FP1 Calculus Questions

2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}\,dx$; *(3 marks)*

(ii) $\displaystyle\int_0^9 \frac{1}{x\sqrt{x}}\,dx$. *(3 marks)*

(b) Explain briefly why the integrals in part (a) are improper integrals. *(1 mark)*

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8 (a) The function f is defined for all real values of x by

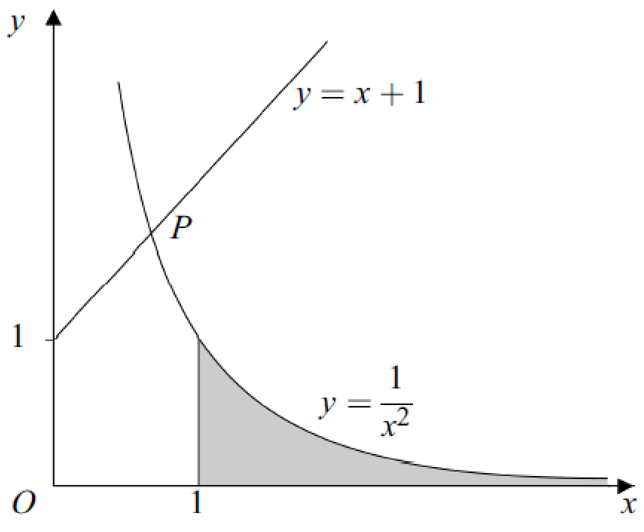
$$f(x) = x^3 + x^2 - 1$$

(i) Express $f(1+h) - f(1)$ in the form

$$ph + qh^2 + rh^3$$

where p, q and r are integers. *(4 marks)*

(ii) Use your answer to part (a)(i) to find the value of $f'(1)$. *(2 marks)*



(c) The region enclosed by the curve $y = \dfrac{1}{x^2}$, the line $x = 1$ and the x-axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. *(3 marks)*

The function f is defined for all real numbers by

$$f(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(b) The quadratic function g is defined for all real numbers by

$$g(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $g(x)$ gives a good approximation to $f(x)$ for small values of x.

(i) Show that $g(0.05)$ and $f(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = g(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $g'(0)$. *(1 mark)*

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8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\displaystyle\int_0^1 (x^{\frac{1}{3}} + x^{-\frac{1}{3}})\,dx$; *(4 marks)*

(b) $\displaystyle\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x}\,dx$. *(4 marks)*

FP1 Calculus Answers

2(a)(i)	$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}\ (+c)$	M1A1		M1 for $kx^{\frac{1}{2}}$
	$\int_0^9 \frac{1}{\sqrt{x}}\,dx = 6$	A1√	3	ft wrong coeff of $x^{\frac{1}{2}}$
(ii)	$\int x^{-\frac{3}{2}}\,dx = -2x^{-\frac{1}{2}}\ (+c)$	M1A1		M1 for $kx^{-\frac{1}{2}}$
	$x^{-\frac{1}{2}} \to \infty$ as $x \to 0$, so no value	E1	3	'Tending to infinity' clearly implied
(b)	Denominator $\to 0$ as $x \to 0$	E1	1	
	Total		7	

8(a)(i)	$(1+h)^3 = 1 + 3h + 3h^2 + h^3$	B1		
	$f(1+h) = 1 + 5h + 4h^2 + h^3$	M1A1√		PI; ft wrong coefficients
	$f(1+h) - f(1) = 5h + 4h^2 + h^3$	A1√	4	ft numerical errors
(ii)	Dividing by h	M1		
	$f'(1) = 5$	A1√	2	ft numerical errors
(c)	Area $= \int_1^\infty x^{-2}\,dx$	M1		
	$\ldots = \left[-x^{-1}\right]_1^\infty$	M1		Ignore limits here
	$\ldots = 0 - -1 = 1$	A1	3	

(b)(i)	$f(0.05) \approx 0.542\ 66$	B1		either value AWRT 0.5427
	$g(0.05) \approx 0.542\ 68$	B1	2	both values correct to 4DP
(ii)	$\frac{g(h) - g(0)}{h} = \frac{\sqrt{3}}{2} - \frac{1}{4}h$	M1A1	2	M1A0 if num error made
(iii)	As $h \to 0$ this gives $g'(0) = \frac{\sqrt{3}}{2}$	A1F	1	AWRT 0.866; ft num error

8(a)	$\int\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)\mathrm{d}x=\frac{3}{4}x^{\frac{4}{3}}+\frac{3}{2}x^{\frac{2}{3}}\ (+\,c)$	M1A1		M1 for adding 1 to index at least once
	$\int_0^1 \ldots=\left(\frac{3}{4}+\frac{3}{2}\right)-0=\frac{9}{4}$	m1A1	4	Condone no mention of limiting process; m1 if "$-$ 0" stated or implied
(b)	Second term is $x^{-\frac{4}{3}}$	B1		
	Integral of this is $-3x^{-\frac{1}{3}}$	M1A1		M1 for correct index
	$x^{-\frac{1}{3}}\to\infty$ as $x\to 0$, so no value	E1	4	