

OCR Maths FP1

Topic Questions from Papers

Roots of Polynomial Equations

Answers

1	(a) (i) $\alpha + \beta = 2$ $\alpha\beta = 4$	B1B1	2	Values stated
	(ii) <i>EITHER</i> $\alpha^2 + \beta^2 = -4$ OR	M1 A1	2	Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly
	(iii) $x^2 + 4x + 16 = 0$	M1 A1		Find numeric values of roots, square and add Obtain given answer correctly
		B1		State or use $\alpha^2\beta^2 = 16$
	(b) (i) $p = 2$	M1 A1	3	Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square Obtain $x^2 + 4x + 16 = 0$
	(ii) $a = 44$	M1 A1	2	Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$ Obtain $p = 2$
		M1 A1ft	2	Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation Obtain $11p^2$
			11	

(Q8, June 2005)

2	$u^2 + 4u + 4$	B1		$u + 2$ squared and cubed correctly
	$u^3 + 6u^2 + 12u + 8$			
		M1		Substitute these and attempt to simplify
		A1		Obtain $u^3 - 5 = 0$ or equivalent
	$u = \sqrt[3]{5}$	A1ft		Correct solution to their equation
	$x = 2 + \sqrt[3]{5}$	A1ft		Obtain 2 + their answer [Decimals score 0/2 of final A marks]
			5	
			5	

(Q4, Jan 2006)

3	(i)	$\alpha + \beta + \gamma = 9$	B1	1	
	(ii)	$p = \frac{9 - \alpha}{2}$	B1 M1 A1 A1	4	State or use other root is $p - iq$ Substitute into (i) Obtain $2p + \alpha = 9$ Obtain correct answer a.e.f.
	(iii)	$\alpha\beta\gamma = 29$	B1	1	
	(iv)	$\alpha(p^2 + q^2) = 29$	M1 A1ft		Substitute into (iii) Obtain unsimplified expression with no i's
		$q = \sqrt{\frac{29}{\alpha} - \frac{(9 - \alpha)^2}{4}}$	M1 M1 A1	5	Rearrange to obtain q or q^2 Substitute their expression for p a.e.f. Obtain correct answer a.e.f.
	(iv) Alternative method	$2p\alpha + p^2 + q^2 = 27$	M1 A1	11	Substitute into $\alpha\beta + \beta\gamma + \gamma\alpha = 27$ Obtain unsimplified expression with no i's
		$q = \sqrt{27 - \frac{(9 - \alpha)^2}{4} - \alpha(9 - \alpha)}$	M1 A1		Rearrange to obtain q or q^2 Substitute their expression for p a.e.f. Obtain correct answer a.e.f.

(Q10, Jan 2006)

4	(i)	$2 + 3i$	B1	1	Conjugate seen
	(ii)	$p = -4$ $q = 13$	M1 A1 M1 A1	4	Attempt to sum roots or consider x terms in expansion or substitute $2 - 3i$ into equation and equate imaginary parts Correct answer Attempt at product of roots or consider last term in expansion or consider real parts Correct answer
				5	

(Q3, June 2006)

5	(i) $\alpha + \beta + \gamma = 2$ $\alpha\beta\gamma = -4$	B1 B1	3	Write down correct values
	$\alpha\beta + \beta\gamma + \gamma\alpha = 3$	B1		Sum new roots
	(ii)	M1	3	Obtain numeric value using their (i)
	$\alpha + 1 + \beta + 1 + \gamma + 1 = 5$	A1ft		p is negative of their answer
	$p = -5$	A1ft	5	Expand three brackets
	(iii)	M1*		$\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma + 1$
		A1	5	Use their (i) results
		DM1		Obtain 2
		A1ft	5	q is negative of their answer
	$q = -2$	A1ft		11 Alternative for (ii) & (iii)
		M2		Substitute $x = u - 1$ in given equation
		A1		Obtain correct unsimplified equation for u
		M1		Expand
		A2		Obtain $u^3 - 5u^2 + 10u - 2 = 0$
		A1 A1		State correct values of p and q .

