Exercise 3D

1 a $f(x) = x^3 - 2x - 1$

$f(1) = -2$

$f(2) = 3$

There is a change of sign, so there is a root α in the interval [1, 2].

b $f(x) = x^3 - 2x - 1$

$f'(x) = 3x^2 - 2$

Using $x_0 = 1.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.5 - \frac{(-0.625)}{4.75}$$

$$x_1 = 1.6316$$

$x_1 = 1.632$ correct to 3 d.p.

2 a $f(x) = x^2 - \frac{4}{x} + 6x - 10$

$$f'(x) = 2x + \frac{4}{x^2} + 6 = 2\left(x + \frac{2}{x^2} + 3\right)$$

b $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Using $x_0 = -0.4$

$$x_1 = -0.4 - \frac{0.4^2 - \frac{4}{-0.4} + 6\times(-0.4) - 10}{2\left(-0.4 + \frac{2}{-0.4^2} + 3\right)}$$

$$= -0.4 - \frac{-2.24}{30.2}$$

$$= -0.4 + 0.07417...$$

$$= -0.3258...$$

$x_1 = -0.326$ correct to 3 d.p.

3 a $f(x) = x^2 - \frac{3}{x^2}$

$$f(1.3) = 1.69 - \frac{3}{1.69} = -0.0851...$$

$$f(1.4) = 1.96 - \frac{3}{1.96} = 0.429...$$

There is a change of sign in the interval [1.3, 1.4] so there must be a root α in this interval.

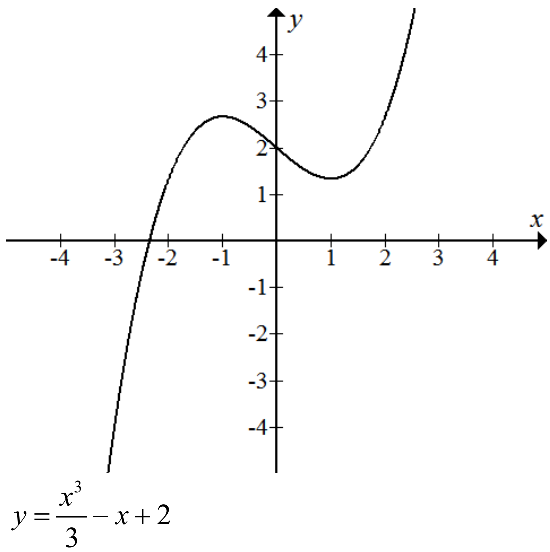
b $f'(x) = 2x + \frac{6}{x^3}$

c $f'(1.3) = 2.6 + \frac{6}{1.3^3} = 5.3309...$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow$$

$$x_1 = 1.3 + \frac{0.0851...}{5.3309...} = 1.316 \text{ to 3 d.p.}$$

4 a



4 b $f(x)=\frac{x^3}{3}-x+2$

$f'(x)=x^2-1$

Using $x_0=-2$

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=-2-\frac{1.3333333}{3}$

$=-2.\dot{4}$

$x_2=x_1-\frac{f(x_0)}{f'(x_0)}$

$=-2.\dot{4}-\frac{-0.4243256}{4.9753086}$

$=-2.3591581$

$x_3=x_2-\frac{f(x_0)}{f'(x_0)}$

$=-2.3591581-\frac{-0.0175732}{4.5656269}$

$=-2.3553091$

$x_4=x_3-\frac{f(x_0)}{f'(x_0)}$

$=-2.3553091-\frac{-0.0000349}{4.5475810}$

$=-2.3553014$

Therefore $x=-2.355$ (3 d.p.)

c $f'(1)=0$ hence the Newton-Raphson method will not work.

5 a $f(x)=x^4-7x^3+1$

$f(0)=1$

$f(1)=-5$

There is a change of sign so there is a root between $x=0$ and $x=1$.

5 b $f'(x)=4x^3-21x^2$

Using $x_0=0.5$

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=0.5-\frac{0.1875}{-4.75}$

$=0.5394737$

$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

$=0.5394737-\frac{-0.0143287}{-5.4836532}$

$=0.5368607$

$x_3=x_2-\frac{f(x_2)}{f'(x_2)}$

$=0.5368607-\frac{-0.0000653}{-5.4336729}$

$=0.5368487$

$x_4=x_3-\frac{f(x_3)}{f'(x_3)}$

$=0.5368487-\frac{-0.0000000013...}{-5.4334435}$

$=0.5368486$

Therefore $x=0.5368$ (4 d.p.)

c $f'(0)=0$ hence the Newton-Raphson method will not work.