FP1 Roots & Coefficients Questions

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - \mathrm{i}$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

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1 The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots α and β.

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) (i) Expand $(\alpha + \beta)^3$. *(1 mark)*

(ii) Show that $\alpha^3 + \beta^3 = 4$. *(3 marks)*

(c) Find a quadratic equation with roots α^3 and β^3, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers. *(3 marks)*

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3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

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4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{4}$. *(2 marks)*

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta}$$ *(3 marks)*

FP1 Roots & Coefficients of Equations Answers

(b)(i)	Other root is $\sqrt{5}$ + i	B1	1	
(ii)	Sum of roots is $2\sqrt{5}$	B1		
	Product is 6	M1A1	3	
(iii)	$p = -2\sqrt{5}$, $q = 6$	B1		
		B1√	2	ft wrong answers in (ii)

1(a)	$\alpha + \beta = 2,\ \alpha\beta = \frac{2}{3}$	B1B1	2	**SC** 1/2 for answers 6 and 2
(b)(i)	$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	B1	1	Accept unsimplified
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	M1		
	Substitution of numerical values	m1		
	$\alpha^3 + \beta^3 = 4$	A1	3	convincingly shown AG
(c)	$\alpha^3\beta^3 = \frac{8}{27}$	B1		
	Equation of form $px^2 \pm 4px + r = 0$	M1		
	Answer $27x^2 - 108x + 8 = 0$	A1√	3	ft wrong value for $\alpha^3\beta^3$
	Total		**9**	

3(a)	$\alpha + \beta = -2,\ \alpha\beta = \frac{3}{2}$	B1B1	2	
(b)	Use of expansion of $(\alpha + \beta)^2$	M1		
	$\alpha^2 + \beta^2 = (-2)^2 - 2\left(\frac{3}{2}\right) = 1$	m1A1	3	convincingly shown (**AG**); m1A0 if $\alpha + \beta = 2$ used
(c)	$\alpha^4 + \beta^4$ given in terms of $\alpha + \beta, \alpha\beta$ and/or $\alpha^2 + \beta^2$	M1A1		M1A0 if num error made
	$\alpha^4 + \beta^4 = -\frac{7}{2}$	A1	3	OE
	Total		**8**	

4(a)	$\alpha + \beta = \frac{1}{2},\ \alpha\beta = 2$	B1B1	2	
(b)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$	M1		
	$\ldots = \frac{\frac{1}{2}}{2} = \frac{1}{4}$	A1	2	Convincingly shown (AG)
(c)	Sum of roots = 1	B1F		PI by term $\pm x$; ft error(s) in (a)
	Product of roots = $\frac{16}{\alpha\beta} = 8$	B1F		ft wrong value of $\alpha\beta$
	Equation is $x^2 - x + 8 = 0$	B1F	3	ft wrong sum/product; "= 0" needed
	Total		**7**	