FP1 Graphs Rational Functions Questions

4 A curve has equation

$$y = \frac{6x}{x-1}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve and the two asymptotes. *(4 marks)*

(c) Solve the inequality

$$\frac{6x}{x-1} < 3$$ *(4 marks)*

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9 A curve C has equation

$$y = \frac{(x+1)(x-3)}{x(x-2)}$$

(a) (i) Write down the coordinates of the points where C intersects the x-axis. *(2 marks)*

(ii) Write down the equations of all the asymptotes of C. *(3 marks)*

(b) (i) Show that, if the line $y = k$ intersects C, then

$$(k-1)(k-4) \geqslant 0$$ *(5 marks)*

(ii) Given that there is only one stationary point on C, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(3 marks)*

(c) Sketch the curve C. *(3 marks)*

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5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0$$ *(3 marks)*

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7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. *(5 marks)*

(c) Hence, or otherwise, solve the inequality

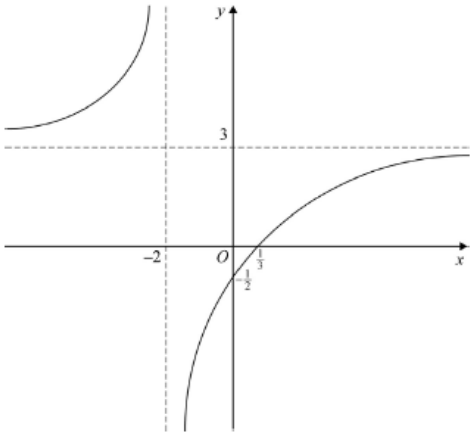
$$0 < \frac{3x - 1}{x + 2} < 3$$ *(2 marks)*

FP1 Graphs Rational Functions Answers

4(a)	Asymptotes $x = 1$, $y = 6$	B1B1	2	
(b)	Curve (correct general shape)	M1		SC Only one branch:
	Curve passing through origin	A1		B1 for origin
	Both branches approaching $x = 1$	A1		B1 for approaching both asymptotes
	Both branches approaching $y = 6$	A1	4	(Max 2/4)
(c)	Correct method	M1		
	Critical values ±1	B1B1		From graph or calculation
	Solution set $-1 < x < 1$	A1√	4	ft one error in CVs; NMS 4/4 after a good graph
	Total		**10**	

9(a)(i)	Intersections at (–1, 0), (3, 0)	B1B1	2	Allow $x = -1$, $x = 3$
(ii)	Asymptotes $x = 0$, $x = 2$, $y = 1$	B1 × 3	3	
(b)(i)	$y = k \Rightarrow kx^2 - 2kx = x^2 - 2x - 3$	M1A1		M1 for clearing denominator
	$\ldots \Rightarrow (k-1)x^2+(-2k+2)x+3=0$	A1√		ft numerical error
	$\Delta = 4(k-1)(k-4)$, hence result	m1A1	5	convincingly shown (AG)
(ii)	$y = 4$ at SP	B1		
	$3x^2 - 6x + 3 = 0$, so $x = 1$	M1A1	3	A0 if other point(s) given
(c)	Curve with three branches	B1		approaching vertical asymptotes
	Middle branch correct	B1		Coordinates of SP not needed
	Other two branches correct	B1	3	3 asymptotes shown
	Total		**16**	

5(a)	Asymptotes $y = 0$, $x = -1$, $x = 1$	B1 × 3	3	
(b)	Three branches approaching two vertical asymptotes	B1		Asymptotes not necessarily drawn
	Middle branch passing through O	B1		with no stationary points
	Curve approaching $y = 0$ as $x \to \pm\infty$	B1		
	All correct	B1	4	with asymptotes shown and curve approaching all asymptotes correctly
(c)	Critical values $x = -1$, 0 and 1	B1		
	Solution set $-1 < x < 0$, $x > 1$	M1A1	3	M1 if one part correct or consistent with c's graph
	Total		**10**	

7(a)	Asymptotes $x = -2$, $y = 3$	B1,B1	2	
(b)		B1		Curve approaching asymptotes
		B1,B1		Passing through $\left(\frac{1}{3}, 0\right)$ and $\left(0, -\frac{1}{2}\right)$
		B1,B1	5	Both branches generally correct B1 if two branches shown
(c)	Solution set is $x > \frac{1}{3}$	B2,1F	2	B1 for good attempt; ft wrong point of intersection
	Total		**9**	