Exercise 3C

1 a $x^3 - 3x - 5 = 0$

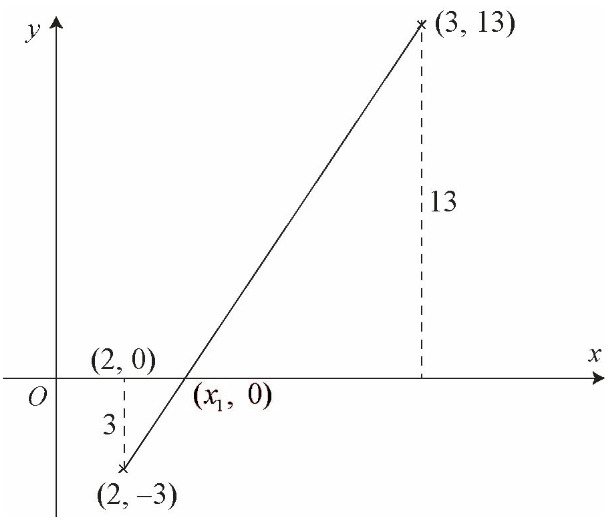
Let $f(x) = x^3 - 3x - 5$

$f(2) = -3$

$f(3) = 13$

There is a sign change between f (2) and f (3) so a root of the equation lies in the interval [2, 3].

b



By similar triangles:

$$\frac{3 - x_1}{x_1 - 2} = \frac{13}{3}$$

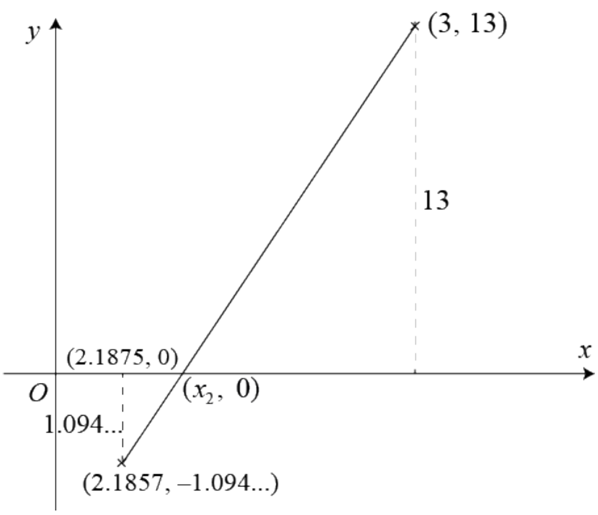
$$9 - 3x_1 = 13x_1 - 26$$

$$16x_1 = 35$$

$$x_1 = \frac{35}{16} = 2.1875$$

$f(2.1875) = -1.094...$

There is a sign change, therefore a root lies between $x = 2.1875$ and $x = 3$.



By similar triangles:

$$\frac{3-x_2}{x_2-2.1875...}=\frac{13}{1.094...}$$

$$1.094...(3-x_2)=13(x_2-2.1875)$$

$$14.084...x_2=31.722...$$

$$x_2=2.250...$$

$$\text{f}(2.250...)=-0.351...$$

There is a sign change, therefore a root lies between $x = 2.250...$ and $x = 3$.
By similar triangles:

$$\frac{3-x_3}{x_3-2.250...}=\frac{13}{0.351...}$$

$$0.351...(3-x_3)=13(x_3-2.250)$$

$$13.351...x_3=30.313...$$

$$x_3=2.270...$$

$$\text{f}(2.270...)=-0.108...$$

Two successive approximations give $x = 2.3$, accurate to 1 d.p.

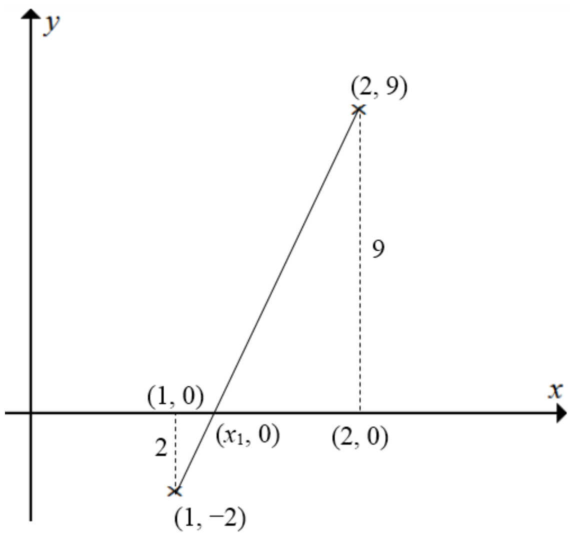
2 a $5x^3-8x^2+1=0$

Let $\text{f}(x)=5x^3-8x^2+1$

$\text{f}(1)=-2$

$\text{f}(2)=9$

There is a sign change, therefore a root lies between $x = 1$ and $x = 2$.



