Exercise 8C

1 Basis: $u_n = 5^n - 1 \Rightarrow n = 1$: $u_1 = 5^1 - 1 = 4$ as given

For $n = 2$: $u_2 = 5^2 - 1 = 24$ from the general statement

$u_2 = 5u_2 + 4 = 5(5) + 4 = 24$ from the recurrence relation.

So u_n is true when $n = 1$ and $n = 2$.

Assumption: Assume u_n is true when $n = k$ for $k \in \mathbb{Z}^+$

$u_k = 5^k - 1$

Induction: Using the recurrence relation

$$u_{k+1} = 5u_k + 4$$
$$= 5(5^k - 1) + 4$$
$$= 5^{k+1} - 1$$

This is the same expression that the general statement gives for u_{k+1}.

Therefore, the general statement is true for $n = k + 1$.

Conclusion: If u_n is true when $n = k$, then it has been shown that u_n is also true when $n = k + 1$. As u_n is true for $n = 1$ and $n = 2$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

2 Basis: $u_n = 2^{n+2} - 5 \Rightarrow n = 1$: $u_1 = 2^3 - 5 = 3$ as given

For $n = 2$: $u_2 = 2^4 - 5 = 11$ from the general statement

$u_2 = 2u_1 + 5 = 2(3) + 5 = 11$ from the recurrence relation.

So u_n is true when $n = 1$ and $n = 2$.

Assumption: Assume u_n is true when $n = k$ for $k \in \mathbb{Z}^+$

$u_k = 2^{k+2} - 5$

Induction: Using the recurrence relation

$$u_{k+1} = 2u_k + 5$$
$$= 2(2^{k+2} - 5) + 5$$
$$= 2^{k+3} - 5$$

This is the same expression that the general statement gives for u_{k+1}.

Therefore, the general statement is true for $n = k + 1$.

Conclusion: If u_n is true when $n = k$, then it has been shown that u_n is also true when $n = k + 1$. As u_n is true for $n = 1$ and $n = 2$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

3 Basis: $u_n = 5^{n-1} + 2 \Rightarrow n = 1:\ u_1 = 5^0 + 2 = 3$ as given

For $n = 2$: $u_2 = 5^1 + 2 = 7$ from the general statement

$u_2 = 5u_1 - 8 = 5(3) - 8 = 7$ from the recurrence relation.

So u_n is true when $n = 1$ and $n = 2$.

Assumption: Assume u_n is true when $n = k$ for $k \in \mathbb{Z}^+$

$u_k = 5^{k-1} + 2$

Induction: Using the recurrence relation

$$\begin{aligned} u_{k+1} &= 5u_k - 8 \\ &= 5\left(5^{k-1} + 2\right) - 8 \\ &= 5^k + 10 - 8 \\ &= 5^k + 2 \end{aligned}$$

This is the same expression that the general statement gives for u_{k+1}.

Therefore, the general statement is true for $n = k + 1$.

Conclusion: If u_n is true when $n = k$, then it has been shown that u_n is also true when $n = k + 1$. As u_n is true for $n = 1$ and $n = 2$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

4 Basis: $u_n = 3^n - 2^n \Rightarrow n = 1:\ u_1 = 3^1 - 2^1 = 1$ as given

$n = 2:\ u_2 = 3^2 - 2^2 = 5$ as given

For $n = 3$: $u_3 = 3^3 - 2^3 = 19$ from the general statement

$u_3 = 5u_2 - 6u_1 = 5 \times 5 - 6 \times 1 = 19$ from the recurrence relation.

So u_n is true when $n = 1$, $n = 2$ and $n = 3$.

Assumption: Assume u_n is true when $n = k$ and $n = k + 1$ for $k \in \mathbb{Z}^+$

$u_k = 3^k - 2^k$

$u_{k+1} = 3^{k+1} - 2^{k+1}$

Induction: Using the recurrence relation

$$\begin{aligned} u_{k+2} &= 5u_{k+1} - 6u_k \\ &= 5\left(3^{k+1} - 2^{k+1}\right) - 6\left(3^k - 2^k\right) \\ &= 5(3^{k+1}) - 6(3^k) - 5(2^{k+1}) + 6(2^k) \\ &= 5(3^{k+1}) - 2(3^{k+1}) - 5(2^{k+1}) + 3(2^{k+1}) \\ &= 3(3^{k+1}) - 2(2^{k+1}) \\ &= 3^{k+2} - 2^{k+2} \end{aligned}$$

This is the same expression that the general statement gives for u_{k+2}.

Therefore, the general statement is true for $n = k + 2$.

Conclusion: If u_n is true when $n = k$ and $n = k + 1$, then it has been shown that u_n is also true when $n = k + 2$. As u_n is true for $n = 1$, $n = 2$ and $n = 3$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

5 <u>Basis:</u> $u_n = (n-2)3^{n-1} \Rightarrow n=1:\ u_1 = (1-2)3^{1-1} = -1$ as given

$n=2:\ u_2 = (2-2)3^{2-1} = 0$ as given

For $n = 3$: $u_3 = (3-2)3^{3-1} = 9$ from the general statement

$u_3 = 6u_2 - 9u_1 = 6\times 0 - 9\times(-1) = 9$ from the recurrence relation.

