FP1 Conics Questions

8 A curve has equation $y^2 = 12x$.

(a) Sketch the curve. *(2 marks)*

(b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*

(ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*

(c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x-coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0$$ *(3 marks)*

(ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*

(iii) Using this value of c, find the coordinates of the point where the line touches the curve. *(2 marks)*

(iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

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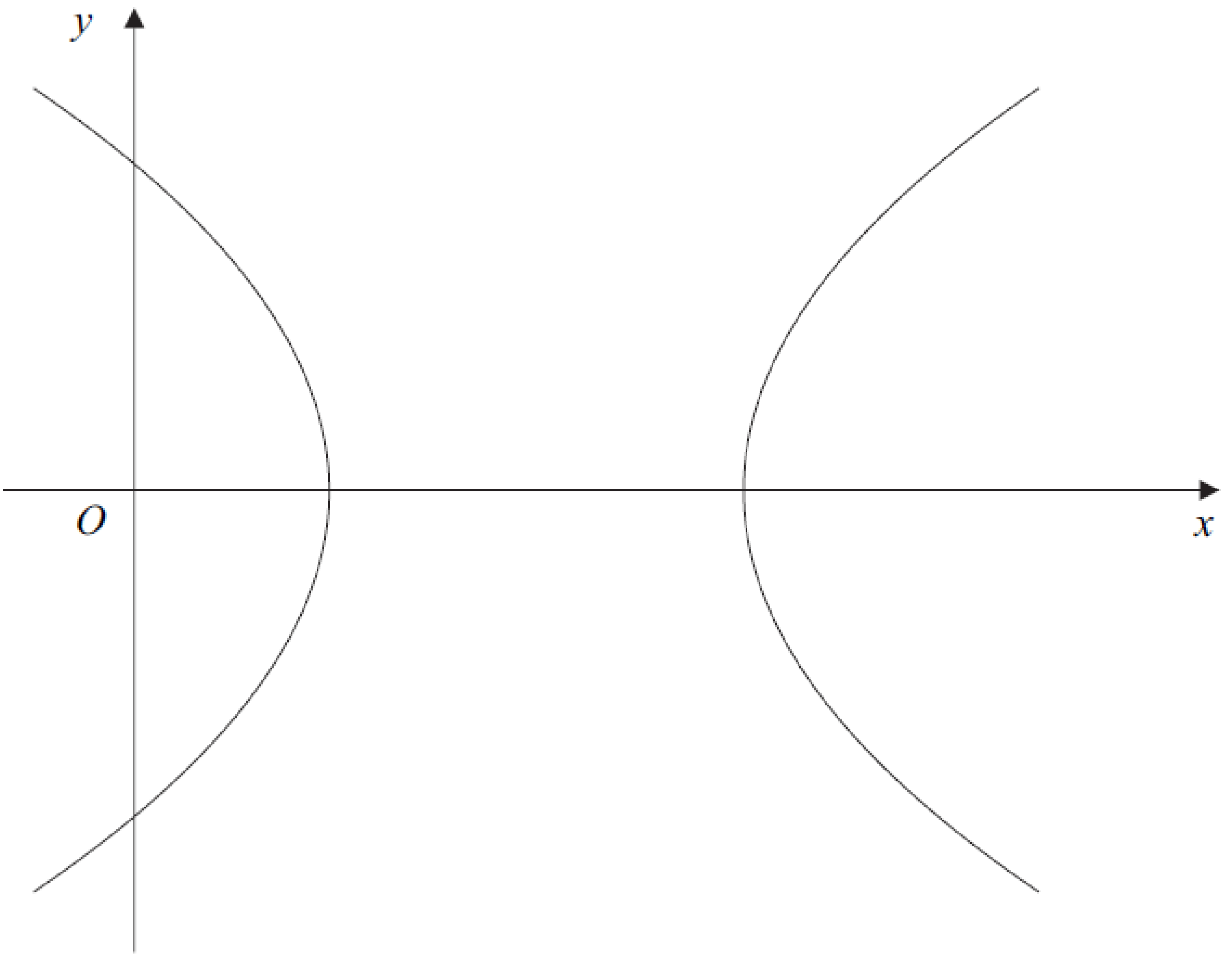
7 (a) Describe a geometrical transformation by which the hyperbola

$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$. *(2 marks)*

(b) The diagram shows the hyperbola H with equation

$$x^2 - y^2 - 4x + 3 = 0$$



By completing the square, describe a geometrical transformation by which the hyperbola H can be obtained from the hyperbola $x^2 - y^2 = 1$. *(4 marks)*

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8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

(a) Find the y-coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. *(3 marks)*

(c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. *(1 mark)*

(d) (i) Show that, if the line $y = x - 4$ intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0$$ *(3 marks)*

(ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C. *(2 marks)*

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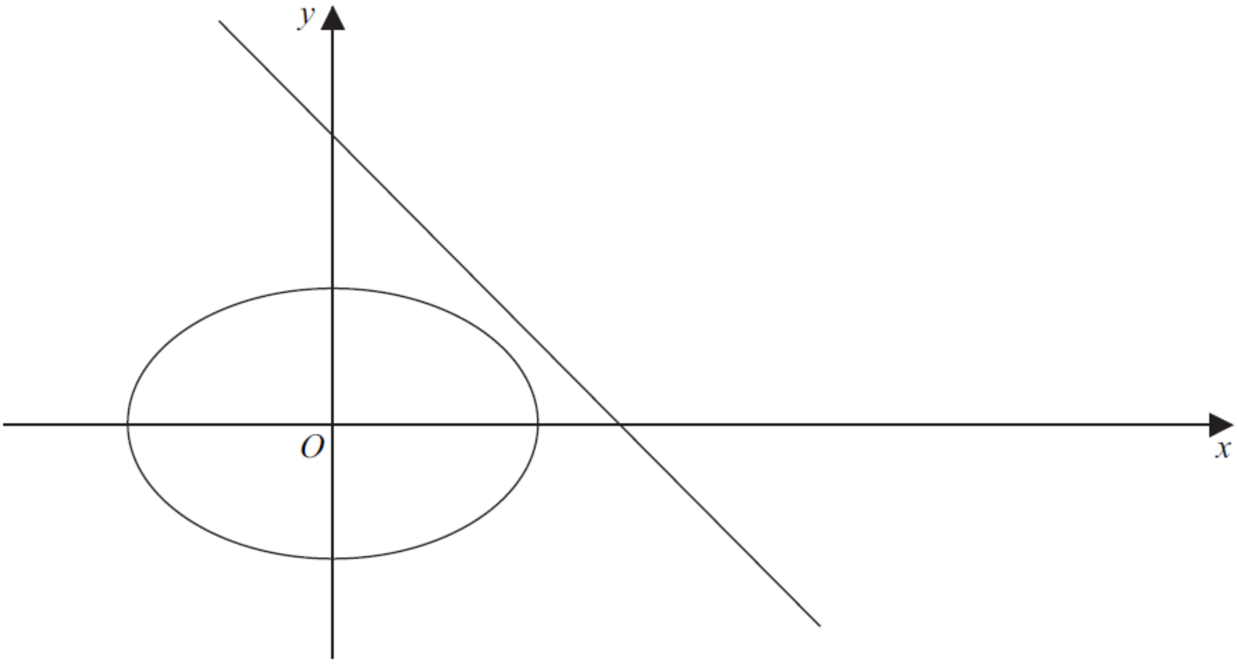
9 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



(a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. *(2 marks)*

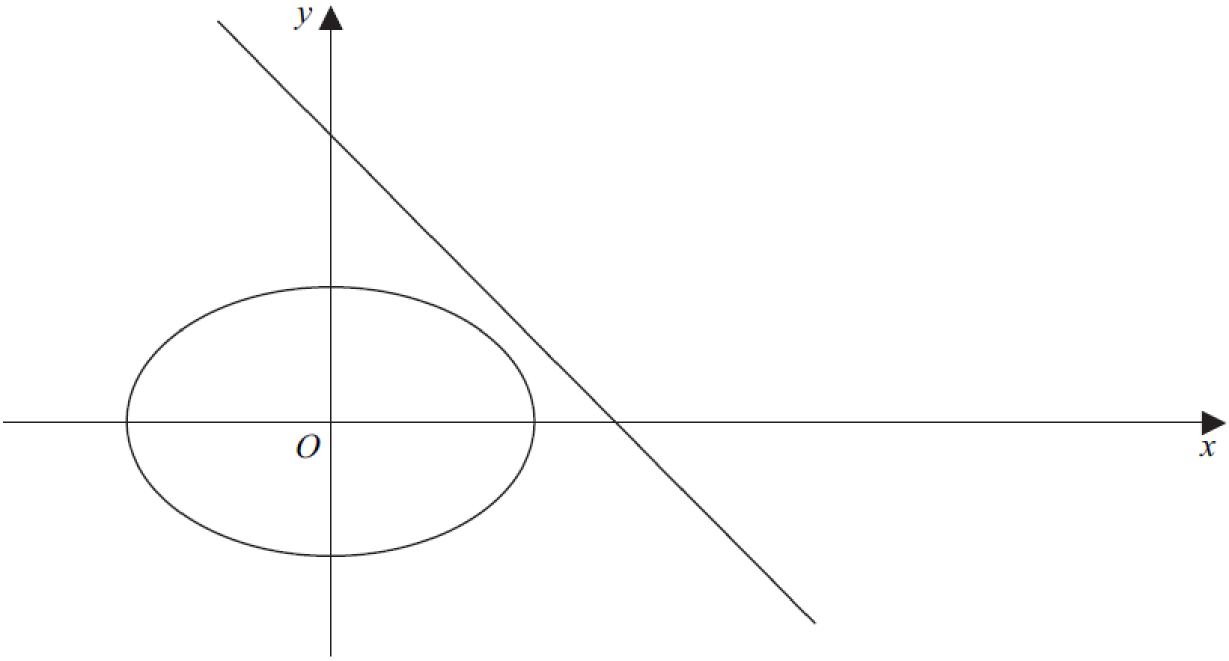
(b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k, the equation of the curve after this translation. *(2 marks)*

