OCR Maths FP1

Topic Questions from Papers

Summation of Series

Answers

1	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	**6**	Obtain given answer correctly

(Q1, June 2005)

2	(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$	M1		Show correct process for subtracting fractions
	$\dfrac{1}{(r+1)(r+2)}$	A1	2	Obtain given answer correctly
	(ii) *EITHER*			
	$\dfrac{2}{3} - \dfrac{1}{2} + \dfrac{3}{4} - \dfrac{2}{3} \ \dots \ \dfrac{n+1}{n+2} - \dfrac{n}{n+1}$	M1		Express terms as differences using (i)
		A1		At least first two and last term correct
	$\dfrac{n+1}{n+2} - \dfrac{1}{2}$	M1		Show or imply that pairs of terms cancel
		A1	4	Obtain correct answer in any form
	OR	M2		State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$
		A1A1		Each term correct
	(iii) $\dfrac{1}{2}$	B1 ft	1	Obtain value from their sum to n terms
			7	

(Q5, June 2005)

3	$8\Sigma r^3 - 6\Sigma r^2 + 2\Sigma r$	M1		Consider the sum of three separate terms
	$8\Sigma r^3 = 2n^2(n+1)^2$	A1		Correct formula stated or used a.e.f.
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated or used a.e.f.
	$2\Sigma r = n(n+1)$	A1		Correct term seen
	$2n^3(n+1)$ **AG**	M1	6	Attempt to factorise or expand and simplify
		A1	**6**	Obtain given answer correctly

(Q5, Jan 2006)

4	(i) $\dfrac{r+2-r}{r(r+2)}$	M1		Show correct process for subtracting fractions
	$\dfrac{2}{r(r+2)}$ **AG**	A1	2	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Express 1st 3 (or last 3) terms so that cancelling occurs
		A1		Obtain $1+\dfrac{1}{2}$
		A1		Obtain $-\dfrac{1}{n+2}$, $-\dfrac{1}{n+1}$
	$\dfrac{3}{2}-\dfrac{1}{n+1}-\dfrac{1}{n+2}$	A1	5	Obtain correct answer in any form
	(iii) (a) $\dfrac{3}{2}$	B1ft	1	Obtain value from their sum to *n* terms
	(b) $\dfrac{1}{n+1}+\dfrac{1}{n+2}$	M1		Using (iii) (a) – (ii) or method of differences again [$n \to \infty$ is a method error]
		A1 ft	2	Obtain answer in any form
			10	

(Q9, Jan 2006)

5	$\Sigma r^3 + \Sigma r^2$	M1		Consider the sum as two separate parts
	$\Sigma r^2 = \dfrac{1}{6}n(n+1)(2n+1)$	A1		Correct formula stated
	$\Sigma r^3 = \dfrac{1}{4}n^2(n+1)^2$	A1		Correct formula stated
	$\dfrac{1}{12}n(n+1)(n+2)(3n+1)$	M1		Attempt to factorise and simplify or expand both expressions
		A1	5	Obtain given answer correctly or complete verification
			$\boxed{5}$	

(Q4, June 2006)

6	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii) $(n+1)^3 - 1 - \frac{3}{2}n(n+1) - n$ $\frac{1}{2}n(n+1)(2n+1)$	B1 B1 M1 M1 A1 A1	6 $\boxed{\mathbf{10}}$	Correct Σr stated $\Sigma\, 1 = n$ Consider sum of three separate terms on RHS Required sum is LHS – two terms Correct unsimplified expression Obtain given answer correctly

(Q9, June 2006)

7		M1		Expand to obtain $r^3 - r$
		M1		Consider difference of two standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	A1		Obtain correct unfactorised answer
		M1		Attempt to factorise
		A1		Obtain factor of $\frac{1}{4}n(n+1)$
	$\frac{1}{4}n(n-1)(n+1)(n+2)$	A1	6 **6**	Obtain correct answer

(Q3, Jan 2007)

8	(i)	M1		Factor of $r!$ or $(r + 1)!$ seen
		A1		Factor of $(r + 1)$ found
	$(r + 1)^2 r!$	A1	3	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		A1		At least 1^{st} two and last term correct
		M1		Show that pairs of terms cancel
	$(n + 2)! - 2!$	A1	4	Obtain correct answer in any form
	(iii)	B1ft	1	Convincing statement for non-converging, ft their (ii)
			8	

(Q8, Jan 2007)

