Exercise 4B

1 a A parabola has focus (5, 0) and directrix $x + 5 = 0$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, $a = 5$ for this parabola, giving the equation $y^2 = 20x$

b A parabola has focus (8, 0) and directrix $x + 8 = 0$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, $a = 8$ for this parabola, giving the equation $y^2 = 32x$

c A parabola has focus (1, 0) and directrix $x = -1$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, $a = 1$ for this parabola, giving the equation $y^2 = 4x$

d A parabola has focus $\left(\frac{3}{2}, 0\right)$ and directrix $x = -\frac{3}{2}$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, $a = \frac{3}{2}$ for this parabola, giving the equation $y^2 = 6x$

e A parabola has focus $\left(\frac{\sqrt{3}}{2}, 0\right)$ and directrix $x + \frac{\sqrt{3}}{2} = 0$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, $a = \frac{\sqrt{3}}{2}$ for this parabola, giving the equation $y^2 = 2\sqrt{3}x$

2 a A parabola has the equation $y^2 = 12x$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$
and directrix at $x + a = 0$
Therefore, for this parabola, $a = 3$, the focus is at (3, 0) and the directrix is at $x + 3 = 0$

b A parabola has the equation $y^2 = 20x$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$
and directrix at $x + a = 0$
Therefore, for this parabola, $a = 5$, the focus is at (5, 0) and the directrix is at $x + 5 = 0$

c A parabola has the equation $y^2 = 10x$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, for this parabola, $a = 2.5$, the focus is at (2.5, 0) and the directrix is at $x + 2.5 = 0$

d A parabola has the equation $y^2 = 4\sqrt{3}x$
The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$
Therefore, for this parabola, $a = \sqrt{3}$, the focus is at $\left(\sqrt{3}, 0\right)$ and the directrix is at $x + \sqrt{3} = 0$

2 e A parabola has the equation $y^2 = \sqrt{2}x$

The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$

Therefore, for this parabola, $a = \frac{\sqrt{2}}{4}$, the focus is at $\left(\frac{\sqrt{2}}{4}, 0\right)$ and the directrix is at $x + \frac{\sqrt{2}}{4} = 0$

f A parabola has the equation $y^2 = 5\sqrt{2}x$

The general equation of a parabola is $y^2 = 4ax$ with focus at $(a, 0)$ and directrix at $x + a = 0$

Therefore, for this parabola, $a = \frac{5\sqrt{2}}{4}$, the focus is at $\left(\frac{5\sqrt{2}}{4}, 0\right)$ and the directrix is at

$x + \frac{5\sqrt{2}}{4} = 0$

3 The distance from P to the point (3, 0) is the same as the distance from P to the directrix:

$(x-3)^2 + (y-0)^2 = (x+3)^2$

$x^2 - 6x + 9 + y^2 = x^2 + 6x + 9$

$y^2 = 12x$, so $y^2 = 4ax$ where $a = 3$

4 The distance from P to the point $\left(2\sqrt{5}, 0\right)$ is the same as the distance from P to the directrix:

$\left(x - 2\sqrt{5}\right)^2 + (y-0)^2 = \left(x + 2\sqrt{5}\right)^2$

$x^2 - 4\sqrt{5}x + 20 + y^2 = x^2 + 4\sqrt{5}x + 20$

$y^2 = 8\sqrt{5}x$, so $y^2 = 4ax$ where $a = 2\sqrt{5}$

5 a The distance from P to the point $(0, 2)$ is the same as the distance from P to the directrix:

$(x-0)^2 + (y-2)^2 = (y+2)^2$

$x^2 + y^2 - 4y + 4 = y^2 + 4y + 4$

$x^2 = 8y$

This equation can be written as $y = \frac{x^2}{8}$, so the equation is in the form $y = kx^2$ where $k = \frac{1}{8}$

b The equation for the locus of P is $x^2 = 8y$, which has the form of the equation for a parabola $x^2 = 4ay$ where $a = 2$

Therefore, the focus of P is (0, 2) and the directrix is $y + 2 = 0$ or $y = -2$

c