(Q10, June 2006)

6	(i) $\alpha + \beta = -5$ $\alpha\beta = 10$	B1 B1	2	State correct values
	(ii) $\alpha^2 + \beta^2 = 5$	M1		Use $(\alpha + \beta)^2 - 2\alpha\beta$
		A1	2	Obtain given answer correctly, using value of -5
	(iii)	B1		Product of roots = 1
		M1	4	Attempt to find sum of roots
		A1		Obtain $\frac{5}{10}$ or equivalent
	$x^2 - \frac{1}{2}x + 1 = 0$	B1ft	4	Write down required quadratic equation, or any multiple.
			8	

(Q7, Jan 2007)

7	(i) (a) $\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 2$	B1 B1	2	State correct values
	(b) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= 9 - 4 = 5$	M1		State or imply the result and use their values
	(ii) (a) $\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0$	A1 ft	2	Obtain correct answer
	(b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3$	M1 A1 M1 A1ft	2 2 2	Use given substitution to obtain an equation Obtain correct answer Obtain correct answer Required expression is related to new cubic stated or implied -(their "b" / their "a")

(Q6, June 2007)

8	(i) $7u^3 + 24u^2 - 3u + 2 = 0$	M1 A1	2	Use given substitution Obtain correct equation a.e.f.
	(ii) <i>EITHER</i> correct value is $-\frac{3}{7}$	M1 A1ft	2	Required expression related to new cubic Their c / their a
	<i>OR</i> correct value is $-\frac{3}{7}$	M1 A1		Use $\frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$ or equivalent Obtain correct answer
			4	

(Q3, Jan 2008)

9	(i) $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	M1 A1	2	Correct binomial expansion seen Obtain given answer with no errors seen
	(ii) <i>Either</i> $\alpha + \beta = 5, \alpha\beta = 7$ $\alpha^3 + \beta^3 = 20$	B1 B1 M1 A1		6 8
	$x^2 - 20x + 343 = 0$	M1 A1ft		
	<i>Or</i> $u^{\frac{2}{3}} - 5u^{\frac{1}{3}} + 7 = 0$	M1 A1		Substitute $x = u^{\frac{1}{3}}$ Obtain correct answer
	$u^3 - 20u + 343 = 0$	M2 A2		Complete method for removing fractional powers Obtain correct answer

(Q9, Jan 2008)

10 (i) $3 - i$	B1	Conjugate stated
	1	
(ii) <i>EITHER</i>	M1	Use sum of roots
	A1	Obtain correct answer
	M1	Use sum of pairs of roots
	A1	Obtain correct answer
	M1	Use product of roots
	A1	Obtain correct answers
	6	
	M1	Attempt to find a quadratic factor
	A1	Obtain correct factor
	M1	Expand linear and quadratic factors
	A1A1A1	Obtain correct answers
	M1	Substitute 1 imaginary & the real root into eqn
	M1	Equate real and imaginary parts
	M1	Attempt to solve 3 eqns.
	A1A1A1	Obtain correct answers

(Q6, June 2008)

<p>11 $\alpha + \beta = -k$ $\alpha\beta = 2k$</p> <p>$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$</p> <p>$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1}{2}(k-4)$</p> <p>$\alpha'\beta' = 1$</p> <p>$x^2 - \frac{1}{2}(k-4)x + 1 = 0$</p>	<p>B1 State or use correct value</p> <p>B1 State or use correct value</p> <p>M1 Attempt to express sum of new roots in terms of $\alpha + \beta$, $\alpha\beta$</p> <p>A1 Obtain correct expression</p> <p>A1 Obtain correct answer a.e.f.</p> <p>B1 Correct product of new roots seen</p> <p>B1ft Obtain correct answer, must be an eqn.</p> <p style="text-align: center;">7</p> <p style="text-align: center;">Alternative for last 5 marks</p> <p>M1 Obtain expression for $u = \frac{\alpha}{\beta}$ in terms of k and α or k and β</p> <p>A1 Obtain a correct expression</p> <p>A1 rearrange to get α in terms of u</p> <p>M1 Substitute into given equation</p> <p>A1 Obtain correct answer</p>
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(Q8, June 2008)

