OCR Maths FP1

Topic Questions from Papers

Summation of Series

1 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$ [6]

(Q1, June 2005)

2 **(i)** Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}.$$ [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots + \frac{1}{(n+1)(n+2)}.$$ [4]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$. [1]

(Q5, June 2005)

3 Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(8r^3 - 6r^2 + 2r) = 2n^3(n+1).$$ [6]

(Q5, Jan 2006)

4 **(i)** Show that $\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \ldots + \frac{2}{n(n+2)}.$$ [5]

(iii) Hence find the value of

(a) $\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$, [1]

(b) $\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$. [2]

(Q9, Jan 2006)

5 Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(r^3 + r^2) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

(Q4, June 2006)

6 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n}\left\{(r+1)^3 - r^3\right\} = (n+1)^3 - 1.$$ [2]

(ii) Show that $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$. [2]

(iii) Use the results in parts **(i)** and **(ii)** and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3\sum_{r=1}^{n} r^2 = \tfrac{1}{2}n(n+1)(2n+1).$$ [6]

(Q9, June 2006)

7 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to find

$$\sum_{r=1}^{n} r(r-1)(r+1),$$

expressing your answer in a fully factorised form. [6]

(Q3, Jan 2007)

8 **(i)** Show that $(r+2)! - (r+1)! = (r+1)^2 \times r!$. [3]

(ii) Hence find an expression, in terms of n, for

$$2^2 \times 1! \ + \ 3^2 \times 2! \ + \ 4^2 \times 3! \ + \ \ldots \ + \ (n+1)^2 \times n!.$$ [4]

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! \ + \ 3^2 \times 2! \ + \ 4^2 \times 3! \ + \ \ldots$$

converges. [1]

(Q8, Jan 2007)

9 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(3r^2 - 3r + 1) = n^3.$$ [6]

(Q3, June 2007)

10 **(i)** Show that

$$\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}.$$ [1]

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(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)}.$$ [3]

(iii) Hence find the value of $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. [3]

(Q5, June 2007)

11 Given that $\sum_{r=1}^{n} (ar^2 + b) \equiv n(2n^2 + 3n - 2)$, find the values of the constants a and b. [5]

(Q2, Jan 2008)

12 **(i)** Show that $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2} = \frac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$ [6]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$. [1]

(iv) Given that $\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$, find the value of N. [4]

(Q10, Jan 2008)

13 **(i)** Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!}.$$ [4]

(Q3, June 2008)

14 Find $\sum_{r=1}^{n} r^2(r-1)$, expressing your answer in a fully factorised form. [6]

(Q5, June 2008)

15 Find $\sum_{r=1}^{n} (4r^3 + 6r^2 + 2r)$, expressing your answer in a fully factorised form. **[6]**

(Q3, Jan 2009)

16 **(i)** Show that $\frac{1}{2r-3} - \frac{1}{2r+1} = \frac{4}{4r^2-4r-3}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=2}^{n} \frac{4}{4r^2-4r-3}.$$ **[6]**

(iii) Show that $\sum_{r=2}^{\infty} \frac{4}{4r^2-4r-3} = \frac{4}{3}$. **[1]**

(Q9, Jan 2009)

17 Evaluate $\sum_{r=101}^{250} r^3$. **[3]**

(Q1, June 2009)

18 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^4 - r^4\} = (n+1)^4 - 1.$$ **[2]**

(ii) Show that $(r+1)^4 - r^4 \equiv 4r^3 + 6r^2 + 4r + 1$. **[2]**

(iii) Hence show that

$$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2.$$ **[6]**

(Q7, June 2009)

19 Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. **[6]**

(Q4, Jan 2010)

20 **(i)** Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$. **[4]**

(iii) Find $\sum_{r=2}^{\infty} \frac{2r+1}{r^2(r+1)^2}$. **[2]**

(Q7, Jan 2010)

21 Find $\sum_{r=1}^{n} (2r-1)^2$, expressing your answer in a fully factorised form. **[6]**

(Q3, June 2010)

22 **(i)** Show that $\dfrac{1}{\sqrt{r+2}+\sqrt{r}} \equiv \dfrac{\sqrt{r+2}-\sqrt{r}}{2}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2}+\sqrt{r}}.$$ **[6]**

(iii) State, giving a brief reason, whether the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2}+\sqrt{r}}$ converges. **[1]**

(Q8, June 2010)

23 Given that $\displaystyle\sum_{r=1}^{n}(ar^3+br) \equiv n(n-1)(n+1)(n+2)$, find the values of the constants a and b. **[6]**

(Q4, Jan 2011)

24 **(i)** Show that $\dfrac{1}{r}-\dfrac{2}{r+1}+\dfrac{1}{r+2} \equiv \dfrac{2}{r(r+1)(r+2)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$ **[6]**

(iii) Show that $\displaystyle\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$. **[3]**

(Q10, Jan 2011)

25 Find $\displaystyle\sum_{r=1}^{2n}\left(3r^2-\tfrac{1}{2}\right)$, expressing your answer in a fully factorised form. **[6]**

(Q4, June 2011)

26 **(i)** Show that $\dfrac{1}{r-1}-\dfrac{1}{r+1} \equiv \dfrac{2}{r^2-1}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=2}^{n} \frac{2}{r^2-1}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=1000}^{\infty} \frac{2}{r^2-1}$. **[3]**

(Q7, June 2011)

27 Find $\displaystyle\sum_{r=1}^{n} r(r^2-3)$, expressing your answer in a fully factorised form. **[6]**

(Q4, Jan 2012)

28 **(i)** Show that $\dfrac{r}{r+1} - \dfrac{r-1}{r} \equiv \dfrac{1}{r(r+1)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)}.$$ **[4]**

(iii) Hence find $\displaystyle\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. **[2]**

(Q8, Jan 2012)

29 Find $\displaystyle\sum_{r=1}^{n} (3r^2 - 3r + 2)$, expressing your answer in a fully factorised form. **[7]**

(Q4, June 2012)

30 **(i)** Show that $\dfrac{1}{r} - \dfrac{1}{r+2} \equiv \dfrac{2}{r(r+2)}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=1}^{n} \frac{2}{r(r+2)}$. **[6]**

(iii) Given that $\displaystyle\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$, find the value of N. **[4]**

(Q8, June 2012)

31 Find $\displaystyle\sum_{r=1}^{n} (r-1)(r+1)$, giving your answer in a fully factorised form. **[6]**

(Q2, Jan 2013)

32 **(i)** Show that $\dfrac{1}{r} - \dfrac{3}{r+1} + \dfrac{2}{r+2} \equiv \dfrac{2-r}{r(r+1)(r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$. **[2]**

(Q8, Jan 2013)

33 Find $\displaystyle\sum_{r=1}^{n} (4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form. **[6]**

(Q5, June 2013)

34 **(i)** Show that $\dfrac{1}{3r-1} - \dfrac{1}{3r+2} \equiv \dfrac{3}{(3r-1)(3r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}$. **[6]**

(Q9, June 2013)