OCR Maths FP1

Topic Questions from Papers

Complex Numbers

1 The complex numbers $2 + 3\text{i}$ and $4 - \text{i}$ are denoted by z and w respectively. Express each of the following in the form $x + \text{i}y$, showing clearly how you obtain your answers.

(i) $z + 5w$, [2]

(ii) z^*w, where z^* is the complex conjugate of z, [3]

(iii) $\frac{1}{w}$. [2]

(Q3, June 2005)

2 Use an algebraic method to find the square roots of the complex number $21 - 20\text{i}$. [6]

(Q4, June 2005)

3 The loci C_1 and C_2 are given by

$$|z - 2\text{i}| = 2 \qquad \text{and} \qquad |z + 1| = |z + \text{i}|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2. [2]

(Q6, June 2005)

4 **(i)** Express $(1 + 8\text{i})(2 - \text{i})$ in the form $x + \text{i}y$, showing clearly how you obtain your answer. [2]

(ii) Hence express $\frac{1 + 8\text{i}}{2 + \text{i}}$ in the form $x + \text{i}y$. [3]

(Q1, Jan 2006)

5 **(a)** The complex number $3 + 2\text{i}$ is denoted by w and the complex conjugate of w is denoted by w^*. Find

(i) the modulus of w, [1]

(ii) the argument of w^*, giving your answer in radians, correct to 2 decimal places. [3]

(b) Find the complex number u given that $u + 2u^* = 3 + 2\text{i}$. [4]

(c) Sketch, on an Argand diagram, the locus given by $|z + 1| = |z|$. [2]

(Q7, Jan 2006)

6 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3\text{i}$.

(i) Write down the other root. [1]

(Q3, June 2006)

7 The complex numbers $3 - 2\mathrm{i}$ and $2 + \mathrm{i}$ are denoted by z and w respectively. Find, giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(\mathrm{i}z)^2$, [3]

(iii) $\dfrac{z}{w}$. [3]

(Q5, June 2006)

8 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \qquad \text{and} \qquad \arg z = \tfrac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number representing the point of intersection of C_1 and C_2. [2]

(Q6, June 2006)

9 Use an algebraic method to find the square roots of the complex number $15 + 8\mathrm{i}$. [6]

(Q2, Jan 2007)

10 **(i)** Sketch, on an Argand diagram, the locus given by $|z - 1 + \mathrm{i}| = \sqrt{2}$. [3]

(ii) Shade on your diagram the region given by $1 \leqslant |z - 1 + \mathrm{i}| \leqslant \sqrt{2}$. [3]

(Q4, Jan 2007)

11 **(i)** Verify that $z^3 - 8 = (z - 2)(z^2 + 2z + 4)$. [1]

(ii) Solve the quadratic equation $z^2 + 2z + 4 = 0$, giving your answers exactly in the form $x + \mathrm{i}y$. Show clearly how you obtain your answers. [3]

(iii) Show on an Argand diagram the roots of the cubic equation $z^3 - 8 = 0$. [3]

(Q5, Jan 2007)

12 The complex number $a + \mathrm{i}b$ is denoted by z. Given that $|z| = 4$ and $\arg z = \frac{1}{3}\pi$, find a and b. [4]

(Q1, June 2007)

13 The loci C_1 and C_2 are given by $|z - 3| = 3$ and $\arg(z - 1) = \frac{1}{4}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3| \leqslant 3 \quad \text{and} \quad 0 \leqslant \arg(z - 1) \leqslant \tfrac{1}{4}\pi.$$ [2]

(Q8, June 2007)

14 **(i)** Use an algebraic method to find the square roots of the complex number $16 + 30\text{i}$. [6]

(ii) Use your answers to part **(i)** to solve the equation $z^2 - 2z - (15 + 30\text{i}) = 0$, giving your answers in the form $x + \text{i}y$. [5]

(Q10, June 2007)

15 The complex number $3 - 4\text{i}$ is denoted by z. Giving your answers in the form $x + \text{i}y$, and showing clearly how you obtain them, find

(i) $2z + 5z^*$, [2]

(ii) $(z - \text{i})^2$, [3]

(iii) $\frac{3}{z}$. [3]

(Q4, Jan 2008)

16 The loci C_1 and C_2 are given by

$$|z| = |z - 4\text{i}| \qquad \text{and} \qquad \arg z = \tfrac{1}{6}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \text{i}y$, the complex number represented by the point of intersection of C_1 and C_2. [3]

(Q6, Jan 2008)

17 The complex number $3 + 4\text{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = |a|$, [2]

(b) $\arg(z - 3) = \arg a$. [3]

(Q2, June 2008)

18 **(i)** Use an algebraic method to find the square roots of the complex number $5 + 12\text{i}$. [5]

(ii) Find $(3 - 2\text{i})^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

(Q9, June 2008)

19 Express $\dfrac{2+3\text{i}}{5-\text{i}}$ in the form $x + \text{i}y$, showing clearly how you obtain your answer. **[4]**

(Q1, Jan 2009)

20 **(i)** Use an algebraic method to find the square roots of the complex number $2 + \text{i}\sqrt{5}$. Give your answers in the form $x + \text{i}y$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + \text{i}y$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**

(Q10, Jan 2009)

21 The complex numbers z and w are given by $z = 5 - 2\text{i}$ and $w = 3 + 7\text{i}$. Giving your answers in the form $x + \text{i}y$ and showing clearly how you obtain them, find

(i) $4z - 3w$, **[2]**

(ii) z^*w. **[2]**

(Q3, June 2009)

22 The complex number $3 - 3\text{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = 3\sqrt{2}$, **[3]**

(b) $\arg(z - a) = \frac{1}{4}\pi$. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geqslant 3\sqrt{2} \quad \text{and} \quad 0 \leqslant \arg(z - a) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

