k+3}{(k+3)!}$	M1		
	Result	A1	2	
(b)	Assume true for $n = k$ For $n = k + 1$			
	$\sum_{r=1}^{k+1} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{k+1}}{(k+2)!} + \frac{(k+1)2^{k+1}}{(k+3)!}$	M1A1		If no LHS of equation, M1A0
	$= 1 - 2^{k+1}\left(\frac{1}{(k+2)!} - \frac{k+1}{(k+3)!}\right)$	m1		m1 for a suitable combination clearly shown
	$= 1 - \frac{2^{k+2}}{(k+3)!}$	A1		clearly shown or stated true for $n = k + 1$
	True for $n = 1$	B1		Shown
	Method of induction set out properly	E1	6	Provided previous 5 marks all earned
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$1+\sqrt{3}\,\mathrm{i} = 2\mathrm{e}^{\frac{\pi \mathrm{i}}{3}}$	B1		B1 both correct
	$1-\mathrm{i} = \sqrt{2}\,\mathrm{e}^{-\frac{\pi \mathrm{i}}{4}}$	B1B1	3	OE
(ii)	$2^{\frac{21}{2}}$ or equivalent single expression	B1F		No decimals; must include fractional powers
	Raising and adding powers of e	M1		
	$\frac{17\pi}{12}$ or equivalent angle	AIF	3	Denominators of angles must be different
(b)	$z = \sqrt[3]{2^{10}\sqrt{2}}\,\mathrm{e}^{\frac{17\pi \mathrm{i}}{36}+\frac{2k\pi \mathrm{i}}{3}}$	M1		
	$\sqrt[3]{2^{10}\sqrt{2}} = 8\sqrt{2}$	B1		CAO
	$\theta = \frac{17\pi}{36}, -\frac{7\pi}{36}, -\frac{31\pi}{36}$	A2,1F	4	Correct answers outside range: deduct 1 mark only
	Total		**10**	
	TOTAL		**75**	

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General Certificate of Education (A-level)
January 2011

Mathematics MFP2

(Specification 6360)

Further Pure 2

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CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

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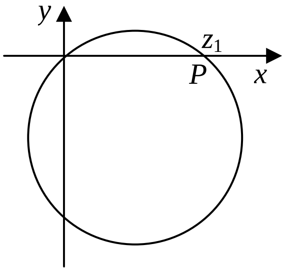
Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	 Circle correct centre through (0, 0)	B1 B1 B1	3	
(b)(i)	z_1 correctly chosen	B1F	1	ft if circle encloses (0, 0)
(ii)	$\lvert z_1 \rvert = 8$	B1F	1	ft if centre misplotted
	Total		**5**	
2(a)	$u_r - u_{r-1} =$ $\frac{1}{6}r(r+1)(4r+11) - \frac{1}{6}(r-1)r(4r+7)$	M1		
	Correct expansion in any form, eg $\frac{1}{6}r\left(4r^2+15r+11-4r^2-3r+7\right)$	A1		
	$= r(2r+3)$	A1	3	AG
(b)	Attempt to use method of differences $S_{100} = u_{100} - u_0$ $= 691850$	M1 A1 A1	3	CAO
	Total		**6**	
3(a)	$(1+\mathrm{i})^2 = 2\mathrm{i}$ or $(1+\mathrm{i}) = \sqrt{2}\,\mathrm{e}^{\frac{\pi \mathrm{i}}{4}}$	B1		
	$2\mathrm{i}(1+\mathrm{i}) = 2\mathrm{i}-2$	B1	2	AG **Alternative method:** $(1+\mathrm{i})^3 = 1+3\mathrm{i}+3\mathrm{i}^2+\mathrm{i}^3$ B1 $= 2\mathrm{i}-2$ B1
(b)(i)	Substitute $z = 1+\mathrm{i}$ Correct expansion	M1 A1		allow for correctly picking out either the real or the imaginary parts
	$k = -5$	A1	3	
(ii)	$\beta+\gamma = 5+\mathrm{i}-\alpha = 4$	B1	1	AG
(iii)	$\alpha\beta\gamma = 5(1+\mathrm{i})$ $\beta\gamma = 5$	M1 A1F		allow if sign error ft incorrect k
	$z^2 - 4z + 5 = 0$	M1		
	Use of formula or $(z-2)^2 = -1$	A1F		No ft for real roots if error in k
	$z = 2 \pm \mathrm{i}$ NB allow marks for (b) in whatever order they appear	A1F	5	
	Total		**11**	

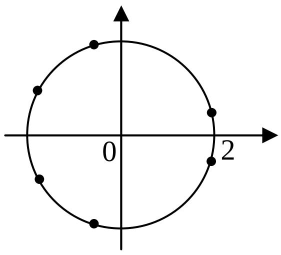
MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\frac{dy}{dx} = 12\sinh x - 8\cosh x - 1$	B1		The B1 and M1 could be in reverse order if put in terms of e first
	$12\frac{\left(e^x - e^{-x}\right)}{2} - 8\frac{\left(e^x + e^{-x}\right)}{2} - 1 = 0$	M1		M0 if $\sinh x$ and $\cosh x$ in terms of e^x are interchanged
	$2e^{2x} - e^x - 10 = 0$	A1F		ft slips of sign
	$\left(2e^x - 5\right)\left(e^x + 2\right) = 0$	M1A1F		ft provided quadratic factorises
	$e^x \neq -2$	E1		some indication of rejection needed
	$x = \ln\frac{5}{2}$ one stationary point	A1F	7	Condone $e^x = \frac{5}{2}$ with statement provided quadratic factorises **Special Case** If $\frac{dy}{dx} = 12\sinh x - 8\cosh x$ B0 For substitution in terms of e^x M1 leading to $e^{2x} = 5$ A1 Then M0
(b)	$b = 12\frac{\left(\frac{5}{2}+\frac{2}{5}\right)}{2} - 8\frac{\left(\frac{5}{2}-\frac{2}{5}\right)}{2} - \ln\frac{5}{2}$	M1A1F		for substitution into original equation
	$= \frac{174}{10} - \frac{84}{10} - \ln\frac{5}{2}$	A1		CAO
	$= 9 - a$	A1	4	AG; accept $b = 9 - a$
	Total		**11**	
5(a)	$\frac{du}{dx} = \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}$	B1		
	$\times(-2x)$	B1	2	
(b)	$\int \sin^{-1} x\ dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\ dx$	M1 A1A1		A1 for each part of the integration by parts
	$\int -\frac{x}{\sqrt{1-x^2}}\ dx = \sqrt{1-x^2}$ used	A1F		ft sign error in $\frac{du}{dx}$
	$\frac{\sqrt{3}}{2}\frac{\pi}{3} + \sqrt{1-\frac{3}{4}} - 1$	m1		substitution of limits
	$\frac{1}{6}\sqrt{3}\pi - \frac{1}{2}$	A1	6	CAO
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\dfrac{dx}{dt} = \sec t - \cos t$	B1,B1		use of FB for $\sec t$; if done from first principles, allow B1 when $\sec t$ is arrived at
	Use of $1 - \cos^2 t = \sin^2 t$	M1		
	$\dfrac{dx}{dt} = \sin t \tan t$	A1	4	AG
(b)	$\dot{x}^2 + \dot{y}^2 = \sin^2 t \tan^2 t + \sin^2 t$	M1A1		sign error in $\dfrac{dy}{dt}$ A0
	Use of $1 + \tan^2 t = \sec^2 t$	m1		
	$\sqrt{\dot{x}^2 + \dot{y}^2} = \tan t$	A1F		ft sign error in $\dfrac{dy}{dt}$
	$\int_0^{\frac{\pi}{3}} \tan t \ dt = [\ln \sec t]_0^{\frac{\pi}{3}}$	A1F		ft sign error in $\dfrac{dy}{dt}$
	$= \ln 2$	A1	6	CAO
	Total		**10**	
7(a)	$f(k+1) - 5f(k)$			
	$= 12^{k+1} + 2\times5^k - 5(12^k + 2\times5^{k-1})$	M1		
	$= 12^{k+1} + 2\times5^k - 5\times12^k - 2\times5^k$	A1		for expansion of bracket $5\times5^{k-1} = 5^k$ used
	$= 12\times12^k - 5\times12^k = 7\times12^k$	A1	3	clearly shown
(b)	Assume $f(k) = M(7)$			
	Then $f(k+1) = 5f(k) + M(7)$	M1		Not merely a repetition of part (a)
	$= M(7)$	A1		clearly shown
	$f(1) = 12 + 2 = 14 = M(7)$	B1		
	Correct inductive process	E1	4	(award only if all 3 previous marks earned)
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$4\left(1+\mathrm{i}\sqrt{3}\right)=8\left(\frac{1}{2}+\mathrm{i}\frac{\sqrt{3}}{2}\right)$	M1		for either $4\left(1+\mathrm{i}\sqrt{3}\right)$ or $4\left(1-\mathrm{i}\sqrt{3}\right)$ used
	$=8\mathrm{e}^{\frac{\pi \mathrm{i}}{3}}$	A1		If either r or θ is incorrect but the same value in both (i) and (ii) allow A1 but for θ only if it is given as $\frac{\pi}{6}$
(ii)	$4\left(1-\mathrm{i}\sqrt{3}\right)=8\mathrm{e}^{\frac{-\pi \mathrm{i}}{3}}$	A1	3	
(b)	$z^3-4=\pm\sqrt{-48}$	M1		taking square root
	$z^3=4\pm4\sqrt{3}\,\mathrm{i}$	A1	2	AG
(c)(i)	$z=2\mathrm{e}^{\frac{\frac{\pi \mathrm{i}}{3}+2k\pi \mathrm{i}}{3}}$ or $z=2\mathrm{e}^{\frac{\frac{-\pi \mathrm{i}}{3}+2k\pi \mathrm{i}}{3}}$	B1F M1		for the 2; ft incorrect 8, but no decimals for either, PI
	$z=2\mathrm{e}^{\frac{\pi \mathrm{i}}{9}},\,2\mathrm{e}^{\frac{7\pi \mathrm{i}}{9}},\,2\mathrm{e}^{\frac{5\pi \mathrm{i}}{9}}$ $=2\mathrm{e}^{\frac{-\pi \mathrm{i}}{9}},\,2\mathrm{e}^{\frac{-7\pi \mathrm{i}}{9}},\,2\mathrm{e}^{\frac{-5\pi \mathrm{i}}{9}}$	A3,2,1F	5	Allow A1 for any 2 roots not +/– each other Allow A2 for any 3 roots not +/– each other Allow A3 for all 6 correct roots Deduct A1 for each incorrect root in the interval; ignore roots outside the interval ft incorrect r
(ii)	 Radius 2	B1F		clearly indicated; ft incorrect r allow B1 for 3 correct points condone lines
	Plotting roots	B2,1	3	
(d)(i)	Sum of roots = 0 as coefficient of $z^5=0$	E1	1	OE
(ii)	Use of, say, $\frac{1}{2}\left(\mathrm{e}^{\frac{\pi \mathrm{i}}{9}}+\mathrm{e}^{\frac{-\pi \mathrm{i}}{9}}\right)=\cos\frac{\pi}{9}$	M1		
	$\cos\frac{3\pi}{9}=\frac{1}{2}$ used	A1		
	$\cos\frac{\pi}{9}+\cos\frac{3\pi}{9}+\cos\frac{5\pi}{9}+\cos\frac{7\pi}{9}=\frac{1}{2}$	A1	3	AG
	Total		**17**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MFP2

(Specification 6360)

Further Pure 2

Final

Mark Scheme

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Set and published by the Assessment and Qualifications Alliance.

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Registered address: AQA, Devas Street, Manchester M15 6EX.