So u_n is true when $n = 1$, $n = 2$ and $n = 3$.

<u>Assumption:</u> Assume u_n is true when $n = k$ and $n = k + 1$ for $k \in \mathbb{Z}^+$

$u_k = (k-2)3^{k-1}$

$u_{k+1} = (k-1)3^k$

<u>Induction:</u> Using the recurrence relation

$$\begin{aligned} u_{k+2} &= 6u_{k+1} - 9u_k \\ &= 6(k-1)3^k - 9(k-2)3^{k-1} \\ &= 3^k\left[6(k-1) - 3(k-2)\right] \\ &= 3^k(3k) \\ &= (k)3^{k+1} \\ &= (k+2-2)3^{k+2-1} \end{aligned}$$

This is the same expression that the general statement gives for u_{k+2}.

Therefore, the general statement is true for $n = k + 2$.

<u>Conclusion:</u> If u_n is true when $n = k$ and $n = k + 1$, then it has been shown that u_n is also true when $n = k + 2$. As u_n is true for $n = 1$, $n = 2$ and $n = 3$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

6 <u>Basis:</u> $u_n = 2\left(5^{n-1}\right) - 2^{n-1} \Rightarrow n = 1:\ u_1 = 2\left(5^0\right) - 2^0 = 1$ as given

$n = 2:\ u_2 = 2\left(5^1\right) - 2^1 = 8$ as given

For $n = 3$: $u_3 = 2\left(5^2\right) - 2^2 = 46$ from the general statement

$u_3 = 7u_2 - 10u_1 = 7(8) - 10(1) = 46$ from the recurrence relation.

So u_n is true when $n = 1$, $n = 2$ and $n = 3$.

<u>Assumption:</u> Assume u_n is true when $n = k$ and $n = k + 1$ for $k \in \mathbb{Z}^+$

$u_k = 2\left(5^{k-1}\right) - 2^{k-1}$

$u_{k+1} = 2\left(5^k\right) - 2^k$

<u>Induction:</u> Using the recurrence relation

$$\begin{aligned}
u_{k+2} &= 7u_{k+1} - 10u_k \\
&= 7\left[2(5^k) - 2^k\right] - 10\left[2(5^{k-1}) - 2^{k-1}\right] \\
&= 14(5^k) - 20(5^{k-1}) - 7(2^k) + 10(2^{k-1}) \\
&= 5^k(14 - 4) - 2^k(7 - 5) \\
&= 10(5^k) - 2(2^k) \\
&= 2\left(5^{k+1}\right) - 2^{k+1}
\end{aligned}$$

This is the same expression that the general statement gives for u_{k+2}.

Therefore, the general statement is true for $n = k + 2$.

<u>Conclusion:</u> If u_n is true when $n = k$ and $n = k + 1$, then it has been shown that u_n is also true when $n = k + 2$. As u_n is true for $n = 1$, $n = 2$ and $n = 3$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.

7 <u>Basis:</u> $u_n = (3n-2)3^n \Rightarrow n=1:\ u_1 = (3-2)3^1 = 3$ as given

$n=2:\ u_2 = (6-2)3^2 = 36$ as given

For $n = 3$: $u_3 = (9-2)3^3 = 189$ from the general statement

$u_3 = 6u_2 - 9u_1 = 6 \times 36 - 9 \times 3 = 189$ from the recurrence relation.

So u_n is true when $n = 1$, $n = 2$ and $n = 3$.

<u>Assumption:</u> Assume u_n is true when $n = k$ and $n = k + 1$ for $k \in \mathbb{Z}^+$

$u_k = (3k-2)3^k$

$u_{k+1} = (3(k+1)-2)3^{k+1} = (3k+1)3^{k+1}$

<u>Induction:</u> Using the recurrence relation

$$\begin{aligned}
u_{k+2} &= 6u_{k+1} - 9u_k \\
&= 6(3k+1)3^{k+1} - 9(3k-2)3^k \\
&= 3^{k+1}[6(3k+1) - 3(3k-2)] \\
&= 3^{k+1}[18k+6-9k+6] \\
&= 3^{k+1}(9k+12) \\
&= (3k+4)3^{k+2} \\
&= [3(k+2)-2]3^{k+2}
\end{aligned}$$

This is the same expression that the general statement gives for u_{k+2}.

Therefore, the general statement is true for $n = k + 2$.

<u>Conclusion:</u> If u_n is true when $n = k$ and $n = k + 1$, then it has been shown that u_n is also true when $n = k + 2$. As u_n is true for $n = 1$, $n = 2$ and $n = 3$ then u_n is true for all $n \geq 1$ and $n \in \mathbb{Z}^+$ by mathematical induction.