

Exercise 6C

$$1 \text{ a } \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{b } \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\text{c } \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\text{d } \begin{pmatrix} 5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

$$2 \text{ a } 4$$

$$\text{b } 3$$

$$\text{c } 4$$

$$\text{d } 2.5$$

$$3 \text{ a } (0, 0)$$

$$\text{b } (0,0), (3,0), (3,4), (0,4)$$

So area = $3 \times 4 = 12$

$$4 \text{ a } \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{b } \begin{pmatrix} -3 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\text{c } \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$5 \text{ a } \text{Stretch parallel of the } x\text{-axis, scale factor } 2 \text{ and stretch parallel to the } y\text{-axis, scale factor } -3$$

$$\text{b } \det \mathbf{M} = -6 \text{ so the area has been multiplied by } 6. \text{ So } k = 4.$$

$$6 \text{ a } \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

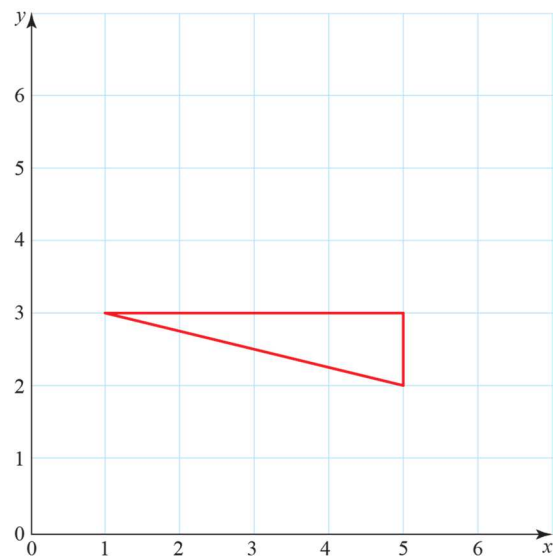
$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 15 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ 6 \end{pmatrix}$$

So vertices are (3,9), (15,9), (15,6)

$$\text{b } \text{Area of original triangle} = \frac{1}{2} \times 4 \times 1 = 2$$

$\det \mathbf{M} = 9$ so new area = 18



$$7 \text{ a } \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ -15 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ -15 \end{pmatrix}$$

So vertices are (4,0), (8,0), (8,-15), (4,-15)

$$\text{b } \text{Area of rectangle} = 4 \times 15 = 60$$

$$8 \text{ a } \text{Enlargement, centre } (0, 0), \text{ scale factor } 2\sqrt{5}$$

$$\text{b } \text{Area of original triangle}$$

$$= \left| \frac{1}{2} \times (4-a) \times 2 \right| = |4-a|$$

$$\det \mathbf{M} = 20 \Rightarrow |4-a| = 3$$

$$\Rightarrow a = 7 \text{ or } 1$$

$$\begin{aligned} 9 \text{ a } \mathbf{M}^2 &= \begin{pmatrix} p & 1 \\ p & q \end{pmatrix} \begin{pmatrix} p & 1 \\ p & q \end{pmatrix} \\ &= \begin{pmatrix} p^2 + p & p + q \\ p^2 + pq & p + q^2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{b } \mathbf{M}^2 &= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \Rightarrow p^2 + p = 6 \\ &\Rightarrow (p+3)(p-2) = 0 \\ &\Rightarrow p = -3 \text{ or } 2 \\ &\text{But also } p+q = 0 \text{ so } p \text{ must be } -3 \\ &\text{because } q > 0 \\ &\text{So } p = -3, q = 3 \end{aligned}$$

$$10 \text{ a } \mathbf{M} = \begin{pmatrix} 5 & -1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$$

b Stretch parallel to the x -axis, scale factor 8; and stretch parallel to the y -axis, scale factor -8 . Or enlargement scale factor 8 and centre $(0, 0)$ and reflection in the x -axis.

$$\begin{aligned} \text{c } \det \mathbf{M} &= -6 \\ \text{so original area} &= 320 \div 64 = 5 \\ &\Rightarrow |2(k-1)| = 5 \\ &\Rightarrow k = -\frac{3}{2} \text{ or } \frac{7}{2} \end{aligned}$$

$$11 \quad \det \mathbf{M} = -1 \times -1 - (-\sqrt{2} \times \sqrt{2}) = 1 - 2 = 3$$

$$\text{So area of } P' = 12 \times 3 = 36$$

$$12 \quad \det \mathbf{M} = 8 + k$$

$$\text{Area of } T = \frac{1}{2}(4-k)(k-1)$$

$$\Rightarrow \frac{1}{2}(4-k)(k-1)(k+8) = 10$$

$$\Rightarrow (4-k)(k-1)(k+8) = 20$$

$$\Rightarrow (k^2 - 5k + 4)(k+8) = -20$$

$$\Rightarrow k^3 + 3k^2 - 36k + 52 = 0$$

$$\Rightarrow (k-2)(k^2 + 3k^2 + 5k - 26) = 0$$

$$\Rightarrow k = 2$$

for integer values of k .

$$13 \text{ a } \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\text{b } \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 7 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 7\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

So vertices of T' are

$$(0, 0), (0, 7\sqrt{2}), \left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\begin{aligned} \text{c } \det \mathbf{M} &= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right) \\ &= \frac{1}{2} - \left(-\frac{1}{2}\right) = 1 \end{aligned}$$

$$\text{d } \text{Area of } T' = \frac{1}{2} \times 7\sqrt{2} \times \frac{5}{\sqrt{2}} = 17.5$$

So area of $T = 17.5$

Challenge

$$\text{a } 0$$

$$\text{b } \text{For any } x, y, \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7x \\ 0 \end{pmatrix}$$

The y -coordinate is 0, so all points (x, y) map onto the x -axis.