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M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

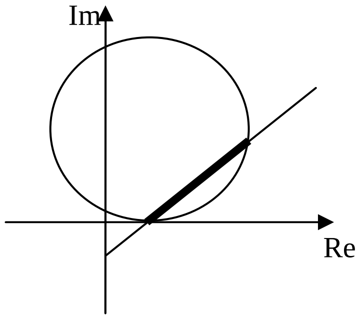
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)				Use average of whole question if 2 diagrams used
(i)	Circle correct centre touching x-axis	B1 B1 B1F	 3	Circle in any position Must be shown ft incorrect centre
(ii)	half-line through (0, –2) through point of contact of circle with x-axis	B1 B1 B1	 3	 Can be inferred
(b)	Inside circle On line	B1 B1F	 2	 ft errors in position of line and circle
	Total		**8**	
2(a)	$\frac{(e^x+e^{-x})}{2}\frac{(e^y+e^{-y})}{2}-\frac{(e^x-e^{-x})}{2}\frac{(e^y-e^{-y})}{2}$	M1A1		M0 if sinh and cosh confused M1 for formula quoted correctly
	Correct expansions	A1		Use of e^{xy} A0
	$=\frac{1}{2}\left(e^{x-y}+e^{-(x-y)}\right)=\cosh(x-y)$	A1	4	AG
(b)(i)	$\cosh(x-\ln 2)=\cosh x\cosh(\ln 2)-\sinh x\sinh(\ln 2)$	M1		**Alternative:** $\frac{e^{x-\ln 2}+e^{-x+\ln 2}}{2}=\frac{e^x-e^{-x}}{2}$ M1
	$\left.\begin{array}{l}\cosh(\ln 2)=\frac{5}{4}\\ \sinh(\ln 2)=\frac{3}{4}\end{array}\right\}$ any method	B1		Both correct $e^{x-\ln 2}=\frac{e^x}{2}$ or $e^{-x+\ln 2}=2e^{-1}$ used B1
	$\frac{5}{4}\cosh x=\frac{7}{4}\sinh x$	A1F		$e^x=\sqrt{6}$ A1
	$\tanh x=\frac{5}{7}$	A1	4	AG $\tanh x=\frac{5}{7}$ A1
(ii)	$x=\frac{1}{2}\ln\left(\frac{1+\frac{5}{7}}{1-\frac{5}{7}}\right)$ or $\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{5}{7}$	M1		Could be embedded in (b)(i)
	$=\frac{1}{2}\ln 6$	A1	2	
	Total		**10**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(r+1)! = (r+1)r(r-1)!$	M1		
	Result	A1	2	AG
(b)	**Attempt** to use method of differences	M1		
	$\sum_{r=1}^{n}(r^2+r-1)(r-1)! = (n+1)! + n! - 1! - 0!$	A1		
	$(n+1)! = (n+1)n!$	m1		Must be seen
	$(n+2)n! - 2$	A1	4	AG
	Total		**6**	
4(a)(i)	$\sum\alpha = 2$	B1		
	$\sum\alpha\beta = 0$	B1	2	
(ii)	$\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$	M1		Used. Watch $\sum\alpha = -2$ (M1A0)
	$= 4$	A1	2	AG
(iii)	Clear explanation	E1	1	eg α satisfies the cubic equation since it is a root. Accept $z = \alpha$
(iv)	$\sum\alpha^3 = 2\sum\alpha^2 - 3k$	M1		Or $\sum\alpha^3 = \left(\sum\alpha\right)^3 - 3\sum\alpha\sum\alpha\beta + 3\alpha\beta\gamma$
	$= 8 - 3k$	A1	2	AG
(b)(i)	$\alpha^4 = 2\alpha^3 - k\alpha$	B1		
	$\sum\alpha^4 = 2\sum\alpha^3 - k\sum\alpha$	M1		Or $\sum\alpha^4 = \left(\sum\alpha^2\right)^2 - 2\left(\sum\alpha\beta\right)^2 + 4\alpha\beta\gamma\sum\alpha$
	$= 2(8-3k) - 2k$	A1		ft on $\sum\alpha = -2$
	$k = 2$	A1	4	AG
(ii)	$\sum\alpha^5 = 2\sum\alpha^4 - k\sum\alpha^2$	M1		
	Substitution of values	A1		
	$= -8$	A1	3	
	Total		**14**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$2y\frac{dy}{dx} = 2x$	B1		Or $\frac{dy}{dx} = x\left(x^2+8\right)^{-\frac{1}{2}}$
	$S = 2\pi\int_0^6 y\sqrt{1+\frac{x^2}{y^2}}\ dx$	M1 A1F		M1 for use of formula provided $\frac{dy}{dx}$ is a function of x A1 for substitution for $\frac{dy}{dx}$ (one slip)
	Eliminating all y	m1		
	$= 2\sqrt{2}\pi\int_0^6 \sqrt{x^2+4}\ dx$	A1	5	AG
(b)	$dx = 2\cosh\theta\ d\theta$ or $\frac{dx}{d\theta} = 2\cosh\theta$	B1		
	$S = 2\sqrt{2}\,\pi\int\sqrt{4\sinh^2 x+4}.\ 2\cosh\theta\ d\theta$	M1		For eliminating x completely and use of $d\theta$, ie $d\theta$ attempted
	$S = \left(2\sqrt{2}\right)\pi\int 2\cosh\theta.\ 2\cosh\theta\ d\theta$	m1		Use of $\cosh^2\theta - \sinh^2\theta = 1$ (ignore limits)
	$= 4\sqrt{2}\,\pi\int\left(\cosh 2\theta + 1\right)\ d\theta$	m1		Use of formula for $\cosh 2\theta$; must be correct
	$= 4\sqrt{2}\,\pi\left[\frac{\sinh 2\theta}{2} + \theta\right]$	B1F		Correct integration of $a\cosh 2\theta + b$
	$= 4\sqrt{2}\,\pi\left[\sinh\theta\cosh\theta + \theta\right]$	m1		Use of $\sinh 2\theta = 2\sinh\theta\cosh\theta$ Must be seen
	$= 4\sqrt{2}\,\pi\left[\frac{x}{2}\sqrt{\frac{x^2}{4}+1} + \sinh^{-1}\frac{x}{2}\right]_0^6$	M1		Or change limits
	$= \pi\left[24\sqrt{5} + 4\sqrt{2}\ \sinh^{-1}3\right]$	A1	8	AG
	Total		**13**	
6(a)	Expansion of $(k+1)\left(4(k+1)^2 - 1\right)$	M1		Any valid method – first step correct
	$= 4k^3 + 12k^2 + 11k + 3$	A1	2	AG
(b)	Assume true for $n = k$ For $n = k+1$:			
	$\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k\left(4k^2-1\right) + (2k+1)^2$	M1A1		No LHS M1A0
	$= \frac{1}{3}\left(4k^3 + 12k^2 + 11k + 3\right)$	A1F		ft error in $(2k + 1)$
	$= \frac{1}{3}(k+1)\left(4(k+1)^2 - 1\right)$	A1		Using part (a)
	True for $n = 1$ shown	B1		
	Proof by induction set out properly (if factorised by 3 linear factors, allow A1 at this particular point)	E1	6	Dependent on all marks correct
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$	M1		Attempt to expand 3 correct terms
	Expansion in any form	A1		Correct simplification
	Equate real parts:	m1		
	$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	A1		AG
	Equate imaginary parts:			
	$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$	A1	5	CAO
(ii)	$\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta}$	M1		Used
	Division by $\cos^5\theta$ or by $\cos^4\theta$	m1		
	$\tan 5\theta = \dfrac{\tan\theta\left(5 - 10\tan^2\theta + \tan^4\theta\right)}{1 - 10\tan^2\theta + 5\tan^4\theta}$	A1	3	AG
(b)	$\theta = \dfrac{\pi}{5} \Rightarrow \tan 5\theta = 0$	M1		Or for $\tan^4\theta - 10\tan^2\theta + 5 = 0$
	$\therefore \tan\dfrac{\pi}{5}$ satisfies $t^4 - 10t^2 + 5 = 0$	A1		Or for $\tan 5\theta = 0$
	Other roots $\tan\dfrac{k\pi}{5}$ $k = 2, 3, 4$	B1	3	OE
(c)	Product of roots = 5	M1		
	$\tan\dfrac{\pi}{5} = -\tan\dfrac{4\pi}{5}$	B1		Or $\tan\dfrac{2\pi}{5} = -\tan\dfrac{3\pi}{5}$
	$\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5$	A1		
	$\tan\dfrac{\pi}{5}\tan\dfrac{2\pi}{5} = +\sqrt{5}$	A1		
	– sign rejected with reason	E1	5	**Alternative (c)** Use of quadratic formula M1 $t^2 = 5 \pm 2\sqrt{5}$ A1 $t = \pm\sqrt{5 \pm 2\sqrt{5}}$ B1 Correct selection of +ve values E1 Multiplied together to get $\sqrt{5}$ A1
	Total		**16**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
January 2012

Mathematics **MFP2**

(Specification 6360)

Further Pure 2

Final

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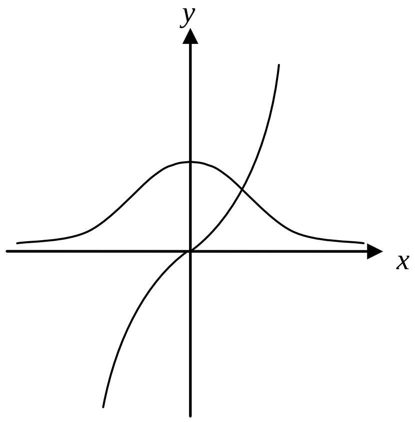
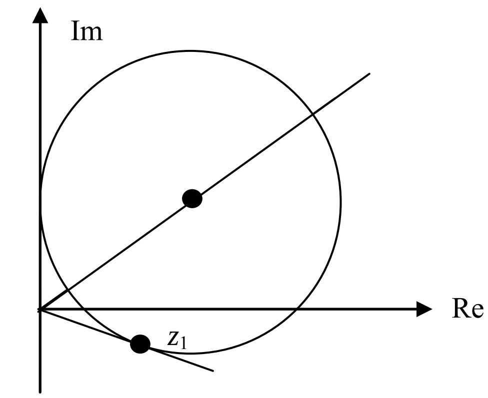
1(a)				
	Sketch $y = \sinh x$	B1		gradient > 0 at (0, 0); no asymptotes
	Sketch $y = \text{sech}\, x$:			
	Symmetry about $x = 0$ with max point	B1		
	Asymptote $y = 0$	B1		must not cross *x*-axis
	Point (0, 1) marked or implied	B1	4	
(b)	$\sinh x = \dfrac{1}{\cosh x}$	M1		
	$\sinh 2x = 2$	M1		use of double angle formula
	Use of ln	m1		dependent on previous M2
	$x = \dfrac{1}{2}\ln\left(2+\sqrt{5}\right)$	A1	4	
	or			
	$\dfrac{1}{2}\left(e^{2x} - e^{-2x}\right) = 2$ OE	(M1)		incorrect sinh *x*, cosh *x* M0 (no marks)
	$e^{4x} - 4e^{2x} - 1 = 0$	(M1)		ie multiply by e^{2x} and rewrite
	Correct use of formula	(m1)		
	Result	(A1)	(4)	
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)				
	Half-line with gradient < 1	B1	1	condone a short line, ie it stops at or inside circle
(b)(i)	Circle centre on L, x-coord 6 indicated	B1		
	touching $\text{Re } z = 0$ not at (0, 0)	B1	2	not touching Re axis
(ii)	y-coord of centre is $2\sqrt{3}$ or $\frac{6}{\sqrt{3}}$	B1		OE; PI
	$z_0 = 6 + 2\sqrt{3}\,\text{i}$,	B1F,		ft error in coords of centre
	$k = 6$	B1	3	
(iii)	Point z_1 shown	B1		PI
	$\arg z_1 = -\frac{1}{6}$	B1	2	
	Total		**8**	
3(a)	$\frac{dy}{dx} = \frac{1}{2\tanh x}$	B1		
	$\times \text{sech}^2 x$	B1		
	$= \frac{1}{2\sinh x \cosh x}$	M1		for expressing in terms of $\sinh x$ and $\cosh x$
	$= \frac{1}{\sinh 2x}$	A1	4	AG; PI by previous line
(b)	$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{1}{\sinh^2 2x}}$	M1		use of formula; accept √ inserted at any stage
	$= \sqrt{\frac{\cosh^2 2x}{\sinh^2 2x}}$	m1		relevant use of $\cosh^2 - \sinh^2 = 1$
	$= \frac{\cosh 2x}{\sinh 2x}$	A1		OE
	Integral is $\frac{1}{2}\ln \sinh 2x$	M1A1		M1 for ln sinh
	$\sinh(2\ln 4) = \frac{255}{32}$ $\quad \sinh(2\ln 2) = \frac{15}{8}$	B1B1		PI
	$s = \frac{1}{2}\ln\left(\frac{17}{4}\right)$	A1F	8	ft error in $\frac{1}{2}$
	Total		**12**	