(c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x-coordinates of the points of intersection must satisfy the equation

$$3x^2 - 2(k + 4)x + (k^2 + 6) = 0$$ *(4 marks)*

(d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. *(4 marks)*

(e) On **Figure 3**, show the translated curves corresponding to these two values of k. *(3 marks)*

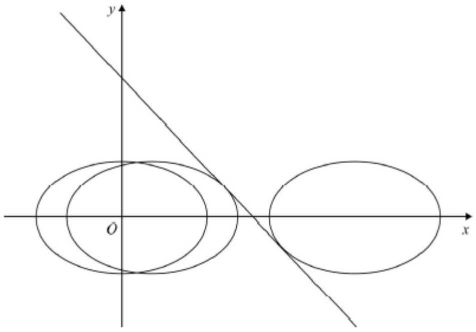

y
O
x

FP1 Conics Answers

8(a)	Good attempt at sketch	M1		
	Correct at origin	A1	2	
(b)(i)	y replaced by $y - 2$	B1		
	Equation is $(y-2)^2 = 12x$	B1√	2	ft $y+2$ for $y-2$
(ii)	Equation is $x^2 = 12y$	B1	1	
(c)(i)	$(x+c)^2 = x^2 + 2cx + c^2$	B1		
	$\ldots = 12x$	M1		
	Hence result	A1	3	convincingly shown (AG)
(ii)	Tangent if $(2c-12)^2 - 4c^2 = 0$	M1		
	ie if $-48c + 144 = 0$ so $c = 3$	A1	2	
(iii)	$x^2 - 6x + 9 = 0$	M1		
	$x = 3, y = 6$	A1	2	
(iv)	$c = 4 \Rightarrow$ discriminant $= -48 < 0$	M1A1		OE
	So line does not intersect curve	A1	3	
	Total		**15**	

7(a)	Stretch parallel to y axis ...	B1		
	... scale-factor $\frac{1}{2}$ parallel to y axis	B1	2	
(b)	$(x-2)^2 - y^2 = 1$	M1A1		
	Translation in x direction ...	A1		
	... 2 units in positive x direction	A1	4	
	Total		**6**	

8(a)	$x = 10 \Rightarrow 4 - \frac{y^2}{9} = 1$	M1		
	$\Rightarrow y^2 = 27$	A1		PI
	$\Rightarrow y = \pm 3\sqrt{3}$	A1	3	
(b)	One branch generally correct	B1		Asymptotes not needed
	Both branches correct	B1		With implied asymptotes
	Intersections at $(\pm 5, 0)$	B1	3	
(c)	Required tangent is $x = 5$	B1F	1	ft wrong value in (b)
(d)(i)	y correctly eliminated	M1		
	Fractions correctly cleared	m1		
	$16x^2 - 200x + 625 = 0$	A1	3	convincingly shown (**AG**)
(ii)	$x = \frac{25}{4}$	B1		No need to mention repeated root, but B0 if other values given as well
	Equal roots $\Rightarrow$ tangency	E1	2	Accept 'It's a tangent'
	Total		**12**	

9(a)	Intersections $\left(\pm\sqrt{2}, 0\right)$, $(0, \pm 1)$	B1B1	2	Allow B1 for $\left(\sqrt{2}, 0\right)$, $(0, 1)$
(b)	Equation is $\dfrac{(x-k)^2}{2} + y^2 = 1$	M1A1	2	M1 if only one small error, eg $x + k$ for $x - k$
(c)	Correct elimination of y Correct expansion of squares Correct removal of denominator Answer convincingly established	M1 M1 M1 A1	4	AG
(d)	Tgt $\Rightarrow 4(k+4)^2 - 12(k^2+6) = 0$ ... $\Rightarrow k^2 - 4k + 1 = 0$ ... $\Rightarrow k = 2 \pm \sqrt{3}$	M1 m1A1 A1	4	 OE
(e)		B1 B2	3	Curve to left of line Curve to right of line Curves must touch the line in approx correct positions SC 1/3 if both curves are incomplete but touch the line correctly
	Total		**15**	