Exercise 5a

1 a $\begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$

The size is 2×2.

b $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

The size is 2×1.

c $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

The size is 2×3.

d $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

The size is 1×3.

e $\begin{pmatrix} 3 & -1 \end{pmatrix}$

The size is 1×2.

f $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The size is 3×3.

2 Compare corresponding elements:
$a = 6$
$b = 2$

3 a $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}+\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 8 & -1 \\ 1 & 4 \end{pmatrix}$

b $\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}-\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}=\begin{pmatrix} 2 & 2 \\ -2 & -5 \end{pmatrix}$

c $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}+\begin{pmatrix} 4 & 1 \\ -1 & -2 \end{pmatrix}-\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

4 a $A + B$ is $(2\times1)+(1\times2)$ Not possible

b $A-E=\begin{pmatrix} 1 \\ 2 \end{pmatrix}-\begin{pmatrix} 3 \\ -1 \end{pmatrix}=\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

c $F - D + C = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix}-\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$
$+\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}=\begin{pmatrix} 1 & 1 & 4 \end{pmatrix}$

4 d $B + C$ is $(1\times2)+(1\times3)$ Not possible

e $F-(D+C)=\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}$
$-\left[\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}+\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}\right]$
$=\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}-\begin{pmatrix} -1 & 2 & -1 \end{pmatrix}$
$=\begin{pmatrix} 3 & -1 & 4 \end{pmatrix}$

f $A - F = (2\times1)-(1\times3)$ Not possible.

g $C-(F-D)=\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$
$-\left[\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}-\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}\right]$
$=\begin{pmatrix} -1 & 1 & 0 \end{pmatrix}-\begin{pmatrix} 2 & 0 & 4 \end{pmatrix}$
$=\begin{pmatrix} -3 & 1 & -4 \end{pmatrix}$

5

$a-1 = 5 \Rightarrow a = 6$
$2-c = 0 \Rightarrow c = 2$
$-1-d = 0 \Rightarrow d = -1$
$b-(-2) = 5 \Rightarrow b = 3$

6

$1+a=c \quad (1)$
$2+b=5 \Rightarrow b=3$
$0+c=c$
$a+1=c$
$b+2=c \quad (2)$
$c+0=c$
Use $b = 3$ in (2) $\Rightarrow c = 5$
Use $c = 5$ in (1) $\Rightarrow a = 4$

7

$5+a = 7 \Rightarrow a = 2$
$3+b = 1 \Rightarrow b = -2$
$0+c = 2 \Rightarrow c = 2$
$-1+d = 0 \Rightarrow d = 1$
$2+e = 1 \Rightarrow e = -1$
$1+f = 4 \Rightarrow f = 3$

8 a $3\begin{pmatrix} 2 & 0 \\ 4 & -6 \end{pmatrix}=\begin{pmatrix} 6 & 0 \\ 12 & -18 \end{pmatrix}$

8 b $\frac{1}{2}\begin{pmatrix}2 & 0\\ 4 & -6\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2 & -3\end{pmatrix}$

c $2\begin{pmatrix}1\\ -1\end{pmatrix}=\begin{pmatrix}2\\ -2\end{pmatrix}$

d **A** and **B** are not the same size, so you can't subtract them.

9 a

$$3\mathbf{A}+2\mathbf{B}=3\begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix}+2\begin{pmatrix}2 & 1\\ -2 & 3\end{pmatrix}$$
$$=\begin{pmatrix}9 & -6\\ 3 & 0\end{pmatrix}+\begin{pmatrix}4 & 2\\ -4 & 6\end{pmatrix}$$
$$=\begin{pmatrix}13 & -4\\ -1 & 6\end{pmatrix}$$

b $2\mathbf{A}-4\mathbf{B}=2\begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix}-4\begin{pmatrix}2 & 1\\ -2 & 3\end{pmatrix}$
$$=\begin{pmatrix}6 & -4\\ 2 & 0\end{pmatrix}-\begin{pmatrix}8 & 4\\ -8 & 12\end{pmatrix}$$
$$=\begin{pmatrix}-2 & -8\\ 10 & -12\end{pmatrix}$$

c $5\mathbf{A}-2\mathbf{B}=5\begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix}-2\begin{pmatrix}2 & 1\\ -2 & 3\end{pmatrix}$
$$=\begin{pmatrix}15 & -10\\ 5 & 0\end{pmatrix}-\begin{pmatrix}4 & 2\\ -4 & 6\end{pmatrix}$$
$$=\begin{pmatrix}11 & -12\\ 9 & -6\end{pmatrix}$$

d $\frac{1}{2}\mathbf{A}+\frac{3}{2}\mathbf{B}=\frac{1}{2}\begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix}+\frac{3}{2}\begin{pmatrix}2 & 1\\ -2 & 3\end{pmatrix}$
$$=\begin{pmatrix}\frac{3}{2} & -1\\ \frac{1}{2} & 0\end{pmatrix}+\begin{pmatrix}3 & \frac{3}{2}\\ -3 & \frac{9}{2}\end{pmatrix}$$
$$=\begin{pmatrix}\frac{9}{2} & \frac{1}{2}\\ -\frac{5}{2} & \frac{9}{2}\end{pmatrix}$$

10
$$1+2k=7$$
$$\Rightarrow \quad 2k=6$$
$$\Rightarrow \quad k=3.$$
$$2-k=x$$
$$\Rightarrow \quad 2-3=x$$
$$\therefore \quad x=-1$$

11
$$2a-3=3 \quad \Rightarrow \quad 2a=6$$
$$\Rightarrow \quad a=3$$
$$0-3c=3 \quad \Rightarrow \quad c=-1$$
$$2-3d=-4 \quad \Rightarrow -3d=-6$$
$$\Rightarrow \quad d=2$$
$$2b+3=-4 \quad \Rightarrow \quad 2b=-7$$
$$\Rightarrow \quad b=-3.5$$

12
$$5-2c=9$$
$$\Rightarrow \quad -4=2c$$
$$\Rightarrow \quad c=-2.$$
$$a-4=1$$
$$\Rightarrow \quad a=5.$$
$$b-2=3$$
$$\Rightarrow \quad b=5.$$
$$0+2=d$$
$$\Rightarrow \quad d=2$$

13
$$-3+2k^2=k$$
$$\Rightarrow \quad 2k^2-k-3=0$$
$$(2k-3)(k+1)=0$$
$$\therefore \quad k=\tfrac{3}{2} \text{ or } -1$$
AND $k+2k^2=6$
$$\Rightarrow \quad 2k^2+k-6=0$$
$$(2k-3)(k+2)=0$$
$$\therefore \quad k=\tfrac{3}{2} \text{ or } -2$$
So common value is $k=\frac{3}{2}$