Q	Solution	Marks	Total	Comments
4	Assume result true for $n = k$			
	Then $u_{k+1} = \dfrac{3}{4 - \left(\dfrac{3^{k+1} - 3}{3^{k+1} - 1}\right)}$	M1		
	$= \dfrac{3(3^{k+1} - 1)}{4(3^{k+1} - 1) - (3^{k+1} - 3)}$	A1		
	$4 \times 3^{k+1} - 3^{k+1} = 3^{k+2}$	A1		clearly shown
	$u_{k+1} = \dfrac{3^{k+2} - 3}{3^{k+2} - 1}$	A1		
	$n = 1 \quad \dfrac{3^2 - 3}{3^2 - 1} = \dfrac{3}{4} = u_1$	B1		
	Induction proof set out properly	E1	6	must have earned previous 5 marks
	Total		**6**	
5	Numerator $= e^{\frac{p\pi i}{8}}$	B1		
	Denominator $= e^{\frac{-q\pi i}{12}}$	B1		
	Fraction $= e^{\frac{p\pi i}{8} + \frac{q\pi i}{12}}$	M1		allow for attempt to subtract powers
	$= e^{\frac{\pi i}{24}(3p+2q)}$	A1		
	$i = e^{\frac{12\pi i}{24}}$	m1		OE
	$3p + 2q = 12$	A1F		ft errors of sign or arithmetic slips
	$p = 2,\ q = 3$	A1	7	CAO
	Alternative 1			
	Numerator $= \cos\frac{p\pi}{8} + i\sin\frac{p\pi}{8}$	(B1)		
	Denominator $= \cos\frac{-q\pi}{12} + i\sin\frac{-q\pi}{12}$	(B1)		needs more than just $\cos\frac{q\pi}{12} - \sin\frac{p\pi}{12}$
	Fraction = $\left(\cos\frac{p\pi}{8} + i\sin\frac{p\pi}{8}\right)\left(\cos\frac{q\pi}{12} + i\sin\frac{q\pi}{12}\right)$	(M1)		
	$= \cos\frac{\pi}{24}(3p + 2q) + i\sin\frac{\pi}{24}(3p + 2q)$	(A1)		
	$= i$ if $\cos\frac{\pi}{24}(3p + 2q) = 0$ or $\sin\frac{\pi}{24}(3p + 2q) = 1$	(m1)		
	$3p + 2q = 12$	(A1F)		
	$p = 2,\ q = 3$	(A1)	(7)	CAO
	Alternative 2			
	LHS $\cos\frac{p\pi}{8} + i\sin\frac{p\pi}{8}$	(B1)		
	RHS $i\cos\frac{q\pi}{12} + \sin\frac{q\pi}{12}$	(B1)		
	$\cos\frac{p\pi}{8} = \sin\frac{q\pi}{12}$ or $\sin\frac{p\pi}{8} = \cos\frac{q\pi}{12}$	(M1)		
	Introduction of $\dfrac{\pi}{2}$	(m1)		
	$\dfrac{p\pi}{8} = \dfrac{\pi}{2} - \dfrac{q\pi}{12}$	(A1)		
	$3p + 2q = 12$	(A1F)		
	$p = 2,\ q = 3$	(A1)	(7)	CAO (correct answers, insufficient working 3/7 only)
	Total		**7**	

Q	Solution	Marks	Total	Comments
6(a)	$7+4x-2x^2=9-2(x-1)^2$	M1A1	2	
(b)	Put $u=\sqrt{2}(x-1)$	M1		allow $u=k(x-1)$ any k
	$\mathrm{d}u=\sqrt{2}\,\mathrm{d}x$	A1F		
	$\mathrm{I}=\frac{1}{\sqrt{2}}\int\frac{\mathrm{d}u}{\sqrt{9-u^2}}$	A1F		ft error in (a); must have u^2 only, ie $\frac{1}{\sqrt{2}}$ outside integrand
	$=\frac{1}{\sqrt{2}}\sin^{-1}\frac{u}{3}$	A1		for $\sin^{-1}\frac{u}{p}$
	Change limits or replace u	m1		provided $\sin^{-1}$
	$=\frac{\pi}{4\sqrt{2}}$ or $\frac{\pi\sqrt{2}}{8}$	A1	6	CAO
	Alternative – if integration is attempted without substitution:			
	$\sin^{-1}$	(M1)		
	$\frac{1}{\sqrt{2}}$	(A1F)		
	$(x-1)$	(A1)		
	$\frac{\sqrt{2}}{3}$	(A1F)		
	Substitution of limits	(m1)		
	$\frac{\pi}{4\sqrt{2}}$	(A1)	(6)	CAO
	Total		**8**	
7(a)	Use of $\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum\alpha\beta$	M1		
		A1	2	AG
(b)	$p=0,\ \ q=5+6\mathrm{i}$	B1,B1	2	
(c)(i)	Substitute $3\mathrm{i}$ for z **or** use $3\mathrm{i}\beta\gamma=-r$	M1		allow for $3\mathrm{i}\beta\gamma=r$
	$-27\mathrm{i}+15\mathrm{i}-18+r=0$ **or** $\beta\gamma=5+6\mathrm{i}+\alpha^2$	A1		any form
	$r=18+12\mathrm{i}$	A1F	3	one error
(ii)	Cubic is $(z-3\mathrm{i})(z^2+3\mathrm{i}z-4+6\mathrm{i})$ or use of $\beta\gamma$ and $\beta+\gamma$	M1A1	2	clearly shown
(iii)	$\mathrm{f}(-2)=0$ or equate imaginary parts	M1		
	$\beta=-2,\ \gamma=2-3\mathrm{i}$	A1,A1F	3	correct answers no working and no check B1 only
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	$1, e^{\frac{2\pi i}{5}}, e^{\frac{4\pi i}{5}}, e^{\frac{-2\pi i}{5}}, e^{\frac{-4\pi i}{5}}$	B1	1	accept e^0
(b)	$\frac{z^5-1}{z-1} = z^4+z^3+z^2+z+1$	B1		B0 if assumed
	$=\left(z-e^{\frac{2\pi i}{5}}\right)\left(z-e^{\frac{4\pi i}{5}}\right)\left(z-e^{\frac{-2\pi i}{5}}\right)\left(z-e^{\frac{-4\pi i}{5}}\right)$	M1A1	3	accept if $e^{\frac{6\pi i}{5}}, e^{\frac{8\pi i}{5}}$ used here
(c)	Correct grouping of linear factors	M1		
	$e^{\frac{2\pi i}{5}} + e^{\frac{-2\pi i}{5}} = 2\cos\frac{2\pi}{5}$	A1		clearly shown
	$\left(z^2 - 2\cos\frac{2\pi}{5}z+1\right)\left(z^2-2\cos\frac{4\pi}{5}z+1\right)$	A1		
	$\div z^2$ to give answer	A1	4	AG
(d)	Substitute into LHS to give w^2+w-1	B1		
	RHS $\left(w-2\cos\frac{2\pi}{5}\right)\left(w-2\cos\frac{4\pi}{5}\right)$	B1		
	Solve $w^2+w-1=0$	M1		
	$w=\frac{-1\pm\sqrt{5}}{2}$	A1		
	$\cos\frac{2\pi}{5}=\frac{\sqrt{5}-1}{4}$	A1		
	with reasons for choice	E1	6	
	Total		**14**	
	TOTAL		**75**	

General Certificate of Education (A-level)
June 2012

Mathematics MFP2

(Specification 6360)

Further Pure 2

Mark Scheme

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Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

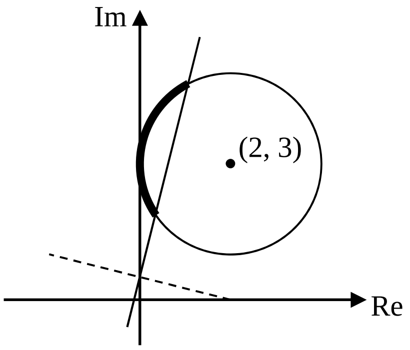
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	Sketch of $y = \cosh x$	B1	1	approximately correct with minimum point above the *x*-axis, symmetrical about *y*-axis
(b)	Attempt to factorise	M1		or complete square or use (correct unsimplified) formula
	$(3\cosh x - 5)(2\cosh x + 1) = 0$	A1		
	$\cosh x \neq -\frac{1}{2}$	E1		indicated or stated (not merely neglected)
	$x = \ln\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}\right)$	M1		evidence of use of formula. Must see –1 or equivalent
		A1F		ft incorrect factorisation
	$= \pm \ln 3$	A1F	6	A1 for $\pm$
	Alternative:			
	$3\left(\frac{e^x + e^{-x}}{2}\right) = 5$			
	$3e^{2x} - 10e^x + 3 = 0$	(M1)		
	$(3e^x - 1)(e^x - 3) = 0$	(A1F)		Correct factors
	$x = \ln\frac{1}{3}$ or $\ln 3$	(A1F)		for both
	NB if $\cosh x = \frac{e^x + e^{-x}}{2}$ used initially, M0 until quartic in e^x is factorised			M1 for e^x–3 is a factor A1 if correct M1 for $3e^x$–1is a factor A1 if correct A1 for $x = \pm \ln 3$ E1 for showing remaining quadratic has no real roots
	Total		**7**	

MFP2

Q	Solution	Marks	Total	Comments
2(a)				
(i)	Circle	B1		Convex loop
	Correct centre	B1		Some indication of position of centre
	Touching Im axis	B1	3	
(ii)	Straight line well to left of centre	B1		$\frac{1}{2}$ line through $(0, \frac{1}{2})$ B0
	through $(0, \frac{1}{2})$	B1		Point approximately between 0 and 1
	$\perp$ to line joining $(-2,1)$ and $(2,0)$	B1	3	
	NB 0/3 for line parallel to *x*-axis 0/3 for line joining the two points (–2, 1) and (2,0) 0/3 for line joining (0,0) to centre of circle			
(b)	Minor arc indicated	B1F	1	ft incorrect position of line or circle
	Total		**7**	

MFP2

Q	Solution	Marks	Total	Comments
3(a)	Attempt to put LHS over common denominator	M1		
	$\frac{2^{r+1}(r+1)-2^r(r+2)}{(r+1)(r+2)}$	A1		any form
	$=\frac{r(2^{r+1}-2^r)}{(r+1)(r+2)}$			
	$=\frac{r2^r}{(r+1)(r+2)}$ must see $r2^{r+1}=2r2^r$	A1	3	clearly shown as AG
(b)	$\frac{2^2}{3}-\frac{2}{2}$ $\frac{2^3}{4}-\frac{2^2}{3}$ ……… $\frac{2^{31}}{32}-\frac{2^{30}}{31}$	M1		3 rows indicated (PI)
	$S_{30}=\frac{2^{31}}{32}-1$ or $S_n=\frac{2^{n+1}}{n+2}-1$	A1		
	$=2^{26}-1$	A1	3	CAO
	Total		**6**	
4(a)(i)	$\alpha+\beta+\gamma=0$	B1	1	
(ii)	$\alpha\beta\gamma=-q$	B1	1	
(b)	$\alpha^3+p\alpha+q=0$	M1		
	$\sum\alpha^3+p\sum\alpha+3q=0$	m1		
	$\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma$	A1	3	AG
	Alternative to (b) Use of $\left(\sum\alpha\right)^3=\left(\sum\alpha^3\right)+6\alpha\beta\gamma+3\left(\sum\alpha\sum\alpha\beta-3\alpha\beta\gamma\right)$	(M1)		
	Substitution of $\sum\alpha=0$	(m1)		
	Result	(A1)		
(c)(i)	$\beta=4-7\text{i},\ \gamma=-8$	B1,B1	2	
(ii)	Attempt at either p or q	M1		
	$p=1$	A1F		
	$q=520$	A1F	3	ft incorrect roots provided p and q are real
(d)	Replace z by $\frac{1}{z}$ in cubic equation	M1 A1F		or $\sum\frac{1}{\alpha}=-\frac{p}{q}, \sum\frac{1}{\alpha\beta}=0, \frac{1}{\alpha\beta\gamma}=-\frac{1}{q}$ ft on incorrect p and/or q
	$520z^3+z^2+1=0$ coefficients must be integers	A1	3	CAO
	Total		**13**	

MFP2

Q	Solution	Marks	Total	Comments
5(a)	$\frac{1}{x} = \cos y$ or $\frac{1}{y} = \cos x$	M1		
	$y = \cos^{-1}\frac{1}{x}$ ie result	A1	2	CSO
(b)	$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{d}{dx}\left(\cos^{-1}\frac{1}{x}\right)$	M1		
	$= -\frac{1}{\sqrt{1-\frac{1}{x^2}}}$ if in terms of u A0	A1		
	$\times\left(-\frac{1}{x^2}\right)$	A1		
	$= \frac{1}{\sqrt{x^4 - x^2}}$	A1	4	clearly shown (AG)
	Alternative			
	$\cos y = \frac{1}{x}$			Use of sec $y = x$ M0
	$-\sin y \frac{dy}{dx} = \frac{-1}{x^2}$	(M1) (A1)		
	Substitute for sin y	(A1)		
	Result	(A1)		
	Total		**6**	