12	<p>(i)</p> <p>(ii) $\alpha + \beta = 6k, \alpha\beta = k^2$ $\alpha - \beta = (4\sqrt{2})k$</p> <p>(iii) $\sum \alpha' = 6k$ $\alpha' \beta' = \alpha\beta - (\alpha - \beta) - 1$ $\alpha' \beta' = k^2 - (4\sqrt{2})k - 1$ $x^2 - 6kx + k^2 - (4\sqrt{2})k - 1 = 0$</p>	<p>M1 A1</p> <p>B1 B1 M1 A1</p> <p>B1ft</p> <p>M1</p> <p>A1ft</p> <p>B1ft</p>	<p>2</p> <p>4</p> <p>4</p> <p>10</p>	<p>Expand at least 1 of the brackets Derive given answer correctly</p> <p>State or use correct values Find value of $\alpha - \beta$ using (i) Obtain given value correctly (allow if $-6k$ used)</p> <p>Sum of new roots stated or used</p> <p>Express new product in terms of old roots</p> <p>Obtain correct value for new product</p> <p>Write down correct quadratic equation</p>
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(Q8, Jan 2009)

13	Either $p + q = -1, pq = -8$	B1	4 4	Both values stated or used	
		$\frac{p+q}{pq}$		B1	Correct expression seen
		$-\frac{7}{8}$		M1 A1	Use their values in their expression Obtain correct answer
	Or $\frac{1}{p} + \frac{1}{q} = 8$	B1		Substitute $x = \frac{1}{u}$ and use new quadratic	
		$p + q = 1$		B1	Correct value stated
		$-\frac{7}{8}$		M1 A1	Use their values in given expression Obtain correct answer
	Or $\frac{-1 \pm \sqrt{33}}{2}$	$-\frac{7}{8}$		M1	Find roots of given quadratic equation
				A1	Correct values seen
				M1	Use their values in given expression
				A1	Obtain correct answer

(Q4, June 2009)

14	(i) $u^3 = \{(-)(5u + 7)\}^2$	M1	3	Use given substitution and rearrange
		A1		Obtain correct expression, or equivalent
	$u^3 - 25u^2 - 70u - 49 = 0$	A1		Obtain correct final answer
	(ii)	M1		Use coefficient of u of their cubic or identity connecting the symmetric functions and substitute values from given equation
	-70	A1 ft		Obtain correct answer
			5	

(Q5, June 2009)

15 (i)	$u^3 - 3u^2 + 3u - 1$	B1	3	Correct unsimplified expansion of $(u - 1)^3$
	$2u^3 - 6u^2 + 9u - 8 = 0$	M1		Substitute for x
		A1		Obtain correct equation

(ii)	4	M1	Use $(\pm)\frac{d}{a}$ of new equation
		A1ft	Obtain correct answer from their equation

5

(Q2, Jan 2010)

16 (i)		B1	State or use $5 + i$ as a root
		M1	Use $\sum \alpha\beta = 6$
	$x = -2$	A1	3 Obtain correct answer

(ii) Either		M1	Use $p = -\sum \alpha$
	$p = -8$	A1ft	Obtain correct answer, from their root
		M1	Use $q = -\alpha\beta\gamma$
	$q = 52$	A1ft	4 Obtain correct answer, from their root
Or		M1	Attempt to find quadratic factor
		M1	Attempt to expand quadratic and linear
		A1A1	Obtain correct answers
Or		M1	Substitute $(5 - i)$ into equation
		M1	Equate real and imaginary parts
		A1	Obtain correct answer for p
		A1ft	Obtain correct answer for q , ft their p
		7	

(Q6, Jan 2010)

17	<i>Either</i>	B1B1	State or use correct results
	$\alpha + \beta = -2k \quad \alpha\beta = k$	M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
		A1	Obtain $4k$
		B1ft	7 Correct quadratic equation a.e.f.
	$y^2 - 4ky + 4k = 0$		
Or		B1	State or use correct result
$\alpha + \beta = -2k$		B1	State or imply form of new roots
$\frac{-2k}{\alpha}$		B1	State correct substitution
$y = \frac{-2k}{x}$		M1	Rearrange and substitute for x
		A1	Correct unsimplified equation
		M1	Attempt to clear fractions
	$y^2 - 4ky + 4k = 0$	A1	Correct quadratic equation a.e.f.
Or			
	$-k \pm \sqrt{k^2 - k}$	B1	Find roots of original equation
$\frac{\alpha + \beta}{\alpha} = \frac{2k}{k + \sqrt{k^2 - k}}, \frac{\alpha + \beta}{\beta} = \frac{2k}{k - \sqrt{k^2 - k}}$		B1	Express both new roots in terms of k
		M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
		A1	Obtain $4k$
	$y^2 - 4ky + 4k = 0$	B1ft	Correct quadratic equation a.e.f.