9	$3\Sigma r^2 - 3\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$3\Sigma r^2 = \frac{1}{2}n(n + 1)(2n + 1)$	A1		Correct formula stated
	$3\Sigma r = \frac{3}{2}n(n + 1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
		M1		Attempt to simplify
	n^3	A1	6	Obtain given answer correctly
			6	

(Q3, June 2007)

10	(i) $\frac{1}{r(r + 1)}$	B1	1	Show correct process to obtain given result
	(ii)	M1		Express terms as differences using (i)
		M1		Show that terms cancel
	$1 - \frac{1}{n + 1}$	A1	3	Obtain correct answer, must be n not any other letter
	(iii) $S_\infty = 1$	B1ft		State correct value of sum to infinity Ft their (ii)
		M1		Use sum to infinity – their (ii)
	$\frac{1}{n + 1}$	A1 c.a.o.	3	Obtain correct answer a.e.f.
			7	

(Q5, June 2007)

11	$\frac{a}{6}n(n+1)(2n+1)+bn$	M1 A1		Consider sum as two separate parts Correct answer a.e.f.
	$a = 6 \;\; b = -3$	M1 A1 A1	5 **5**	Compare co-efficients Obtain correct answers

(Q2, Jan 2008)

12	(i)	M1 A1	2	Attempt to combine 3 fractions Obtain given answer correctly
	(ii)	M1 A1 M1 A1 M1		Express at least first 3 terms using (i) All terms correct Express at least last 2 terms using (i) All terms correct in terms of *n* Show that correct terms cancel
	$2+1-\frac{1}{2}-\frac{2}{n+1}-\frac{1}{n+2}$	A1	6	Obtain unsimplified correct answer
	(iii) $\frac{5}{2}$	B1ft	1	Obtain correct answer from their (ii)
	(iv) $\frac{2}{N+1}+\frac{1}{N+2}=\frac{7}{10}$	B1ft		Their (iii) – their (ii)
	$7N^2-9N-36=0$	M1		Attempt to clear fractions & solve equation, Obtain correct simplified equation
	$N = 3$	A1 A1	4 **13**	Obtain only the correct answer

(Q10, Jan 2008)

13	**(i)** $\frac{r}{(r+1)!}$	**M1**	Common denominator of $(r+1)!$ or $r!(r+1)!$
		A1 $\boxed{2}$	Obtain given answer correctly
	(ii) $1-\frac{1}{(n+1)!}$	**M1**	Express terms as differences using (i)
		A1 **M1** **A1** $\boxed{4}$	At least 1st two and last term correct Show pairs cancelling Correct answer a.e.f.

(Q3, June 2008)

14		**M1**	Express as difference of two series
		M1	Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	**A1**	Correct unsimplified answer
		M1	Attempt to factorise
		A1	At least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(3n+2)(n-1)$	**A1**	Obtain correct answer
		$\boxed{\mathbf{6}}$	

(Q5, June 2008)

15	$n^2(n+1)^2 + n(n+1)(2n+1) + n(n+1)$	M1		Express as sum of 3 terms
		A1		2 correct unsimplified terms
		A1		3rd correct unsimplified term
	$n(n+1)^2(n+2)$	M1		Attempt to factorise
		A1ft		Two factors found, ft their quartic
		A1	6	Correct final answer a.e.f.
			6	

(Q3, Jan 2009)

16	**(i)**	M1		Use correct denominator
		A1	2	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Do this for at least 1st 3 terms
		A1		First 3 terms all correct
		A1		Last 3 terms all correct (in terms or *n* or *r*)
	$1 + \frac{1}{3} - \frac{1}{2n-1} - \frac{1}{2n+1}$	M1		Show pairs cancelling
		A1	6	Obtain correct answer, a.e.f.(in terms of *n*)
	(iii) $\frac{4}{3}$	B1ft	1	Given answer deduced correctly, ft their (ii)
			9	

(Q9, Jan 2009)

17		B1		State correct value of S_{250} or S_{100}
		M1		Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99})
	984390625 – 25502500 = 958888125	A1	3	Obtain correct exact answer
			3	

(Q1, June 2009)

18	(i)	M1 A1	 2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	 2	Attempt to expand and simplify Obtain given answer correctly
	(iii)	B1 B1		Correct $\sum r$ stated $\sum 1 = n$
		M1* *DM1		Consider sum of 4 separate terms on RHS Required sum is LHS – 3 terms
	$(n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$	A1		Correct unsimplified expression
	$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2$	A1	6 **10**	Obtain given answer correctly

(Q7, June 2009)