MFP2

Q	Solution	Marks	Total	Comments
6(a)	Use of $\cosh 2x = 2\cosh^2 x - 1$	M1		or $\cosh 4x = 2\cosh^2 2x - 1$
	$\text{RHS} = \frac{1}{2}\cosh 2x + \frac{1}{2}\cosh^2 2x$	A1		
	$= \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)$	A1	3	
	If substituted for both cosh 4*x* and cosh 2*x* in LHS M1 only, until corrected If RHS is put in terms of e^x M1 for correct substitution A1 for correct expansion A1 for correct result			
(b)	$\frac{dy}{dx} = 2\cosh x \sinh x = \sinh 2x$	M1A1		allow A1 for $1 + \left(\frac{dy}{dx}\right)^2 = 1 - 4\cosh^2 x + 4\cosh^4 x$ Incorrect form for $\cosh^2 x$ in terms of cosh 2*x* M1 only
	Or $y = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$ $\frac{dy}{dx} = \frac{2e^{2x} - 2e^{x}}{4}$	(M1)		
	= sinh 2*x*	(A1)		
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$	A1	3	AG
(c)	$S = 2\pi\int_{(0)}^{(\ln 2)} \cosh^2 x \cosh 2x \, dx$	M1A1		allow even if limits missing
	$= 2\pi\int_0^{\ln 2} \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)\, dx$	m1		
	$= \frac{2\pi}{4}\left[x + \frac{2\sinh 2x}{2} + \frac{\sinh 4x}{4}\right]$	A1		Integrated correctly
	Correct use of limits	m1		
	a =128 , *b* = 495	A1,A1	7	accept correct answers written down with no working. Only one A1 if 2π not used
	Total		**13**	

MFP2

Q	Solution	Marks	Total	Comments
7(a)	Assume true for $n = k$			
	Then $\sum_{r=1}^{k+1} \frac{2r+1}{r^2(r+1)^2}$			
	$=1-\frac{1}{(k+1)^2}+\frac{2k+3}{(k+1)^2(k+2)^2}$	M1A1		M1A0 if no LHS
	$=1-\frac{1}{(k+1)^2}\left(1-\frac{2k+3}{(k+2)^2}\right)$	m1		attempt to factorise or put over a common denominator
	$=1-\frac{1}{(k+1)^2}\left(\frac{k^2+2k+1}{(k+2)^2}\right)$	A1		any correct combination starting 1–
	$=1-\frac{1}{(k+2)^2}$	A1		
	True for $n=1$ LHS = RHS = $\frac{3}{4}$	B1		
	Method of induction set out properly	E1	7	must score all 6 previous marks for this mark
(b)	$(n+1)^2 > 10^5$ or $\frac{1}{(n+1)^2} > 10^{-5}$	M1		Condone equals
	$n+1 > 316.2$ $n > 315.2$ $n = 316$	A1	2	
	Total		**9**	

MFP2

Q	Solution	Marks	Total	Comments
8(a)	Use of $(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$	M1		Stated or used
	$\cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$	A1		allow $\frac{2}{3}$ if this line is assumed allow if complex conjugate used
	$z^n + \frac{1}{z^n} = 2\cos n\theta$	A1	3	AG
(b)(i)	$z^8 + 4z^4 + 6 + 4z^{-4} + z^{-8}$	B1	1	allow in retrospect
(ii)	$z^2 + \frac{1}{z^2} = 2\cos 2\theta$ used	B1		Can be implied from (b)(i)
	$(2\cos 2\theta)^4 = 2\cos 8\theta + 8\cos 4\theta + 6$	M1A1		M1 for RHS A1 for whole line
	$\cos^4 2\theta = \frac{1}{8}\cos 8\theta + \frac{1}{2}\cos 4\theta + \frac{3}{8}$	A1F	4	ft coefficients on previous line
	Alternative to (b)(ii) $\cos^4 2\theta = \left(\frac{1+\cos 4\theta}{2}\right)^2$	(M1) (A1)		
	$\cos^2 4\theta = \frac{1}{2}(1+\cos 8\theta)$	(B1)		
	Final result	(A1)		
(c)	$8\cos^4 2\theta = \cos 8\theta + 5 \rightarrow \cos 4\theta = \frac{1}{2}$	M1 A1F		ft provided simplifies to cos 4 $\theta = p$
	$k = \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$	A1	3	CAO
(d)	$\int_0^{\frac{\pi}{2}} \cos^4 2\theta \,\mathrm{d}\theta =$			
	$\left[\frac{\sin 8\theta}{64} + \frac{\sin 4\theta}{8} + \frac{3}{8}\theta\right]_0^{\frac{\pi}{2}}$	M1 A1F		ie their $\cos^4 2\theta$
	$= \frac{3\pi}{16}$	A1	3	AG
	Total		**14**	
	TOTAL		**75**	

Version

General Certificate of Education (A-level)
January 2013

Mathematics MFP2

(Specification 6360)

Further Pure 2

Final

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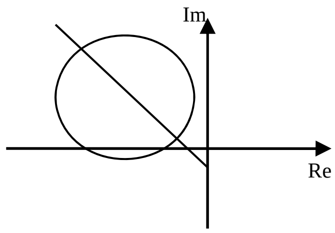
Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	$\cosh x = \frac{1}{2}(e^x + e^{-x})$			*or* $12\cosh x = 6(e^x + e^{-x})$
	or $\sinh x = \frac{1}{2}(e^x - e^{-x})$	M1		*or* $4\sinh x = 2(e^x - e^{-x})$
	$12\cosh x - 4\sinh x =$ $6(e^x + e^{-x}) - 2(e^x - e^{-x})$			
	$12\cosh x - 4\sinh x = 4e^x + 8e^{-x}$	A1 **cso**	2	**AG**
(b)	$4e^x + 8e^{-x} = 33$			
	$\Rightarrow 4e^{2x} - 33e^x + 8 \quad (= 0)$	M1		attempt to multiply by e^x to form quadratic in e^x
	$\Rightarrow (e^x - 8)(4e^x - 1) \quad (= 0)$	m1		factorisation attempt (see below) or correct use of formula
	$\Rightarrow \left(e^x =\right) \; 8, \quad \left(e^x =\right) \; \frac{1}{4}$	A1		correct roots
	$\left(x =\right) \; 3\ln 2$	A1		
	$\left(x =\right) \; -2\ln 2$	A1	5	
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\lvert 4-4\text{i}\rvert = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$	B1		verification that $\lvert -2+\text{i}+6-5\text{i}\rvert = 4\sqrt{2}$
	$\arg(-2+2\text{i}) = \pi - \tan^{-1}(1) = \frac{3\pi}{4}$	B1	2	verification that $\arg(z+\text{i}) = \frac{3\pi}{4}$
				
(b)	Circle	M1		freehand circle sketched
	Centre at $-6+5\text{i}$	A1		clear from diagram or centre stated
	Cutting Re axis but not cutting Im axis	A1		
	"Straight" line	M1		freehand line
	Half line from $0-\text{i}$	A1		not horizontal or vertical but end point at $0-\text{i}$ must be clear from diagram/stated
	gradient –1 (approx)	A1	6	making $45°$ to negative Re axis and positive Im axis
(c)	Calculation based on fact that L_2 passes through centre of L_1	M1		idea of vector $\begin{bmatrix}-4\\4\end{bmatrix}$ from centre
	Q represents $-10+9\text{i}$	A1	2	must write as a complex number
	Total		**10**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\dfrac{1}{5r-2}-\dfrac{1}{5r+3}=\dfrac{5r+3-(5r-2)}{(5r-2)(5r+3)}$	M1		condone omission of brackets for M1
	$=\dfrac{5}{(5r-2)(5r+3)}$	A1cso	2	$A = 5$
(b)	Attempt to use method of differences	M1		at least 2 terms of correct form seen
	$k\left\{\dfrac{1}{3}-\dfrac{1}{5n+3}\right\}$	A1		correct cancellation leaving correct two fractions
	$k\left\{\dfrac{(5n+3)-3}{3(5n+3)}\right\}$	m1		attempt to write with common denominator
	$S_n=\dfrac{1}{5}\left\{\dfrac{(5n+3)-3}{3(5n+3)}\right\}=\dfrac{n}{3(5n+3)}$	A1cso	4	**AG** $k=\dfrac{1}{5}$ used correctly throughout
(c)	$S_\infty=\dfrac{1}{15}$	B1	1	
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\alpha+\beta+\gamma=5$	B1		
	$\alpha\beta\gamma=4$	B1	2	
(ii)	$\alpha\beta\gamma^2+\alpha\beta^2\gamma+\alpha^2\beta\gamma=\alpha\beta\gamma(\alpha+\beta+\gamma)$	M1		
	$=5\times4=20$	A1√	2	FT their results from (a)(i)
(b)(i)	If α,β,γ are all real then $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2\geqslant 0$			
	Hence α,β,γ cannot all be real	E1	1	argument must be sound
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha=k$	B1		$\sum\alpha\beta=k$ PI
	$(\alpha\beta+\beta\gamma+\gamma\alpha)^2$ $=\sum\alpha^2\beta^2+2(\alpha\beta\gamma^2+\alpha\beta^2\gamma+\alpha^2\beta\gamma)$	M1		correct identity for $\left(\sum\alpha\beta\right)^2$
	$=-4+2(20)$	A1√		substituting their result from (a)(ii)
	$k=\pm6$	A1 **cso**	4	must see k=…
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x = \tanh y = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$			
	$xe^y + xe^{-y} = e^y - e^{-y}$	M1		*or* $xe^{2y} + x = e^{2y} - 1$
	$\Rightarrow (x+1)e^{-y} = e^y(1-x)$			
	$\Rightarrow (x+1) = e^{2y}(1-x)$	A1		
	$e^{2y} = \dfrac{1+x}{1-x} \Rightarrow y = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$	A1cso	3	**AG**
(b)	$y = \dfrac{1}{2}\ln(1+x) - \dfrac{1}{2}\ln(1-x)$	M1		
	$\dfrac{dy}{dx} = \dfrac{1}{2(1+x)} + \dfrac{1}{2(1-x)}$	A1		
	$= \dfrac{1-x+1+x}{2(1+x)(1-x)} = \dfrac{2}{2(1-x^2)} = \dfrac{1}{1-x^2}$	A1cso	3	**AG**
				Alternative 1 $\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{(1-x)}{(1+x)} \times \dfrac{d}{dx}\left(\dfrac{1+x}{1-x}\right)$ M1 $\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{(1-x)}{(1+x)} \times \dfrac{(1-x)+(1+x)}{(1-x)^2}$ A1 $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1-x^2}$ A1 cso
(c)	$\int 4\tanh^{-1}x\,dx = 4x\tanh^{-1}x - \int \dfrac{4x}{1-x^2}\,dx$	M1		
	$4x\tanh^{-1}x + 2\ln(1-x^2)$	A1		
	$\tanh^{-1}\frac{1}{2} = \frac{1}{2}\ln 3$	B1		must simplify logarithm to ln3
	Value of integral = $\ln 3 + 2\ln\frac{3}{4}$	A1		any correct form
	$\ln\left(\dfrac{3^3}{2^4}\right)$	A1cso	5	all working must be correct
	Total		**11**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=3t^2 \quad \frac{\mathrm{d}y}{\mathrm{d}t}=12t$	B1		both correct
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 9t^4+144t^2$	M1		'their' $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$
	$s=\int \sqrt{9t^4+144t^2}\,(\mathrm{d}t)$	A1		OE
	$s=\int_0^3 3t\sqrt{t^2+16}\,\mathrm{d}t$	A1cso	4	A = 16
(b)	$k(t^2+A)^{\frac{3}{2}}$	M1		where k is a constant; ft their A
	$(t^2+16)^{\frac{3}{2}}$	A1		
	$25^{\frac{3}{2}}-16^{\frac{3}{2}}$	m1		F(3) – F(0)
	= 61	A1 **cso**	4	**AG**
	Total		**8**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$p(k+1) - p(k) = k^3 + (k+1)^3 + (k+2)^3$ $- (k-1)^3 - k^3 - (k+1)^3$	M1		
	$= (k+2)^3 - (k-1)^3$			
	$= k^3 + 6k^2 + 12k + 8 - (k^3 - 3k^2 + 3k - 1)$	A1		multiplied out & correct unsimplified
	$= 9k^2 + 9k + 9 \quad = 9(k^2 + k + 1)$ which is a multiple of 9 (since $k^2 + k + 1$ is an integer)	A1cso	3	correct algebra plus statement
(ii)	p(1) = 1 + 8 = 9 $\Rightarrow$ p(1) is a multiple of 9	B1		result true for $n = 1$
	$p(k+1) = p(k) + 9(k^2 + k + 1)$ *or* $p(k+1) = p(k) + 9N$	M1		p(k+1) = … **and** result from part (i) considered **and** mention of divisible by 9
	Assume p(k) is a multiple of 9 so p(k) = 9M, where M is integer $\Rightarrow p(k+1) = 9M + 9N = 9(M + N)$			must have word such as "assume" for A1
	$\Rightarrow p(k+1)$ is a multiple of 9	A1		convincingly shown
	Result true for $n = 1$ therefore true for $n = 2$, $n = 3$ etc by induction. (*or* p(n) is a multiple of 9 for all integers $n \geqslant 1$)	E1	4	must earn previous 3 marks before E1 is scored
(b)	$p(n) = (n-1)^3 + n^3 + (n+1)^3$			need to see this OE as evidence
	$= 3n^3 + 6n$	B1		*or* $3n(n^2 + 2)$
	$p(n) = 3n(n^2 + 2)$ & p(n) is a multiple of 9.			both of these required
	Therefore $n(n^2 + 2)$ is a multiple of 3 (for any positive integer n.)	E1	2	plus concluding statement
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$r=8$	B1		
	$\tan^{-1}\pm\frac{4\sqrt{3}}{4}$ or $\pm\frac{\pi}{3}$ seen	M1		or $\frac{\pi}{6}$ marked as angle to Im axis with "vector" in second quadrant on Arg diag
	$\Rightarrow\theta=\frac{2\pi}{3}$	A1	3	$-4+4\sqrt{3}\mathrm{i}=8\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$
(b)(i)	modulus of each root = 2	B1√		
		M1		use of De Moivre – dividing argument by 3
	$\Rightarrow\theta=-\frac{4\pi}{9},\frac{2\pi}{9},\frac{8\pi}{9}$	A2	4	A1 if 3 "correct" values not all in requested interval
				$2\,\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{8\pi}{9}}$
(ii)	Area = $3\times\frac{1}{2}\times PO\times OR\times\sin\frac{2\pi}{3}$	M1		Correct expression for area of triangle PQR
	$=3\times\frac{1}{2}\times2\times2\times\sin\frac{2\pi}{3}$	A1		correct values of lengths in formula
	$=3\sqrt{3}$	A1cso	3	
(c)	Sum of roots (of cubic) = 0	E1		must be stated explicitly
	Sum of 3 roots including Im terms	M1		in form $r(\cos\theta+i\sin\theta)$
	$2\left(\cos\frac{(-)4\pi}{9}+\cos\frac{2\pi}{9}+\cos\frac{8\pi}{9}\right)$	A1		isolating real terms ; correct and with "2"
	$\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}}=\cos\frac{4\pi}{9}-\mathrm{i}\sin\frac{4\pi}{9}$ seen earlier			*or* $\cos\frac{-4\pi}{9}=\cos\frac{4\pi}{9}$ explicitly stated to earn final A1 mark
	$\cos\frac{2\pi}{9}+\cos\frac{4\pi}{9}+\cos\frac{8\pi}{9}=0$	A1**cso**	4	**AG**
	Total		**14**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2013