18 (i) Either			
$\alpha + \beta = \frac{1}{2}, \alpha\beta = \frac{3}{2}$	B1		State or use both correct results in (i) or (ii)
$\alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$ or $\alpha + \beta + \frac{2}{3}(\alpha + \beta)$	M1		Express sum of new roots in terms of $\alpha + \beta$ and $\alpha\beta$
	M1		Substitute their values into their expression
$p = \frac{5}{6}$	A1	4	Obtain given answer correctly
Or			
$3u^2 - u + 2(= 0)$	B1		Substitute $x = \frac{1}{u}$ and obtain correct quadratic (equation)
	M1		Use sum of roots of new equation
	M1		Substitute their values into their expression
$p = \frac{5}{6}$	A1		Obtain given answer correctly

(ii) $\alpha' \beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1		Correct expansion
$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1		Show how to deal with $\alpha^2 + \beta^2$
	A1		Obtain correct expression
$q = \frac{1}{3}$	M1		Substitute their values into $\alpha' \beta'$
	A1	5	Obtain correct answer a.e.f.

(Q8, Jan 2011)

19 (i) $16 + 30i$	B1	1	State correct value
(ii)	M1		Use $a = -$ (sum of roots)
$a = -32$	A1		Obtain correct answer
	M1		Use $b =$ product of roots
$b = 1156$	A1	4	Obtain correct answer
	M1		Substitute, expand and equate imag. parts
	A1		Obtain a = -32
	M1		Equate real parts
	A1		Obtain b = 1156

(Q9, June 2011)

20 (i)

$$\frac{1}{u^{\frac{3}{2}}} + \frac{3}{u} + 2 = 0$$

EITHER

$$\frac{9}{u^2} + \frac{12}{u} + 4 = \frac{1}{u^3}$$

$$4u^3 + 12u^2 + 9u - 1 = 0$$

OR

e. g. $(2u^{\frac{3}{2}} + 3u^{\frac{1}{2}} + 1)(2u^{\frac{3}{2}} + 3u^{\frac{1}{2}} - 1) = 0$

B1	Use substitution correctly
M1	Rearrange
M1	Square
A1	Obtain correct equation
A1	5 Obtain given answer
M2	Multiply their equation in u by appropriate related expression
A2	Obtain given answer

(ii)

-3

$\frac{9}{4}$

B1	Stated or imply that $u = \frac{1}{x^2}$
M1	Use $-\frac{b}{a}$
A1	Obtain correct answer
M1	Use $\frac{c}{a}$
A1	5 Obtain correct answer

10

(Q10, June 2011)

21	(i)	$\alpha + \beta + \gamma = 3$ $\alpha\beta + \beta\gamma + \gamma\alpha = 2$ $\alpha\beta\gamma = -\frac{2}{3}$	B1 B1 B1 [3]	State correct value State correct value State correct value	
	(ii)	EITHER $c = -\frac{4}{9}$ $\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$ 5 $a = -5$ $\sum \alpha^2 \beta^2 = (\sum \alpha\beta)^2 - 2\alpha\beta\gamma \sum \alpha$ $b = 8$ OR $9y^3 - 45y^2 + 72y - 4 = 0$ $c = -\frac{4}{9}$ $a = -5$ $b = 8$	M1 A1FT M1 A1FT A1FT M1* A1 DM1 A1 [9] B1 M1 DM1 DM1 A1 M1 A1 A1FT A1FT [9]	$c = (\pm)\alpha^2 \beta^2 \gamma^2$ Obtain given correct answer Use correct expression Obtain correct value Obtain answer correctly Attempt to find an identity Obtain correct identity Use appropriate values Obtain correct answer cao State or use correct substitution Rearrange, fractional indices isolated Square both sides Expand and simplify Obtain correct equation Use coefficients of their cubic Obtain given answer correctly Obtain correct answer Obtain correct answer SC mixture of methods only A1FT for a and b	FT for sign error in (i) FT for sign error in (i) Sign change done correctly