(Q6, June 2009)

23 The complex number z satisfies the equation $z + 2\text{i}z^* = 12 + 9\text{i}$. Find z, giving your answer in the form $x + \text{i}y$. **[5]**

(Q3, Jan 2010)

24 The complex number a is such that $a^2 = 5 - 12\text{i}$.

(i) Use an algebraic method to find the two possible values of a. **[5]**

(ii) Sketch on a single Argand diagram the two possible loci given by $|z - a| = |a|$. **[4]**

(Q8, Jan 2010)

25 The complex numbers a and b are given by $a = 7 + 6\text{i}$ and $b = 1 - 3\text{i}$. Showing clearly how you obtain your answers, find

(i) $|a - 2b|$ and $\arg(a - 2b)$, **[4]**

(ii) $\dfrac{b}{a}$, giving your answer in the form $x + \text{i}y$. **[3]**

(Q4, June 2010)

26 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z - 3 + 4\text{i}| = 5$, **[2]**

(b) $|z| = |z - 6|$. **[2]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4\text{i}| \leqslant 5 \quad \text{and} \quad |z| \geqslant |z - 6|.$$ **[2]**

(Q6, June 2010)

27 The complex number z, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4\text{i}$.

(i) Use an algebraic method to find z. **[5]**

(ii) Show that $z^3 = 2 + 11\text{i}$. **[1]**

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w. **[5]**

(Q10, June 2010)

28 The complex numbers z and w are given by $z = 4 + 3\text{i}$ and $w = 6 - \text{i}$. Giving your answers in the form $x + \text{i}y$ and showing clearly how you obtain them, find

(i) $3z - 4w$, **[2]**

(ii) $\dfrac{z^*}{w}$. **[4]**

(Q2, Jan 2011)

29 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = |z - 8|$, **[2]**

(b) $\arg(z + 2\text{i}) = \frac{1}{4}\pi$. **[3]**

(ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leqslant |z - 8| \quad \text{and} \quad 0 \leqslant \arg(z + 2\text{i}) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

(Q6, Jan 2011)

30 The complex number $1 + \mathrm{i}\sqrt{3}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by $|z - a| = |a|$ and $\arg(z - a) = \frac{1}{2}\pi$. **[6]**

(Q5, June 2011)

31 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30\mathrm{i}$.

(i) Write down the other root of the quadratic equation. **[1]**

(ii) Find the values of a and b. **[4]**

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. **[7]**

(Q9, June 2011)

32 The complex number $a + 5\mathrm{i}$, where a is positive, is denoted by z. Given that $|z| = 13$, find the value of a and hence find $\arg z$. **[4]**

(Q1, Jan 2012)

33 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

(Q3, Jan 2012)

34 Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

(Q6, Jan 2012)

35 The complex numbers z and w are given by $z = 6 - \mathrm{i}$ and $w = 5 + 4\mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $z + 3w$, **[2]**

(ii) $\dfrac{z}{w}$. **[3]**

(Q1, June 2012)

36 The loci C_1 and C_2 are given by $|z - 3 - 4\mathrm{i}| = 4$ and $|z| = |z - 8\mathrm{i}|$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[6]**

(ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 - 4\mathrm{i}| \leqslant 4 \text{ and } |z| \geqslant |z - 8\mathrm{i}|.$$ **[2]**

(Q7, June 2012)

37 The complex number $2 - \mathrm{i}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. **[2]**

(ii) Given that $az + bz^* = 4 - 8\mathrm{i}$, find the values of the real constants a and b. **[5]**

(Q3, Jan 2013)

38 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = 2$, **[2]**

(b) $\arg(z - 3 - \mathrm{i}) = \pi$. **[3]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leqslant 2 \text{ and } 0 \leqslant \arg(z - 3 - \mathrm{i}) \leqslant \pi.$$ **[2]**

(Q7, Jan 2013)

39 The complex number $3 + a\mathrm{i}$, where a is real, is denoted by z. Given that $\arg z = \frac{1}{6}\pi$, find the value of a and hence find $|z|$ and $z^* - 3$. **[6]**

(Q1, June 2013)

40 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

(Q3, June 2013)

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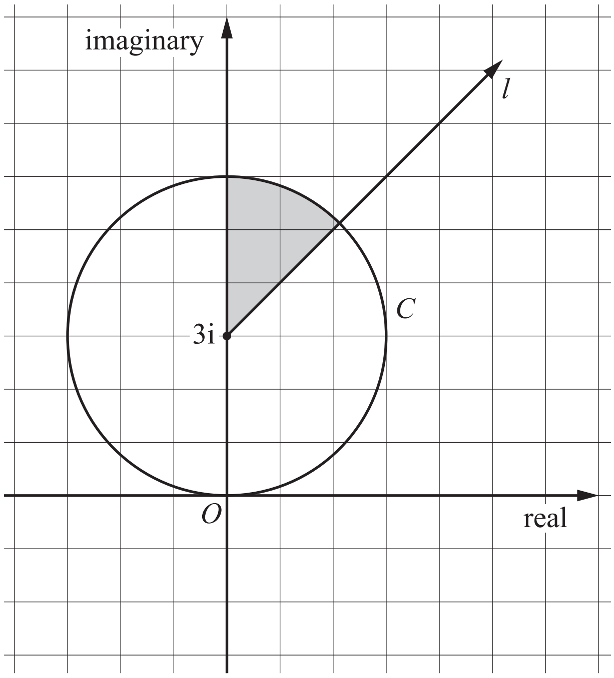

The Argand diagram above shows a half-line l and a circle C. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

(Q6, June 2013)