Mathematics MFP2

(Specification 6360)

Further Pure 2

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

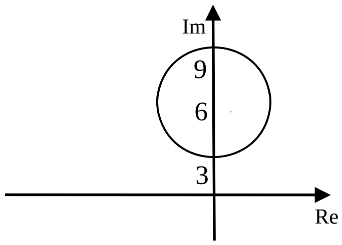
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

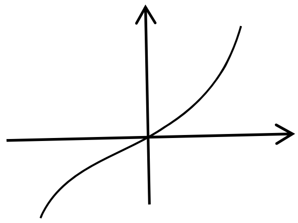
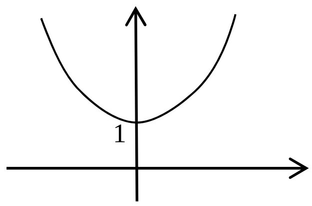
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)				
	Circle	M1		freehand circle
	Centre at 6i	A1		6 marked on Im axis as centre
	Radius 3 & cutting positive Im axis twice	A1	3	radius of 3 clearly indicated with circle in position shown
(b)(i)	(Max $\lvert z \rvert$ is) 9	B1	1	
(ii)	Tangent from *O* to circle	M1		FT their circle position
	Angle of $\frac{\pi}{6}$ or $\frac{\pi}{3}$ ***correctly*** marked	A1		PI ; condone degrees for first A1
	(Max arg z is) $\frac{2\pi}{3}$	A1cso	3	exactly this
	Total		**7**	

Q	Solution	Marks	Total	Comments
2(a)(i)	sinh x graph 	M1		 shape - curve through O, in 1st and 3rd quadrants
	cosh x graph 	M1		 shape - curve all above x-axis
	Gradient of sinh x > 0 at origin **and** cosh x minimum at (0,1)	A1	3	
(ii)	cosh x = 0 has ***no*** solutions **and** sinh x = – k has **one** solution (hence equation has exactly one solution)	E1	1	or cosh x > 0 etc (since y = – k cuts y = sinh x exactly once)
(b)	$\frac{dy}{dx} = 6\cosh x + 2\cosh x \sinh x$	M1 A1		one term correct all correct - may have $6\cosh x + \sinh 2x$
	$(2)\cosh x(3 + \sinh x) = 0$ therefore C has only one stationary point	E1√		putting = 0, factorising **and** concluding statement (may be later)
	$\Rightarrow \sinh x = -3$	m1		finding sinh x from “their” equation
	$\cosh^2 x = 10$			
	$y\,(= -18 + 10) = -8$	A1	5	answer must be integer so do not accept calculator approximation rounded to –8
	Total		**9**	

Q	Solution	Marks	Total	Comments
3	$n = 1$, $\frac{3+1}{3-1} = \frac{4}{2} = 2$ ($u_1 = 2$ so formula is) true when $n = 1$	B1		be convinced they have used $u_n = \frac{3n+1}{3n-1}$
	Assume formula is true for $n = k$ (*)			
	$(u_{k+1} =)\ \dfrac{5\frac{3k+1}{3k-1} - 3}{3\frac{3k+1}{3k-1} - 1}$	M1		clear attempt at RHS of this formula
		m1		clear attempt to remove "double fraction"
	$(u_{k+1} =)\ \frac{5(3k+1) - 3(3k-1)}{3(3k+1) - (3k-1)}$	A1		$\frac{6k+8}{6k+4}$
	$u_{k+1} = \frac{3k+4}{3k+2}$ *or* $u_{k+1} = \frac{3(k+1)+1}{3(k+1)-1}$ Hence formula is true for $n = k+1$ (**)	A1cso		must have "u_{k+1} = " on at least this line
	must have lines (*) & (**) **and** "Result true for $n = 1$ therefore true for $n = 2$, $n = 3$ etc by induction."	E1	6	must also have earned previous 5 marks before E1 is scored
	Total		**6**	
4(a)	$f(r) - f(r-1) =$ $r^2(2r^2 - 1) - (r-1)^2(2(r-1)^2 - 1)$			condone one slip here
	$= 2r^4 - r^2 - (r^2 - 2r + 1)(2r^2 - 4r + 1)$	M1		attempt to multiply out "their" $f(r-1)$
	$= 2r^4 - r^2 - (2r^4 - 8r^3 + 11r^2 - 6r + 1)$	A1		$f(r)$ & $f(r-1)$ expanded correctly condone correct unsimplified
	$= 8r^3 - 12r^2 + 6r - 1$ $= (2r-1)^3$	A1cso	3	**AG**
(b)	Attempt to use method of differences	M1		
	$f(2n) - f(n)$	m1		$(2n)^2\{2(2n^2) - 1\} - n^2(2n^2 - 1)$
	$f(2n) - f(n) = 4n^2(8n^2 - 1) - n^2(2n^2 - 1)$	A1		
	$= 30n^4 - 3n^2$ $= 3n^2(10n^2 - 1)$	A1cso	4	**AG** be convinced
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)(i)	$(\alpha\beta\gamma =) \quad -37+36\text{i}$	B1	1	
(ii)	$(\beta\gamma =) \quad (-2+3\text{i})(1+2\text{i}) = -2+3\text{i}-4\text{i}-6$	M1		correct unsimplified but must simplify i^2
	$(-8-\text{i})\ \alpha = -37+36\,\text{i}$			
	$\Rightarrow (8+\text{i})\ \alpha = 37-36\,\text{i}$	A1cso	2	**AG** be convinced
(iii)	$\Rightarrow \alpha = \dfrac{37-36\text{i}}{8+\text{i}} \times \dfrac{8-\text{i}}{8-\text{i}}$	M1		
	$= \dfrac{296-37\text{i}-288\text{i}-36}{65}$	A1		correct unsimplified
	$= \dfrac{260-325\text{i}}{65}$			
	$= 4-5\text{i}$	A1cao	3	
				Alternative $(8+\text{i})(m+n\,\text{i}) = 37-36\,\text{i}$ $8m-n=37;\ m+8n=-36$ M1 *either* $m=4$ *or* $n=-5$ A1 $\alpha = 4-5\text{i}$ A1
(b)	$\alpha+\beta+\gamma = -p$			
	$-2+3\text{i}+1+2\text{i}+4-5\text{i} = 3$			
	$(\Rightarrow p =)\ -3$	B1	1	
(c)	$\alpha\beta+\beta\gamma+\gamma\alpha = q$			$q = \sum\alpha\beta$ and attempt to evaluate three
	$(7+22\text{i}) + (-8-\text{i}) + (14+3\text{i}) = q$	M1		products FT "their" α
	$q = 13+24\text{i}$	A1cao	2	
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)	$(5\cosh x - 3\sinh x)$			
	$= \frac{5}{2}\left(e^x + e^{-x}\right) - \frac{3}{2}\left(e^x - e^{-x}\right)$	M1		$\cosh x$ **and** $\sinh x$ correct in terms of e^x
	$= e^x + 4e^{-x}$	A1		may be seen as denominator
	$\frac{1}{5\cosh x - 3\sinh x} = \frac{e^x}{4 + e^{2x}}$	A1cso	3	** must have left hand-side ; $m = 4$
(b)	$u = e^x \Rightarrow du = e^x dx$	M1		or $\frac{du}{dx} = e^x$
	$\Rightarrow \int \frac{1}{4+u^2}(du)$	A1√		FT "their" m from part(a) $\Rightarrow \int \frac{1}{m+u^2} du$
	$= \frac{1}{2}\tan^{-1}\frac{u}{2}$	A1√		FT "their" $\frac{1}{\sqrt{m}}\tan^{-1}\frac{u}{\sqrt{m}}$
	$x = 0 \Rightarrow u = 1 \quad x = \ln 2 \Rightarrow u = 2$			
	$\frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}\frac{1}{2}$	A1√		FT "their" $\frac{1}{\sqrt{m}}\left(\tan^{-1}\frac{2}{\sqrt{m}} - \tan^{-1}\frac{1}{\sqrt{m}}\right)$
	$= \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\frac{1}{2}$	A1cso	5	**AG**
	Total		**8**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}u}\left(2u\sqrt{1+4u^2}\right)=\frac{8u^2}{\sqrt{1+4u^2}}+2\sqrt{1+4u^2}$	M1		M1 for clear use of product rule (condone one error in one term)
		A1		correct unsimplified
	$\frac{\mathrm{d}}{\mathrm{d}u}\left(\sinh^{-1}2u\right)=\frac{2}{\sqrt{1+4u^2}}$	B1		
	$\frac{8u^2+2}{\sqrt{1+4u^2}}=\frac{2(1+4u^2)}{\sqrt{1+4u^2}}=2\sqrt{1+4u^2}$			be convinced – must see this line OE
	$\frac{\mathrm{d}}{\mathrm{d}u}\left(2u\sqrt{1+4u^2}+4\sinh^{-1}2u\right)=4\sqrt{1+4u^2}$	A1cso	4	all working must be correct (not enough to just say k = 4)
(ii)	$\frac{1}{\text{“their”}k}\left[2u\sqrt{1+4u^2}+\sinh^{-1}2u\right]_0^1$	M1		anti differentiation
	$=\frac{\sqrt{5}}{2}+\frac{1}{4}\sinh^{-1}2$	A1√	2	FT “their” k or even use of k
(b)(i)	$y=\frac{1}{2}\cos 4x$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=A\sin 4x$			$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 4x$
	substituted into $\int K\,y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,(\mathrm{d}x)$	M1		clear attempt to use formula for CSA
	$(S=)\ \int_0^{\frac{\pi}{8}}2\pi\times\frac{1}{2}\cos 4x\sqrt{1+4\sin^2 4x}\,\mathrm{d}x$ = printed answer (combining $2\times\frac{1}{2}$)	A1cso	2	**AG** $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\sin 4x$ **and** $2\times\frac{1}{2}$ **and** dx must be seen to award A1cso
(ii)	$u=\sin 4x\Rightarrow \mathrm{d}u=4\cos 4x\,\mathrm{d}x$	M1		condone $\mathrm{d}u=B\cos 4x\,\mathrm{d}x$ for M1
	$(S=)\ \frac{\pi}{4}\int_0^1\sqrt{1+4u^2}\,(\mathrm{d}u)$	A1		condone limits seen later
		m1		use of their result from (a)(ii) **correctly** FT “their” B
	$(S=)\ \frac{\pi\sqrt{5}}{8}+\frac{\pi}{16}\sinh^{-1}2$	A1cso	4	OE
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)(i)	$\cos 4\theta + \mathrm{i}\sin 4\theta = (\cos\theta + \mathrm{i}\sin\theta)^4$	M1		De Moivre & attempt to expand RHS
	$\cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta + 6\mathrm{i}^2\cos^2\theta\sin^2\theta$ $+ 4\mathrm{i}^3\cos\theta\sin^3\theta + \mathrm{i}^4\sin^4\theta$	A1		any correct expansion
	Equating "their" real parts	m1		or imaginary parts
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$	A1		**AG** be convinced
	$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	B1	5	correct
(ii)	$\tan 4\theta = \dfrac{\text{"their expression for" } \sin 4\theta}{\text{"their expression for" } \cos 4\theta}$	M1		
	Division by $\cos^4\theta$	m1		
	$\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	A1	3	**AG** be convinced
(b)	$(\tan 4\theta = 1 \Rightarrow) \quad 1 = \dfrac{4t - 4t^3}{1 - 6t^2 + t^4}$	M1		when $\theta = \dfrac{\pi}{16}$
	$1 - 6t^2 + t^4 = 4t - 4t^3$ $\Rightarrow t^4 + 4t^3 - 6t^2 - 4t + 1 = 0$	A1		**AG** be convinced
	$\theta = \dfrac{\pi}{16}$ *satisfies* $\tan 4\theta = 1$ $\Rightarrow \tan\dfrac{\pi}{16}$ is root of quartic equation	E1		both statements required
	(other roots are) $\tan\dfrac{5\pi}{16}, \tan\dfrac{9\pi}{16}, \tan\dfrac{13\pi}{16}$	B1	4	or equivalent tan expressions
(c)	$\sum\alpha = -4$ **and** $\sum\alpha\beta = -6$	B1		watch for minus signs
	$\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$	M1		correct formula
	$(= 16 + 12) = 28$	A1cso		
	$\tan\dfrac{9\pi}{16} = -\tan\dfrac{7\pi}{16}$, $\tan\dfrac{13\pi}{16} = -\tan\dfrac{3\pi}{16}$	B1		explicitly seen
	$\tan^2\dfrac{\pi}{16} + \tan^2\dfrac{3\pi}{16} + \tan^2\dfrac{5\pi}{16} + \tan^2\dfrac{7\pi}{16} = 28$	A1cso	5	**AG** must earn previous 4 marks
	Total		**17**	
	TOTAL		**75**	