19		M1 M1		Express as sum of three series Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1		Obtain correct unsimplified answer
		M1 A1		Attempt to factorise Obtain at least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	**6**	Obtain fully factorised correct answer
		$\boxed{6}$		

(Q4, Jan 2010)

20	**(i)**	B1	**1**	Obtain **given** answer correctly
	(ii)	M1		Express at least 1st two and last term using **(i)**
		A1		All terms correct
		M1		Show that correct terms cancel
	$1 - \frac{1}{(n+1)^2}$	A1	**4**	Obtain correct answer, in terms of n
	(iii) $\frac{1}{4}$	B1		Sum to infinity seen or implied
		B1	**2**	Obtain correct answer **S.C.** -¾ scores B1
		$\boxed{7}$		

(Q7, Jan 2010)

21	Either	M1		Express as a sum of 3 terms
		M1		Use standard sum results
	$\frac{2}{3}n(n+1)(2n+1)-2n(n+1)+n$	A1		Correct unsimplified answer
		M1		Attempt to factorise
		A1		Obtain at least factor of n and a quadratic
	$\frac{1}{3}n(2n-1)(2n+1)$	A1	**6**	Obtain correct answer a.e.f.
	Or			
	$\sum_{r=1}^{2n} r^2 - 4\sum_{r=1}^{n} r^2$	M1		Express as difference of 2 $\sum r^2$ series
		M1		Use standard result
	$\frac{1}{6}\times 2n(2n+1)(4n+1)-4\times\frac{1}{6}n(n+1)(2n+1)$	A1		Correct unsimplified answer
		M1		Attempt to factorise
		A1		Obtain at least factor of n
	$\frac{1}{3}n(2n-1)(2n+1)$	A1		Obtain correct answer
			$\boxed{6}$	

(Q3, June 2010)

22	**(i)**	M1		Attempt to rationalise denominator or cross multiply
		A1	**2**	Obtain **given** answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Attempt this for at least 1st three terms
		A1		1st three terms all correct
		A1		Last two terms all correct
		M1		Show pairs cancelling
	$\frac{1}{2}(\sqrt{n+2}+\sqrt{n+1}-\sqrt{2}-1)$	A1	**6**	Obtain correct answer, in terms of n
	(iii)	B1	**1**	Sensible statement for divergence
			$\boxed{9}$	

(Q8, June 2010)

23	***Either***	B1	Correct value for $\sum r$ stated or used
		M1	Express as sum of two series
	$\frac{a}{4}n^2(n+1)^2 + \frac{bn}{2}(n+1)$	A1	Obtain correct unsimplified answer
		M1	Compare coefficients or substitute values for n
	$a = 4 \quad b = -4$	A1 A1 **6**	Obtain correct answers
	Or		
		M1	Use 2 values for n
	$a + b = 0 \quad 4a + b = 12$	A1 A1	Obtain correct equations
		M1	Solve simultaneous equations
	$a = 4 \quad b = -4$	A1 A1	Obtain correct answers
		$\boxed{\mathbf{6}}$	

(Q4, Jan 2011)

24 (i)		M1		Use correct denominator
		A1	**2**	Obtain **given** answer correctly
(ii)		M1		Express terms as differences using (i)
		M1		Do this for at least 3 terms
		A1		First 3 terms all correct
		A1		Last 2 terms all correct
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1		Show relevant cancelling
		A1	**6**	Obtain correct answer a.e.f.
(iii)	$\frac{1}{2}$	B1ft		S_∞ stated or start at $n + 1$ as in (ii)
	$\frac{1}{n+1} - \frac{1}{n+2}$	M1		S_∞ - their (ii) or show correct cancelling
	$\frac{1}{(n+1)(n+2)}$	A1	**3**	Obtain **given** answer correctly
		$\boxed{\mathbf{11}}$		

(Q10, Jan 2011)

25		M1		Express as sum of two series
	$3 \times \frac{1}{6} \times 2n(2n+1)(4n+1) - \frac{1}{2} \times 2n$	A1 A1		Each term correct a.e.f.
		M1		Attempt to factorise
	$2n^2(4n+3)$	A2	**6**	Completely correct answer, (A1 if one factor not found)
		$\boxed{\mathbf{6}}$		

(Q4, June 2011)

