Exercise 6A

1 a P is not linear because $(0,0) \to (0,1)$ is not linear

b Q is not linear because $x \to x^2$ is not linear

c R is not linear because $y \to x + xy$ is not linear

d S is linear

e T is not linear because $(0,0) \to (3,3)$ is not linear

f U is linear.

2 a S is represented by $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$

b T is not linear because $(0,0) \to (1,-1)$ is not linear

c U is not linear because $x \to xy$ is not linear

d V is represented by $\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$

e W is represented by $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3 a S is not linear because $x \to x^2$ and $y \to y^2$ are not linear

b T is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

c U is represented by $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

d V is represented by $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

e W is represented by $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4 a P: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y+2x \\ -y \end{pmatrix} = \begin{pmatrix} 2x+y \\ 0x-y \end{pmatrix}$ is represented by $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$

b Q: $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 0-y \\ x+2y \end{pmatrix}$ is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

5 a $\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-1 & 2 & 5\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}1 & -2 & -5\\ 1 & 3 & 1\end{pmatrix}$

$\therefore$ vertices of image of T are at $(1,1);(-2,3);(-5,1)$

b $\begin{pmatrix}1 & 4\\ 0 & -2\end{pmatrix}\begin{pmatrix}-1 & 2 & 5\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}3 & 14 & 9\\ -2 & -6 & -2\end{pmatrix}$

$\therefore$ vertices of image of T are at $(3,-2);(14,-6);(9,-2)$

c $\begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix}\begin{pmatrix}-1 & 2 & 5\\ 1 & 3 & 1\end{pmatrix}=\begin{pmatrix}-2 & -6 & -2\\ -2 & 4 & 10\end{pmatrix}$

$\therefore$ vertices of image of T are $(-2,-2);(-6,4);(-2,10)$

6 a $\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}=\begin{pmatrix}-2 & 0 & 2 & 0\\ 0 & 3 & 0 & -3\end{pmatrix}$

$\therefore$ vertices of the image of S are $(-2,0):(0,3);(2,0);(0,-3)$

b $\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}=\begin{pmatrix}-1 & -1 & 1 & 1\\ -1 & 1 & 1 & -1\end{pmatrix}$

$\therefore$ vertices of the image of S are $(-1,-1);(-1,1);(1,1);(1,-1)$

c $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}=\begin{pmatrix}-1 & 1 & 1 & -1\\ -1 & -1 & 1 & 1\end{pmatrix}$

$\therefore$ vertices of the image of S are $(-1,-1);(1,-1);(1,\ 1);(-1,\ 1)$

7 a

$$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}=\begin{pmatrix}-2\\ -1\end{pmatrix}$$

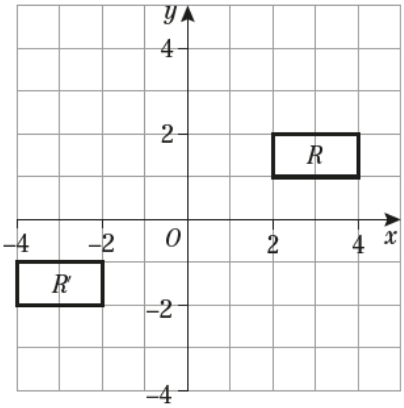
$$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}4\\ 1\end{pmatrix}=\begin{pmatrix}-4\\ -1\end{pmatrix}$$

$$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}4\\ 2\end{pmatrix}=\begin{pmatrix}-4\\ -2\end{pmatrix}$$

$$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}2\\ 2\end{pmatrix}=\begin{pmatrix}-2\\ -2\end{pmatrix}$$

So vertices are $(-2,-1)$, $(-4,-1)$, $(-4,-2)$, $(-2,-2)$

7 b



c Rotation through 180° about (0, 0)

8 a

$$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}2\\0\end{pmatrix}$$

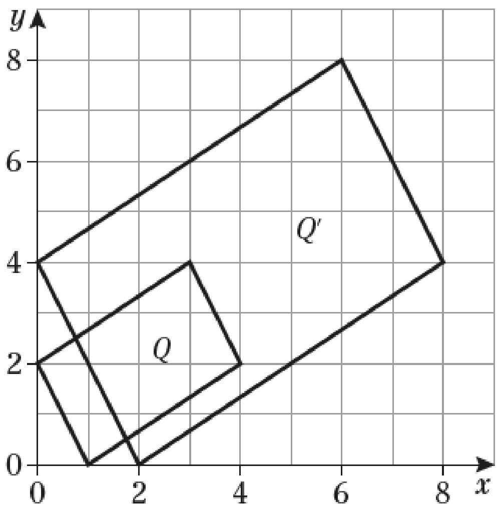
$$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}=\begin{pmatrix}8\\4\end{pmatrix}$$

$$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}6\\8\end{pmatrix}$$

$$\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}$$

so vertices are (2,0), (8,4), (6,8), (0,4)

b



c Enlargement, centre (0, 0), scale factor 2

9 **a**

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

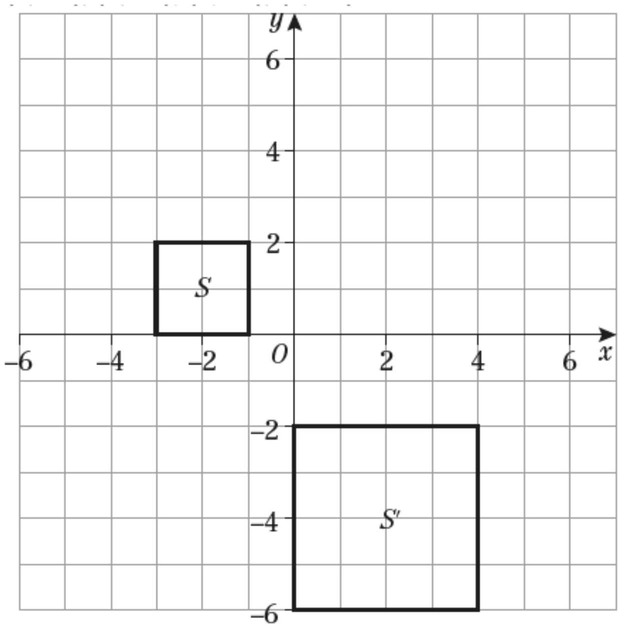
$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

so vertices are $(0,-2)$, $(0,6)$, $(4,-6)$, $(4,-2)$

b



c Reflection in $y = x$ and enlargement, centre (0, 0), scale factor 2

10 **a** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

so vertices are (4,1), (4,3), (1,3)

b The transformation represented by the identity matrix leaves T unchanged.

Challenge

a $\mathbf{T} = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$ so $\mathbf{T}\begin{pmatrix} kx \\ ky \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} kx \\ ky \end{pmatrix} = \begin{pmatrix} 2kx - 3ky \\ kx + ky \end{pmatrix}$

$= k\begin{pmatrix} 2x - 3y \\ x + y \end{pmatrix} = k\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix}$

b $\mathbf{T}\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = \mathbf{T}\begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$

$= \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix} = \begin{pmatrix} 2(x_1 + x_2) - 3(y_1 + y_2) \\ x_1 + x_2 + y_1 + y_2 \end{pmatrix}$

$= \begin{pmatrix} 2x_1 - 3y_1 \\ x_1 + y_1 \end{pmatrix} + \begin{pmatrix} 2x_2 - 3y_2 \\ x_2 + y_2 \end{pmatrix} = \mathbf{T}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \mathbf{T}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$