A-LEVEL

Mathematics

Further Pure 2 – MFP2

Mark scheme

6360

June 2014

Version/Stage: Final

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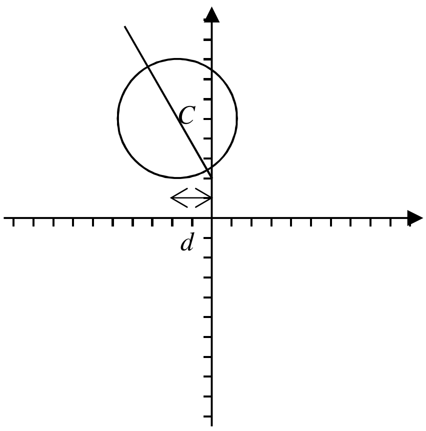
Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a)	$r = 9$	B1		
	$\theta = -\frac{\pi}{2}$	B1		condone $-1.57\ldots$ here only
			2	$-9\text{i} = 9\text{e}^{-\text{i}\frac{\pi}{2}}$
(b)	$r = \sqrt{3}$	B1√		follow through $(their\ r)^{\frac{1}{4}}$; accept $9^{\frac{1}{4}}$ etc
	$(their\ \theta)/4$	M1		generous
	$\theta = -\frac{5\pi}{8}, -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$	A1		two angles correct in correct interval
		A1		**exactly four** angles correct mod 2π
	$\sqrt{3}\,\text{e}^{-\frac{\text{i}5\pi}{8}}, \sqrt{3}\,\text{e}^{-\frac{\text{i}\pi}{8}}, \sqrt{3}\,\text{e}^{\frac{\text{i}3\pi}{8}}, \sqrt{3}\,\text{e}^{\frac{\text{i}7\pi}{8}}$	A1	5	four correct roots in correct interval and in given form; accept $3^{\frac{1}{2}}$ for $\sqrt{3}$
	Total		**7**	

1(a) Accept correct values of r and θ for full marks without candidates actually writing $9\text{e}^{-\text{i}\frac{\pi}{2}}$.
Do not accept angles outside the required interval.
Example "$\theta = -\frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$" scores **B0**

(b) Condone $r = 1.73\ldots$ for **B1** only. Do not follow through a negative value of r for **B1√**.
Example $\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ scores **M1 A1 A1**
Example $\sqrt{3}\,\text{e}^{-\frac{\text{i}\pi}{8}+\frac{\text{i}k\pi}{2}}$ scores **B1 M1** then $k = -1, 0, 1, 2$ scores **A1 A1** with final **A1** only earned when four roots are written in given form

Q	Solution	Mark	Total	Comment
2(a)	Straight line	**M1**		
	Half line from 2 on Im axis	**A1**		not vertical or horizontal
	Making approx. 30° to positive Im axis & 60° to negative Re axis	**A1**	**3**	
(b)(i)	Circle with centre on 'their' L	**M1**		
	Circle correct and touching Im $z = 2$	**A1**	**2**	lowest point of circle at approx 2
				
(b)(ii)	$d = 3\tan\dfrac{\pi}{6}$	**M1**		any correct expression for distance or $\dfrac{b-2}{a} = -\sqrt{3}$ for **M1**
	$a = -\sqrt{3}$	**A1**		condone -1.73 or better
	$b = 5$	**B1**	**3**	centre is $-\sqrt{3} + 5\mathrm{i}$
	Total		**8**	

(a) The two **A1** marks are independent.

(b) (i) If candidate draws a horizontal line at Im $z = 2$ then award **A1** if there is a clear intention for their circle to touch this line.
Allow freehand circle where centre is intended to be on "their" L for **M1** but withhold **A1** if L is in wrong quadrant or drawing of circle is very poor.
Award **A0** if candidate has not scored full marks in **(a)** .

Q	Solution	Mark	Total	Comment
3 (a)	$k^2+7k+14$	B1	1	
(b)	When $n=1$ LHS = $1\times2\times1=2$ RHS = 16 – 14 = 2 Therefore true for $n=1$	B1		
	Assume formula is true for $n=k$ (*) Add (k+1)th term (to both sides)	M1		(k+1)th term must be correct
	$\sum_{r=1}^{k+1} r(r+1)\left(\frac{1}{2}\right)^{r-1}$ $=16-\left(k^2+5k+8\right)\left(\frac{1}{2}\right)^{k-1}$ $+(k+1)(k+2)\left(\frac{1}{2}\right)^{k}$	A1		**A0** if only considering RHS
	$=16-\left(\frac{1}{2}\right)^{k}\left(2k^2+10k+16-k^2-3k-2\right)$ $=16-\left(\frac{1}{2}\right)^{k}\left(k^2+7k+14\right)$	A1		
	$=16-\left(\left(k+1\right)^2+5\left(k+1\right)+8\right)\left(\frac{1}{2}\right)^{k}$	A1		from part (a)
	Hence formula is true when $n=k+1$ (**) but true for $n=1$ so true for $n=2, 3, \ldots$ by induction (***)	E1	6	must have (*), (**) and (***) and must have earned previous 5 marks
	Total		**7**	

(b) For **B1,** accept "n=1 RHS=LHS=2" but must mention here or later that the result is "true when n=1"

Alternative to (***) is "therefore true for all positive integers n" etc
However, " true for all $n \geqslant 1$ " is incorrect and scores **E0**

May define P(k) as the "proposition that the formula is true when $n = k$ " and earn full marks. However, if P(k) is not defined then allow **B1** for showing P(1) is true but withhold **E1** mark.

Q	Solution	Mark	Total	Comment
4 (a) (i)	$\alpha+\beta+\gamma=-2$	B1		
	$\alpha\beta+\beta\gamma+\gamma\alpha=3$	B1	2	
(ii)	$\alpha^2+\beta^2+\gamma^2$			
	$=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$	M1		correct formula
	$=4-6=-2$	A1cso	2	**AG** be convinced; must see $4-6$ **A0** if $\alpha+\beta+\gamma$ or $\alpha\beta+\beta\gamma+\gamma\alpha$ not correct
(b) (i)	$\sum(\alpha+\beta)(\beta+\gamma)=\sum\alpha^2+3\sum\alpha\beta$	M1		or may use $12+4\sum\alpha+\sum\alpha\beta$
	$=-2+9$	m1		ft their $\alpha\beta+\beta\gamma+\gamma\alpha$
	$=7$	A1	3	
(ii)	$\alpha\beta\gamma=4$	B1		**PI** when earning **m1** later
	$(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$ $=\sum\alpha\sum\alpha\beta-\alpha\beta\gamma$	M1		or $(-2-\alpha)(-2-\beta)(-2-\gamma)$ $=-8-4\sum\alpha-2\sum\alpha\beta-\alpha\beta\gamma$
	$=-6-4$	m1		Sub their $\sum\alpha$, $\sum\alpha\beta$ & $\alpha\beta\gamma$
	$=-10$	A1	4	
(c)	Sum of new roots $=2\sum\alpha=-4$	B1		***or* NMS** coefficient of z^2 written as +4
	$z^3\pm4z^2+\text{"their 7"}z-\text{"their }-10\text{"}$ (=0)	M1		correct sub of their results from part **(b)**
	New equation $z^3+4z^2+7z+10=0$	A1	3	**Alternative** $y=-2-z$ **B1** $(-2-y)^3+2(-2-y)^2+3(-2-y)-4=0$ **M1** $y^3+4y^2+7y+10=0$ **A1** **NB** candidate may do this first and then obtain results for part (b)
	Total		14	
(a)(ii)	Accept $\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum\alpha\beta$ etc for **M1**			
(b)(ii)	If **B1** not earned, award **m1** for using $\alpha\beta\gamma=\pm4$ **.**			
(c)	For **M1** the signs of coefficients must be correct FT their results from **(b)** but condone missing "= 0" However, for **A1** the equation must be correct (any variable) including "= 0"			

Q	Solution	Mark	Total	Comment
5(a)	$\left(e^{\theta}-e^{-\theta}\right)^3 = e^{3\theta}-3e^{\theta}+3e^{-\theta}-e^{-3\theta}$ OE	**B1**		correct expansion; terms need not be combined
	$4\sinh^3\theta+3\sinh\theta=$ $\frac{4}{8}\left(e^{3\theta}-3e^{\theta}+3e^{-\theta}-e^{-3\theta}\right)+\frac{1}{2}\left(3e^{\theta}-3e^{-\theta}\right)$	**M1**		correct expression for $\sinh\theta$ and attempt to expand $\left(e^{\theta}-e^{-\theta}\right)^3$
	$=\frac{1}{2}\left(e^{3\theta}-e^{-3\theta}\right)=\sinh 3\theta$	**A1**	**3**	**AG** identity proved
(b)	$16\sinh^3\theta+12\sinh\theta-3=0$ $\Rightarrow 4\sinh 3\theta-3=0$	**M1**		attempt to use previous result
	$\sinh 3\theta=\frac{3}{4}$	**A1**		
	$(3\theta=)\ln\left(\frac{3}{4}+\sqrt{\frac{9}{16}+1}\right)$	**m1**		correct ln form of $\sinh^{-1}$ for "their" $\frac{3}{4}$
	$\theta=\frac{1}{3}\ln 2$	**A1**	**4**	
(c)	$x=\sinh\theta=\frac{1}{2}\left(2^{\frac{1}{3}}-2^{-\frac{1}{3}}\right)$	**M1**		correctly substituting their expression for θ into $\sinh\theta$ removing any ln terms
	$2^{-\frac{2}{3}}-2^{-\frac{4}{3}}$	**A1**	**2**	
	Total		**9**	

(a) For **M1**, must attempt to expand $\left(e^{\theta}-e^{-\theta}\right)^3$ with at least 3 terms and attempt to add expressions for two terms on LHS.
For **A1**, must see both sides of identity connected with at least trailing equal signs.

