Edexcel Maths FP1

Topic Questions from Papers

Proof by Induction

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4. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

(5)

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6. A series of positive integers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \quad \text{for } n \geqslant 1.$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$, for $n \geqslant 1$.

(5)

Leave blank

8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

(7)

Leave blank

3. A sequence of numbers is defined by

$$u_1 = 2,$$
$$u_{n+1} = 5u_n - 4, \qquad n \geqslant 1.$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 5^{n-1} + 1$.

(4)

Leave blank

8. (a) Prove by induction that, for any positive integer *n*,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

Leave blank

7.

$$f(n) = 2^n + 6^n$$

(a) Show that $f(k+1) = 6f(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

Leave blank

9. (a) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

Leave blank

9. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$ is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}\left(4^n - 1\right)$$

(5)

Leave blank

9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$,

(6)

(b) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

Leave blank

6. (a) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

Leave blank

7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geqslant 1, \quad u_1 = 1$$

(a) Find u_2 and u_3.

(2)

(b) Prove by induction that $u_n = 2^n - 1$

(5)

Leave blank

10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by 5.}$$

(6)

Leave blank

8. (a) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(r+3) = \tfrac{1}{3}n(n+1)(n+5)$$

(6)

(b) A sequence of positive integers is defined by

$$u_1 = 1,$$
$$u_{n+1} = u_n + n(3n+1), \qquad n \in \mathbb{Z}^+$$

Prove by induction that

$$u_n = n^2(n-1)+1, \qquad n \in \mathbb{Z}^+$$

(5)

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9. (a) A sequence of numbers is defined by

$$u_1 = 8$$
$$u_{n+1} = 4u_n - 9n, \quad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

Leave blank

8. (a) Prove by induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$$

(6)

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$