(Q10, Jan 2012)

22		EITHER	M1	Use sum of root and conjugate	
		$a = -8$	A1	Obtain correct answer	
		$b = 25$	M1	Use product of root and conjugate	
		OR	A1	Obtain correct answer	
			[4]		
		$a = -8$	M1	Substitute $4 + 3i$ or conjugate into equation	
		$b = 25$	M1	Equate real and imaginary parts	
			A1	Obtain correct answer	
			A1	Obtain correct answer	

(Q3, June 2012)

23	(i)		M1	Attempt to clear fractions	
		$5u^2 + 11u + 8 = 0$	M1	Attempt to expand and simplify to a quadratic	
			A1	Obtain correct answer, must be an equation	
			[3]		
	(ii)	EITHER	B1	State or imply by using roots of new quadratic	
		$u = \frac{1}{x} - 1$	M1	Use their c/a	
		$\frac{8}{5}$	A1 FT	Obtain correct answer	
		OR	[3]		
		$\frac{1}{\alpha\beta} - \frac{\alpha + \beta}{\alpha\beta} + 1$	B1	Express in terms of $\alpha + \beta$ and $\alpha\beta$	
		$\frac{8}{5}$	M1	Use values $-\frac{1}{2}$ and $\frac{5}{2}$ correctly	Must be values from original equation
			A1	Obtain correct answer	

(Q6, June 2012)

24	(i)		M1	Substitute and attempt to simplify	
		$4u^2 + 6u + k + 2 = 0$	A1	Obtain correct answer, must be an equation	
			[2]		
	(ii)	Either	M1	Use products of roots of new quadratic i.e. use $(\pm) c/a$	
		$\frac{k+2}{4}$	A1ft	Obtain correct answer, from their quadratic	
		Or	[2]		
		$\frac{k+2}{4}$	M1	Use sum and product of roots of original equation	
			A1	Obtain correct answer	

(Q4, Jan 2013)

25	(i)		M1	Attempt at complete expansion	
			A1	Obtain correct unsimplified answer	
			A1	Obtain given answer correctly	
			[3]		
	(ii)	Either	B1	State (anywhere) correct values for $\sum\alpha, \sum\alpha\beta, \sum\alpha\beta\gamma$	
		$\sum\alpha = -p, \sum\alpha\beta = -4, \alpha\beta\gamma = -3$	M1	Express given expression as a single fraction	
		$\frac{16-6p}{9}$	A1	Obtain correct expression using (i)	
		Or	M1	Use their values for sum of roots etc. in their expression	
		$9u^3 + (6p-16)u^2 + (8+p^2)u - 1 = 0$	A1	Obtain correct answer	
		$\frac{16-6p}{9}$	[5]		
			B1	Use substitution $1/\sqrt{u}$	
			M1	Rearrange appropriately and square out	
			A1	Obtain correct co-efficients of u^3 and u^2	
			M1	Use $(+/-)b/a$ from their cubic	
			A1	Obtain correct answer	

(Q9, Jan 2013)

26

Either

$$\sum \alpha = -\frac{6}{k}, \quad \sum \alpha\beta = \frac{1}{k}$$

$$\sum \alpha\beta + 2\sum \alpha + 3$$

$$3 - \frac{11}{k}$$

Or

$$ku^3 + (6 - 3k)u^2 + (3k - 11)u + 2 - k = 0$$

$$3 - \frac{11}{k}$$

B1B1 Correct values stated or used

M1 Expand brackets

A1 Obtain correct expression aef

M1 Use their values, in terms of k , for $\sum \alpha$ and $\sum \alpha\beta$

A1 Obtain correct answer aef

[6]B1 State or use substitution $x = u - 1$

M1 Expand and attempt to simplify coefficients

A1 A1 Obtain at least correct 1st and 3rd termsM1 Use their " $\frac{c}{a}$ "

A1 Obtain correct answer a.e.f.

(Q8, June 2013)