(b) Withhold final **A1** if answer is given as $x=\frac{1}{3}\ln 2$.
Alternative: $2e^{3\theta}-2e^{-3\theta}-3=0 \Rightarrow 2e^{6\theta}-3e^{3\theta}-2=0$ *so* $(e^{3\theta}-2)(2e^{3\theta}+1)=0$
scores **M1** for $e^{k\theta}=p$ (quite generous) **A1** for $e^{3\theta}=2$ (and perhaps $e^{3\theta}=-0.5$)
then **m1** for correct ft from $e^{k\theta}=p \Rightarrow k\theta=\ln p$ and final **A1** for $\theta=\frac{1}{3}\ln 2$ and no other solutions

Q	Solution	Mark	Total	Comment
6(a)(i)	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$	M1		
	$z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ $= \cos n\theta - \mathrm{i}\sin n\theta$	E1		or $\frac{1}{\cos n\theta + \mathrm{i}\sin n\theta} \times \frac{\cos n\theta - \mathrm{i}\sin n\theta}{\cos n\theta - \mathrm{i}\sin n\theta}$ **=...** shown – not just stated
	$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$	A1	3	**AG**
(ii)	$\left(z^n + \frac{1}{z^n} =\right) \quad 2\cos n\theta$	B1	1	
(b)(i)	$\left(z - \frac{1}{z}\right)^2 \left(z + \frac{1}{z}\right)^2 = z^4 - 2 + \frac{1}{z^4}$	B1	1	***or*** $z^4 - 2 + z^{-4}$
(ii)	$(2\mathrm{i}\sin\theta)^2 (2\cos\theta)^2 = 2\cos 4\theta - 2$ $-16\sin^2\theta\cos^2\theta = 2\cos 4\theta - 2$	M1		using previous results
	$8\sin^2\theta\cos^2\theta = 1 - \cos 4\theta$	A1cso	2	
(c)	$x = 2\sin\theta \Rightarrow \mathrm{d}x = 2\cos\theta\,\mathrm{d}\theta$	M1		$x = 2\sin\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = k\cos\theta$
	$\int x^2\sqrt{4 - x^2}\,\mathrm{d}x = \int 16\sin^2\theta\cos^2\theta\,\mathrm{d}\theta$	A1		
	$= \int (2 - 2\cos 4\theta)\,(\mathrm{d}\theta)$	m1		correct ***or*** **FT** their (b)(ii) result
	$= 2\theta - \frac{1}{2}\sin 4\theta$	B1√		**FT** integrand of form $k(1 - \cos 4\theta)$
	$= \left[\pi - \frac{1}{2}\sin 2\pi\right] - \left[\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right]$			$x = 1 \Rightarrow \theta = \frac{\pi}{6}; \quad x = 2 \Rightarrow \theta = \frac{\pi}{2};$
	$= \frac{2\pi}{3} + \frac{\sqrt{3}}{4}$	A1cso	5	
	Total		**12**	

(a)(i)	May score **M1 E0 A1** if $z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$ merely quoted and not proved. Condone poor use of brackets for **M1** but not for **A1**.
(b)(ii)	For **M1**, must use $2\mathrm{i}\sin\theta$ and "their" $2\cos\theta$ on LHS but condone poor use of brackets etc when squaring.
(c)	For **A1cso**, must simplify $\sin^{-1}1$ correctly in terms of π. Allow first **A1** for missing $\mathrm{d}\theta$ or incorrect limits used/seen, but withhold final **A1cso.**

Q	Solution	Mark	Total	Comment
7 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1+x}{1-x}\right)=\frac{1-x+1+x}{(1-x)^2}=\frac{2}{(1-x)^2}$	B1		ACF
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+u^2}$	M1		where $u=\frac{1+x}{1-x}$
	$\times\frac{2}{(1-x)^2}$	A1		correct unsimplified
	$=\frac{2}{(1-x)^2+(1+x)^2}=\frac{1}{1+x^2}$	A1	4	AG be convinced
(b)	either $\frac{\mathrm{d}}{\mathrm{d}x}(\tan^{-1}x)=\frac{1}{1+x^2}$ or $\int \frac{1}{1+x^2}\,\mathrm{d}x=\tan^{-1}x \quad (+c)$	B1		
	$\Rightarrow \tan^{-1}\left(\frac{1+x}{1-x}\right)=\tan^{-1}x+C$	M1		
	Putting $x=0$ gives $C=\tan^{-1}1=\frac{\pi}{4}$			
	$\Rightarrow \tan^{-1}\left(\frac{1+x}{1-x}\right)-\tan^{-1}x=\frac{\pi}{4}$	A1	3	AG
	Total		7	

(a) **Alternative** $\tan y=\frac{1+x}{1-x}$

$\sec^2 y\,\frac{\mathrm{d}y}{\mathrm{d}x}$ **M1** $=\frac{2}{(1-x)^2}$ **B1**

$\left(1+\left(\frac{1+x}{1-x}\right)^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}$ **A1** with final **A1** for proving given result

(b) Must see $\frac{\mathrm{d}}{\mathrm{d}x}(\tan^{-1}x)=\frac{1}{1+x^2}$ within attempt at part **(b)** to award **B1**

Q	Solution	Mark	Total	Comment
8(a)	$y=2(x-1)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=(x-1)^{-\frac{1}{2}}$	B1		
	$1+\left(\frac{dy}{dx}\right)^2=1+\frac{1}{x-1}$	M1		ft their $\frac{dy}{dx}$
	$(s=)\int_{(2)}^{(9)}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,(dx)\quad(=)$			$s=\int_2^9\sqrt{1+\frac{1}{x-1}}\,dx$
	$\int_2^9\sqrt{\frac{x}{x-1}}\,dx$	A1	3	(be convinced) **AG** (must have limits & dx on final line)
(b)(i)	$\cosh^{-1}3=\ln\left(3+\sqrt{8}\right)$	M1		
	$\left(1+\sqrt{2}\right)^2=3+2\sqrt{2}=3+\sqrt{8}$			need to see this line OE
	$\cosh^{-1}3=\ln\left(1+\sqrt{2}\right)^2=2\ln\left(1+\sqrt{2}\right)$	A1	2	**AG** (be convinced)
(ii)	$x=\cosh^2\theta \Rightarrow dx=2\cosh\theta\sinh\theta\,d\theta$	M1		$\frac{dx}{d\theta}=k\cosh\theta\sinh\theta$ **OE**
	$(s=)\int\frac{\cosh\theta}{\sinh\theta}2\cosh\theta\sinh\theta\,d\theta$	A1		including $d\theta$ on this or later line
	$2\cosh^2\theta=1+\cosh 2\theta$ **OE**	B1		double angle formula or $\frac{1}{2}\left(e^{2\theta}+2+e^{-2\theta}\right)$
	$(s=)\ \theta+\frac{1}{2}\sinh 2\theta$	A1		or $\left(\frac{1}{4}e^{2\theta}+\theta-\frac{1}{4}e^{-2\theta}\right)$
	$\left.\begin{array}{l}\cosh^{-1}3+\frac{1}{2}\sinh(2\cosh^{-1}3)\\-\cosh^{-1}\sqrt{2}-\frac{1}{2}\sinh\left(2\cosh^{-1}\sqrt{2}\right)\end{array}\right\}$	m1		correct use of correct limits
	$(s=2\ln\left(1+\sqrt{2}\right)-\ln\left(1+\sqrt{2}\right)+6\sqrt{2}-\sqrt{2}$			must see this line OE
	$=5\sqrt{2}+\ln\left(1+\sqrt{2}\right)$	A1	6	**partial AG** (be convinced)
	Total		11	
	TOTAL		75	

(b)(i)	**SC1** for $\cosh\left(2\ln\left(1+\sqrt{2}\right)\right)=\frac{1}{2}\left(\left(1+\sqrt{2}\right)^2+\left(1+\sqrt{2}\right)^{-2}\right)=\frac{1}{2}\left(3+2\sqrt{2}+3-2\sqrt{2}\right)=3\Rightarrow\cosh^{-1}3=2\ln\left(1+\sqrt{2}\right)$
(ii)	Another possible correct form for **m1** is $2\ln\left(1+\sqrt{2}\right)-\ln\left(1+\sqrt{2}\right)+\frac{1}{2}\sinh\left(4\ln\left(1+\sqrt{2}\right)\right)-\frac{1}{2}\sinh\left(2\ln\left(1+\sqrt{2}\right)\right)$

AQA

A-LEVEL
Mathematics

Further Pure 2 – MFP2
Mark scheme

6360
June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$r+1 = A(r+2)+B$ **or** $1 = \frac{A(r+2)}{r+1} + \frac{B}{r+1}$	M1		**OE** with factorials removed
	either $A=1$ or $B=-1$	A1		**correctly** obtained
	$\frac{1}{(r+2)r!} = \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$	A1	3	allow if seen in part **(b)**
(b)	$\frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \ldots$ $\frac{1}{(n+1)!} - \frac{1}{(n+2)!}$	M1		use of their result from part **(a)** at least twice
	Sum $= \frac{1}{2} - \frac{1}{(n+2)!}$	A1	2	must simplify 2! and must have scored at least **M1 A1** in part **(a)**
	Total		**5**	

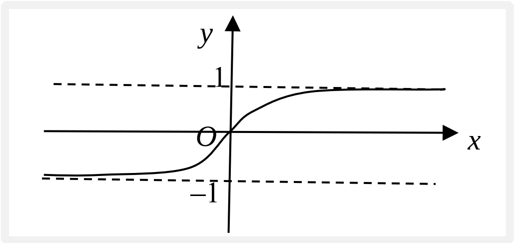
(a) **Alternative Method** Substituting two values of *r* to obtain two correct equations in *A* and *B* with factorials evaluated correctly

$r=0 \Rightarrow \frac{1}{2} = A + \frac{B}{2}$; $r=1 \Rightarrow \frac{1}{3} = \frac{A}{2} + \frac{B}{6}$ earns **M1** then **A1, A1** as in main scheme

NMS $\frac{1}{(r+1)!} - \frac{1}{(r+2)!}$ earns **3 marks**.

However, using an ***incorrect*** expression resulting from poor algebra such as $1 = A(r+2)! + B(r+1)!$ with candidate often fluking $A = 1$, $B = -1$ scores **M0** ie zero marks which you should denote as FIW
These candidates can then score a maximum of **M1** in part **(b).**

(b) **ISW** for incorrect simplification after correct answer seen

Q2	Solution	Mark	Total	Comment
(a)				
	Graph roughly correct through O	M1		condone infinite gradient at O for **M1**
	Correct behaviour as $x \to \pm\infty$ & grad at O	A1		
	Asymptotes have equations $y = 1$ & $y = -1$	B1	3	must state equations
(b)	$\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$ **;** $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	B1		**both** correct **ACF** or **correct** squares of these expressions seen
	($\operatorname{sech}^2 x + \tanh^2 x =$ **)** $\dfrac{2^2 + \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$	M1		attempt to combine their squared terms with correct single denominator
	$\operatorname{sech}^2 x + \tanh^2 x = \dfrac{e^{2x} + 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} = 1$	A1	3	**AG** valid proof convincingly shown to equal 1 **including LHS seen**
(c)	$6(1 - \tanh^2 x) = 4 + \tanh x$	B1		correct use of identity from part **(b)**
	$6\tanh^2 x + \tanh x - 2 \quad (= 0)$	M1		forming quadratic in $\tanh x$
	$\tanh x = \dfrac{1}{2}$, $\tanh x = -\dfrac{2}{3}$	A1		obtained from correct quadratic
	$\tanh x = k \Rightarrow x = \dfrac{1}{2}\ln\left(\dfrac{1+k}{1-k}\right)$	A1F		FT a value of k provided $\lvert k \rvert < 1$
	$x = \dfrac{1}{2}\ln 3$, $x = \dfrac{1}{2}\ln\dfrac{1}{5}$	A1	5	both solutions correct and no others any equivalent form involving ln
	Total		**11**	

(a) Actual asymptotes need not be shown, but if asymptotes are drawn then curve should not cross them for **A1**.
Gradient should not be infinite at O for **A1.**

(b) Condone trailing equal signs up to final line provided "$\operatorname{sech}^2 x + \tanh^2 x =$" is seen on previous line for **A1**
Denominator may be $e^{4x} + 4e^{2x} + 6 + e^{4x} + 4e^{-2x} + e^{-4x}$ etc for **M1** and **A1**

Accept $\operatorname{sech}^2 x + \tanh^2 x = \dfrac{\left(e^x + e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2} = 1$ for **A1**

Alternative : $\left(\dfrac{1}{\cosh^2 x} + \dfrac{\sinh^2 x}{\cosh^2 x} =\right)$ $\dfrac{1 + \left(\frac{1}{2}(e^x - e^{-x})\right)^2}{\left(\frac{1}{2}(e^x + e^{-x})\right)^2}$ scores **B1 M1**

and then **A1** for $\operatorname{sech}^2 x + \tanh^2 x = \dfrac{\frac{1}{4}e^{2x} + \frac{1}{4}e^{-2x} + \frac{1}{2}}{\frac{1}{4}e^{2x} + \frac{1}{2} + \frac{1}{4}e^{-2x}} = 1$, (all like terms combined in any order).