26 (i)		B1	**1**	Obtain **given** answer correctly
(ii)		M1		Express at least 1st two and last two terms using (i)
		A1		1st two terms correct
		A1		Last two terms correct
		M1		Show that correct terms cancel
	$\frac{3}{2}-\frac{1}{n}-\frac{1}{(n+1)}$	A1	**5**	Obtain correct answer, a.e.f. in terms of n
(iii)		B1ft		Sum to infinity stated or implied or start at 1000 as in (ii)
		M1		S_∞ – their (ii) with n = 999 or 1000 or show correct cancelling
	$\frac{1999}{999000}$	A1	**3**	Obtain correct answer, a.e.f.
				(condone 0.002)
		$\boxed{9}$		

(Q7, June 2011)

27				M1 DM1 A1	Express as difference of two series Use standard series results Obtain correct unsimplified answer	
			$\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$			
				M1 A1 A1	Attempt to factorise At least factor of $n(n+1)$ Obtain correct answer	From their unsimplified answer
			$\frac{1}{4}n(n+1)(n+3)(n-2)$	**[6]**		

(Q4, Jan 2012)

28	(i)			M1 A1 **[2]**	Combine with a common denominator Obtain **given** answer correctly	
	(ii)			M1 A1	Express terms using **(i)** At least 1st two and last two correct	
			$\frac{n}{n+1}$	M1	Show terms cancelling	
				A1 **[4]**	Obtain correct answer, in terms of n	
	(iii)		$1-\frac{n}{n+1}$	B1 B1FT **[2]**	$\lim_{n\to\infty}\frac{n}{n+1}=1$ This value – **(ii)**	

(Q8, Jan 2012)

29				M1	Express as sum of 3 series	
				M1	Use standard series results, at least 1 correct	
				A1	Two terms correct	
			$\frac{1}{2}n(n+1)(2n+1)-\frac{3}{2}n(n+1)+2n$	A1	Third term correct	
				M1	Obtain factor of n	
			$n(n^2+1)$	A2	Obtain correct answer c.a.o.	
					Allow A1 for $\frac{1}{2(2n^2+2)}$	
				[7]		

(Q4, June 2012)

30	(i)			B1 **[1]**	Show **given** answer correctly	
	(ii)			M1	Express terms as differences using (i)	
				M1	Attempt this for at least first 3 terms	
				A1	First 3 terms all correct	
				A1	Last 2 terms correct	
				M1	Show terms cancelling	
			$1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	A1	Obtain correct answer, must be in terms of n	
				[6]		
	(iii)		$\frac{3}{2}$	B1ft	State or use correct sum to infinity	
				B1	Their sum to infinity – their (ii) = $\frac{11}{30}$	
				M1	Attempt to solve correct equation	
			$N = 4$	A1	Obtain only $N = 4$	
				[4]		

(Q8, June 2012)

31			$\frac{1}{6}n(n+1)(2n+1)-n$	M1*	Attempt to expand $(r-1)(r+1)$
				DM1	Use standard result for $\sum r^2$
				A1	Obtain correct unsimplified answer
			$\frac{1}{6}n(2n+5)(n-1)$	DM1	Attempt to factorise
				A2	Obtain completely correct answer
					Allow A1 if one bracket still contains a common factor
				[6]	

(Q2, Jan 2013)

32	**(i)**			M1	Obtain correct numerator from addition or partial fractions
				A1	Obtain **given** answer correctly
				[2]	
	(ii)			M1	Express at least three relevent terms using (i)
				A1	1st three terms correct
			$\frac{n}{(n+1)(n+2)}$	A1	Last two terms correct
				M1	Show correct cancelling
				A1	Obtain **given** answer correctly
				[5]	
	(iii)		$-\frac{1}{6}$	M1	Sum 1 to ∞ - 1st term or start process at $r = 2$
				A1	Obtain correct answer
				[2]	

(Q8, Jan 2013)

33			$4\times\frac{1}{4}n^2(n+1)^2-3\times\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$ $n^3(n+1)$	M1 A1 A1 M1 A2 **[6]**	Express as sum of three series Obtain 2 correct (unsimplified) terms Obtain correct 3rd (unsimplified) term Attempt to factorise, at least factor of n Obtain correct answer, A1 if not fully factorised

(Q5, June 2013)

34	**(i)**			M1 A1 **[2]**	Use correct denominator or partial fractions Obtain **given** answer convincingly
	(ii)		$\frac{1}{2}-\frac{1}{6n+2}$	M1 A1 M1 A1 M1 A1 **[6]**	Express at least 1st two and last term using **(i)** All terms correct Show correct terms cancelling Obtain correct unsimplified answer Include $\frac{1}{3}$ and combine their sum as a single fraction Obtain **given** answer

(Q9, June 2013)