FP1 Numerical Solutions of Equations Questions

1 (a) Show that the equation

$$x^3 + 2x - 2 = 0$$

has a root between 0.5 and 1. *(2 marks)*

(b) Use linear interpolation once to find an estimate of this root. Give your answer to two decimal places. *(3 marks)*

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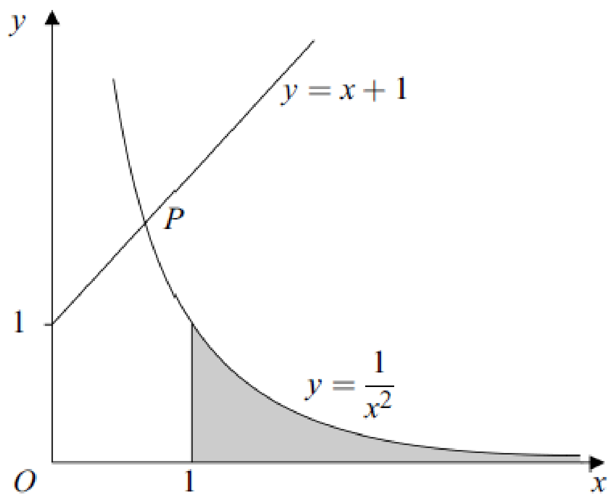
2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \log_{10} x$$

Starting at the point (2, 3) on the curve, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 2.4$. Give your answer to three decimal places. *(6 marks)*

(b) The diagram shows the graphs of

$$y = \frac{1}{x^2} \quad \text{and} \quad y = x + 1 \quad \text{for} \quad x > 0$$



The graphs intersect at the point P.

(i) Show that the x-coordinate of P satisfies the equation $f(x) = 0$, where f is the function defined in part (a). *(1 mark)*

(ii) Taking $x_1 = 1$ as a first approximation to the root of the equation $f(x) = 0$, use the Newton–Raphson method to find a second approximation x_2 to the root.
(3 marks)

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2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. *(3 marks)*

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. *(4 marks)*

FP1 Numerical Solutions of Equations Answers

1(a)	f(0.5) = –0.875, f(1) = 1	B1		
	Change of sign, so root between	E1	2	
(b)	Complete line interpolation method	M2,1		M1 for partially correct method
	Estimated root = $\frac{11}{15} \approx 0.73$	A1	3	Allow $\frac{11}{15}$ as answer
	Total		**5**	

2	1st increment is 0.2 lg 2 ...	M1		or 0.2 lg 2.1 or 0.2 lg 2.2
	... ≈ 0.06021	A1		PI
	$x = 2.2 \Rightarrow y \approx 3.06021$	A1√		PI; ft numerical error
	2nd increment is 0.2 lg 2.2	m1		consistent with first one
	... ≈ 0.06848	A1		PI
	$x = 2.4 \Rightarrow y \approx 3.12869 \approx 3.129$	A1√	6	ft numerical error
	Total		**6**	

	[illegible]	[illegible]	[illegible]	[illegible]
(b)(i)	$x^2(x + 1) = 1$, hence result	B1	1	convincingly shown (AG)
(ii)	$x_2 = 1 - \frac{1}{5} = \frac{4}{5}$	M1A1√ A1√	3	ft c's value of f′(1)
(c)	Area = $\int_1^{\infty} x^{-2}\,dx$	M1		
	... = $\left[-x^{-1}\right]_1^{\infty}$	M1		Ignore limits here
	... = 0 – –1 = 1	A1	3	

2(a)	f(1.6) = −1.304, f(1.8) = 0.632	B1,B1		Allow 1 dp throughout
	Sign change, so root between	E1	3	
(b)	f(1.7) considered first	M1		
	f(1.7) = −0.387, so root > 1.7	A1		
	f(1.75) = 0.109375, so root ≈ 1.7	m1A1	4	m1 for f(1.65) after error
	Total		**7**	