Q3	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 1 - \frac{1}{t^2}$	B1		**OE** eg $\frac{t(2t)-(t^2+1)}{t^2}$ **ACF**
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{2}{t}$	B1		
	$\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 =\right) \quad 1 - \frac{2}{t^2} + \frac{1}{t^4} + \frac{4}{t^2}$	M1		squaring and adding their expressions and attempting to multiply out
	$1 + \frac{2}{t^2} + \frac{1}{t^4} \quad = \left(1 + \frac{1}{t^2}\right)^2$	A1	4	**AG** be convinced
(b)	$2\pi\int_1^2 (2\ln t)\left(1+\frac{1}{t^2}\right)\mathrm{d}t$	B1		must have 2π , limits and dt
		M1		integration by parts - clear attempt to integrate $1+\frac{1}{t^2}$ and differentiate $2\ln t$
	$(2\pi)\left\{(2\ln t)\left(t-\frac{1}{t}\right) - \int \frac{2}{t}\left(t-\frac{1}{t}\right)(\mathrm{d}t)\right\}$	A1		correct (may omit limits, 2π and dt)
	$2\pi\left[(2\ln t)\left(t-\frac{1}{t}\right) - \left(2t+\frac{2}{t}\right)\right]$	A1		correct including 2π (no limits required)
	$= 2\pi(3\ln 2 - 5 + 4)$ $= \pi(6\ln 2 - 2)$	A1	5	
	Total		9	

(b) May have two separate integrals and attempt both using integration by parts for **M1**

Must see $(2\pi)\left\{2t\ln t - \int 2(\mathrm{d}t) \quad -\left(2t^{-1}\ln t - \int 2t^{-2}(\mathrm{d}t)\right)\right\}$ (may omit limits, 2π and dt) for first **A1**

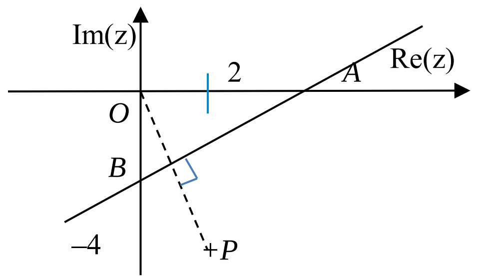
and $2\pi\left[(2t\ln t - 2t) - (2t^{-1}\ln t + 2t^{-1})\right]$ for second **A1**

Condone poor use of brackets if later recovered.

Q4	Solution	Mark	Total	Comment
(a)	$f(k+1) = 2^{4k+7} + 3^{3k+4}$	**M1**		
	convincingly showing $2^{4k+7} = 16 \times 2^{4k+3}$ $f(k+1) - 16f(k)$ $= (81 - 16 \times 3) \times 3^{3k}$	**E1**		must see $16 = 2^4$ OE
	$= 33 \times 3^{3k}$	**A1**	**3**	
(b)	$f(1) = 209$ therefore $f(1)$ is a multiple of 11	**B1**		$f(1) = 209 = 11 \times 19$ *or* $209 \div 11 = 19$ etc therefore true when n=1
	Assume $f(k)$ is a multiple of 11 (*) $f(k+1) = 16f(k) + 33 \times 3^{3k}$	**M1**		attempt at $f(k+1) = \ldots$ using their result from part **(a)**
	$= 11M + 11N = 11(M+N)$			where M and N are integers
	Therefore $f(k+1)$ is a multiple of 11	**A1**		
	Since f(1) is multiple of 11 then f(2), f(3),… are multiples of 11 by induction (or is a multiple of 11 for all integers $n \geq 1$)	**E1**	**4**	must earn previous 3 marks and have (*) before **E1** can be awarded
	Total		**7**	

(a) It is possible to score **M1 E0 A1**

(b) Withhold **E1** for conclusion such as **"**a multiple of 11 for all $n \geq 1$" or poor notation, etc

Q5	Solution	Mark	Total	Comment
(a)				Ignore the line OP drawn in full or circles drawn as part of construction for locus L.
	Straight line Through midpoint of OP, P correct Perpendicular to OP, P correct	**M1** **A1** **A1**	3	P represents $2 - 4\text{i}$
(b)(i)	$(x-2)^2 + (y+4)^2 = x^2 + y^2$	**M1**		
	$2y - x + 5 = 0$	**A1**		
	$A(5,0)$ & $B(0,-2.5)$	**A1**		may have $5 + 0\text{i}$ and $0 - 2.5\text{i}$
	$C\left(\frac{5}{2}, -\frac{5}{4}\right) \Rightarrow$ complex num $= \frac{5}{2} - \frac{5}{4}\text{i}$	**A1**	4	
(ii)	***either*** $\alpha = \frac{5}{2} - \frac{5}{4}\text{i}$ ***or*** $k = \frac{5\sqrt{5}}{4}$	**M1**		allow statement with correct value for centre or radius of circle
	$\left\lvert z - \frac{5}{2} + \frac{5}{4}\text{i} \right\rvert = \frac{5\sqrt{5}}{4}$	**A1**	2	must have exact surd form
	Total		9	

(a) Withhold the final **A1** (if 3 marks earned) if the straight line does not go beyond the Re(z) axis and negative Im(z) axis.
The two **A1** marks can be awarded independently.

(b)(i) **Alternative 1:** grad $OP = -2 \Rightarrow$ grad $L = 0.5$ **M1 ;** $y + 2 = \frac{1}{2}(x-1)$ OE **A1** then **A1, A1** as per scheme

Alternative 2: substituting $z = x$ (or a) then $z = \text{i}y$ (or $\text{i}b$) into given locus equation
Both $(x-2)^2 + 4^2 = x^2$ **and** $2^2 + (y+4)^2 = y^2$ **M1;** $4 - 4x + 16 = 0$ **and** $4 + 8y + 16 = 0$ **OE** for **A1** then **A1, A1** as per scheme.

Q6	Solution	Mark	Total	Comment
(a)	$\sqrt{5+4x-x^2}+\dfrac{(x-2)\frac{1}{2}(4-2x)}{\sqrt{5+4x-x^2}}$	**M1**		product rule (condone one error)
		A1		correct unsimplified
	(+) $\dfrac{9\times\frac{1}{3}}{\sqrt{1-\left(\frac{x-2}{3}\right)^2}}$	**B1**		or $\dfrac{9}{\sqrt{3^2-(x-2)^2}}$ correct unsimplified
	$\dfrac{5+4x-x^2}{\sqrt{5+4x-x^2}}$	**A1**		last two terms above combined correctly
	$\left(\dfrac{dy}{dx}=\right)2\sqrt{5+4x-x^2}$	**A1cso**	**5**	$k=2$
(b)	$\dfrac{1}{k}\left\{(x-2)\sqrt{5+4x-x^2}+9\sin^{-1}\left(\dfrac{x-2}{3}\right)\right\}$	**M1**		ft "their" k
	$\dfrac{1}{\text{"their" } k}\left[\dfrac{3}{2}\sqrt{\dfrac{27}{4}}+9\sin^{-1}\dfrac{1}{2}\right]$	**m1**		correct sub of limits (simplified at least this far)
	$=\dfrac{9}{8}\sqrt{3}+\dfrac{3}{4}\pi$	**A1 cso**	**3**	must have earned **5 marks** in part**(a)** to be awarded this mark
	Total		**8**	
(a)	Second **A1** ; may combine all three terms correctly and obtain $\dfrac{10+8x-2x^2}{\sqrt{5+4x-x^2}}$			

Q7	Solution	Mark	Total	Comment
(a)	$\alpha\beta+\beta\gamma+\gamma\alpha=0$	B1		
	$\alpha\beta\gamma=-\frac{4}{27}$	B1	2	
(b)(i)	$\alpha\beta+\alpha\beta+\beta^2=0$; $\alpha\beta^2=-\frac{4}{27}$	B1		May use γ instead of β throughout **(b)(i)**
		M1		Clear attempt to eliminate either α or β from "their" equations
	$\alpha^3=-\frac{1}{27}$ **or** $\beta^3=\frac{8}{27}$	A1		correct
	either $\alpha=-\frac{1}{3}$ or $\beta=\frac{2}{3}$	A1		
	$\alpha=-\frac{1}{3}$, $\beta=\frac{2}{3}$, $\gamma=\frac{2}{3}$	A1	5	all 3 roots clearly stated
(ii)	$\left(\sum\alpha=1=-\frac{k}{27}\quad\Rightarrow\right)$ $k=-27$	B1	1	or substituting correct root into equation
(c)(i)	$\alpha^2=-2\mathrm{i}$	B1		
	$\alpha^3=-2-2\mathrm{i}$	B1	2	
(ii)	$27(-2-2\mathrm{i})-2\mathrm{i}k+4=0$	M1		correctly substituting "their" $\alpha^2=-2\mathrm{i}$ and "their" $\alpha^3=-2-2\mathrm{i}$
	$k=-27+25\mathrm{i}$	A1	2	
(d)	$y=\frac{1}{z}+1\Rightarrow z=\frac{1}{y-1}$	B1		may use any letter instead of *y*
	$\frac{27}{(y-1)^3}-\frac{12}{(y-1)^2}+4=0$	M1		sub their *z* into cubic equation
	$27-12(y-1)+4(y-1)^3=0$	A1		removing denominators correctly
	$27-12y+12+4(y^3-3y^2+3y-1)=0$	A1		correct and $(y–1)^3$ expanded correctly
	$4y^3-12y^2+35=0$	A1	5	
	Alternative: $\sum\alpha'=3+\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}=3$	(B1)		sum of new roots =3
	$\sum\alpha'\beta'=3+\frac{2(\alpha\beta+\beta\gamma+\gamma\alpha)+\alpha+\beta+\gamma}{\alpha\beta\gamma}$	(M1)		**M1** for either of the other two formulae correct in terms of $\alpha\beta\gamma$, $\alpha\beta+\beta\gamma+\gamma\alpha$ and $\alpha+\beta+\gamma$
	$=0$	(A1)		
	$\prod=1+\frac{\alpha\beta+\beta\gamma+\gamma\alpha+1+\alpha+\beta+\gamma}{\alpha\beta\gamma}$			
	$=\frac{-35}{4}$	(A1)		
	$4y^3-12y^2+35=0$	(A1)	(5)	may use any letter instead of *y*
	Total		**17**	

Q8	Solution	Mark	Total	Comment
(a)(i)	$\left(\omega^5 =\right) \cos 2\pi + \mathrm{i}\sin 2\pi = 1$ So ω is a root of $z^5 = 1$	B1	1	must have conclusion plus verification that $\omega^5 = 1$
(ii)	$\omega^2, \omega^3, \omega^4$.	B1	1	**OE** powers mod 5 (must not include 1)
(b)(i)	$1+\omega+\omega^2+\omega^3+\omega^4 = \dfrac{1-\omega^5}{1-\omega} = 0$	B1	1	or clear statement that sum of roots (of $z^5 - 1 = 0$) is zero
(ii)	$\left(\omega+\dfrac{1}{\omega}\right)^2 + \left(\omega+\dfrac{1}{\omega}\right) - 1$			
	$= \omega^2 + 2 + \dfrac{1}{\omega^2} + \omega + \dfrac{1}{\omega} - 1$	M1		correct expansion
	$= \dfrac{1+\omega+\omega^2+\omega^3+\omega^4}{\omega^2} = 0$	A1	2	**AG** correctly shown to = 0 do not allow simply multiplying by ω^2
(c)	$\dfrac{1}{\omega} = \cos\dfrac{2\pi}{5} - \mathrm{i}\sin\dfrac{2\pi}{5}$	M1		
	$\Rightarrow \omega + \dfrac{1}{\omega} = 2\cos\dfrac{2\pi}{5}$	A1		**SC1** if result merely stated
	Solving quadratic $\left(\omega+\dfrac{1}{\omega}=\right)$ $\dfrac{-1\pm\sqrt{5}}{2}$	M1		must see both values
	Rejecting negative root since $\cos\dfrac{2\pi}{5} > 0$			must see this line for final **A1**
	Hence $\cos\dfrac{2\pi}{5} = \dfrac{\sqrt{5}-1}{4}$	A1	4	
				It is possible to score **SC1 M1 A1**
	Total		9	
(b)(ii)	May replace $\dfrac{1}{\omega^2}$ by ω^3 and $\dfrac{1}{\omega}$ by ω^4 and/or 1 by ω^5 in valid proof. **Alternative:** $1+\omega+\omega^2+\omega^3+\omega^4 = 0 \Rightarrow \dfrac{1}{\omega^2}+\dfrac{1}{\omega}+1+\omega+\omega^2 = 0$ **M1** $\left(\omega+\dfrac{1}{\omega}\right)^2 - 2 + \left(\omega+\dfrac{1}{\omega}\right) + 1 = 0 \Rightarrow \left(\omega+\dfrac{1}{\omega}\right)^2 + \left(\omega+\dfrac{1}{\omega}\right) - 1 = 0$ **A1**			