Edexcel Maths FP1

Topic Questions from Papers

Matrices

Leave blank

7. Given that $\mathbf{X} = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$, where a is a constant, and $a \neq 2$,

(a) find $\mathbf{X}^{-1}$ in terms of a.

(3)

Given that $\mathbf{X} + \mathbf{X}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of a.

(3)

Leave blank

10. $\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(a) Describe fully the transformations described by each of the matrices **A**, **B** and **C**.
(4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(b) Find **D**.
(2)

(c) Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$.
(1)

The triangle *ORS* has vertices at the points with coordinates (0, 0), (–15, 15) and (4, 21). This triangle is transformed onto the triangle *OR′S′* by the transformation described by **E**.

(d) Find the coordinates of the vertices of triangle *OR′S′*.
(4)

(e) Find the area of triangle *OR′S′* and deduce the area of triangle *ORS*.
(3)

Leave blank

5. $\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$.

(a) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(b) find the value of a and the value of b.

(5)

Leave blank

7. $$\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find the value of a for which the matrix **A** is singular.

(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(b) Find $\mathbf{B}^{-1}$.

(3)

The transformation represented by **B** maps the point P onto the point Q.

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$.

(3)

Leave blank

5. $$\mathbf{A} = \begin{pmatrix} a & -5 \\ 2 & a+4 \end{pmatrix}, \text{ where } a \text{ is real.}$$

(a) Find det **A** in terms of *a*.

(2)

(b) Show that the matrix **A** is non-singular for all values of *a*.

(3)

Given that $a = 0$,

(c) find $\mathbf{A}^{-1}$.

(3)

Leave blank

9.

$$\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the geometrical transformation represented by the matrix **M**.
(2)

The transformation represented by **M** maps the point *A* with coordinates (p, q) onto the point *B* with coordinates $(3\sqrt{2}, 4\sqrt{2})$.

(b) Find the value of *p* and the value of *q*.
(4)

(c) Find, in its simplest surd form, the length *OA*, where *O* is the origin.
(2)

(d) Find $\mathbf{M}^2$.
(2)

The point *B* is mapped onto the point *C* by the transformation represented by $\mathbf{M}^2$.

(e) Find the coordinates of *C*.
(2)

Leave blank

2. $$\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}, \text{ where } a \text{ is a real constant.}$$

(a) Given that $a = 2$, find $\mathbf{M}^{-1}$.

(3)

(b) Find the values of a for which $\mathbf{M}$ is singular.

(2)

Leave blank

6. Write down the 2 × 2 matrix that represents

(a) an enlargement with centre $(0, 0)$ and scale factor 8,

(1)

(b) a reflection in the x-axis.

(1)

Hence, or otherwise,

(c) find the matrix **T** that represents an enlargement with centre $(0, 0)$ and scale factor 8, followed by a reflection in the x-axis.

(2)

$$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}, \text{ where } k \text{ and } c \text{ are constants.}$$

(d) Find **AB**.

(3)

Given that **AB** represents the same transformation as **T**,

(e) find the value of k and the value of c.

(2)

Leave blank

2.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find **AB**.

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by **C**,

(2)

(c) write down $\mathbf{C}^{100}$.

(1)

Q2

(Total 6 marks)

Leave blank

8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find det **A**. **(1)**

(b) Find $\mathbf{A}^{-1}$. **(2)**

The triangle *R* is transformed to the triangle *S* by the matrix **A**.
Given that the area of triangle *S* is 72 square units,

(c) find the area of triangle *R*. **(2)**

The triangle *S* has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

(d) Find the coordinates of the vertices of *R*. **(4)**

Leave blank

3. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(b) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

describe fully the geometrical transformation represented by **B**.

(2)

(c) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

Leave blank

5. $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates $(4, 6)$ onto the point with coordinates $(2, -8)$,

(a) find the value of a and the value of b.

(4)

A quadrilateral R has area 30 square units.
It is transformed into another quadrilateral S by the matrix **A**.
Using your values of a and b,

(b) find the area of quadrilateral S.

(4)

Leave blank

4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(a) Find the coordinates of the vertices of T'.

(2)

(b) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(c) Find **QR**.

(2)

(d) Find the determinant of **QR**.

(2)

(e) Using your answer to part (d), find the area of T''.

(3)

Leave blank

8.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Show that **A** is non-singular.

(2)

(b) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

Leave blank

2. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$$

find **AB**.

(2)

(b) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D}$$

find the value of k for which **E** has no inverse.

(4)

Leave blank

9.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

(a) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(b) Find the value of a.

(3)

The point R has coordinates (6, 0).

Given that O is the origin,

(c) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(d) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(f) Find **B**.

(4)

Leave blank

4. The transformation U, represented by the 2×2 matrix **P**, is a rotation through 90° anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$.

(b) Write down the matrix **Q**.

(1)

Given that U followed by V is transformation T, which is represented by the matrix **R**,

(c) express **R** in terms of **P** and **Q**,

(1)

(d) find the matrix **R**,

(2)

(e) give a full geometrical description of T as a single transformation.

(2)

Leave blank

6. $\mathbf{X} = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}$, where a is a constant.

(a) Find the value of a for which the matrix $\mathbf{X}$ is singular.

(2)

$$\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$$

(b) Find $\mathbf{Y}^{-1}$.

(2)

The transformation represented by $\mathbf{Y}$ maps the point A onto the point B.

Given that B has coordinates $(1 - \lambda, 7\lambda - 2)$, where λ is a constant,

(c) find, in terms of λ, the coordinates of point A.

(4)

Leave blank

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(4)

Leave blank

8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and **I** is the 2 × 2 identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by **A** maps the point *P* onto the point *Q*.

Given that *Q* has coordinates $(2k + 8, -2k - 5)$, where *k* is a constant,

(c) find, in terms of *k*, the coordinates of *P*.

(4)

Leave blank

2. (i) $$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}, \quad \text{where } k \text{ is a constant}$$

Given that

$$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where **I** is the 2 × 2 identity matrix, find

(a) **B** in terms of *k*,

(2)

(b) the value of *k* for which **B** is singular.

(2)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = (2 \ \ -1 \ \ 5)$$

and

$$\mathbf{E} = \mathbf{CD}$$

find **E**.

(2)

Leave blank

6. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(a) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(b) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(c) Find the matrix **Q**.

(2)

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$