By similar triangles:

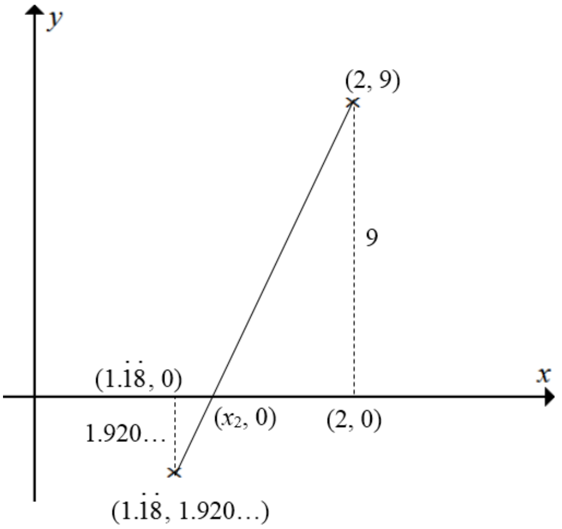
$$\frac{2-x_1}{x_1-1}=\frac{9}{2}$$

$$4-2x_1=9x_1-9$$

$$11x_1=13$$

$$x_1 = \frac{13}{11}$$
$$= 1.\dot{1}\dot{8}$$
$$\mathrm{f}\left(1.\dot{1}\dot{8}\right) = -1.920...$$

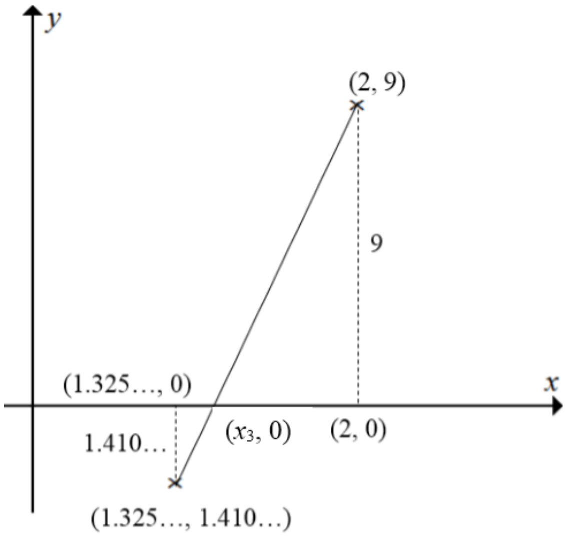
There is a sign change, therefore a root lies between $x = 1.\dot{1}\dot{8}$ and $x = 2$.



By similar triangles:

$$\frac{2 - x_2}{x_2 - 1.\dot{1}\dot{8}} = \frac{9}{1.920...}$$
$$1.920...(2 - x_2) = 9\left(x_2 - 1.\dot{1}\dot{8}\right)$$
$$10.920...x_2 = 14.476...$$
$$x_2 = 1.325...$$
$$\mathrm{f}(1.325...) = -1.410...$$

There is a sign change, therefore a root lies between $x = 1.325...$ and $x = 2$.



By similar triangles:

$$\frac{2-x_3}{x_3-1.325...}=\frac{9}{1.410...}$$

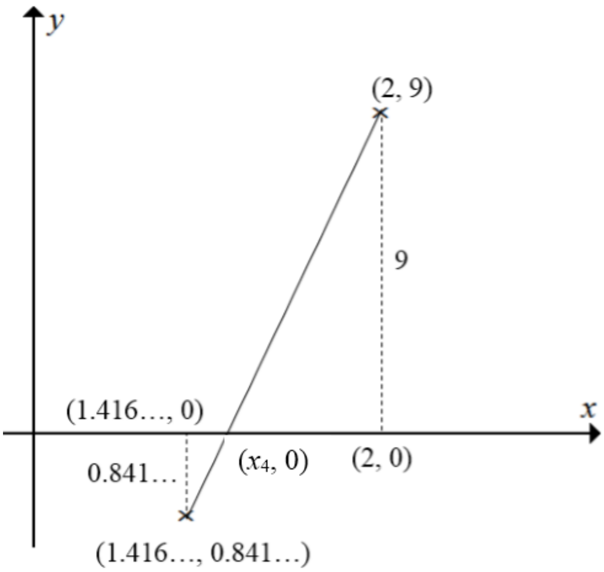
$$1.410...(2-x_3)=9(x_3-1.325...)$$

$$10.410...x_3=14.745...$$

$$x_3=1.416...$$

$$\text{f}(1.416...)=-0.841...$$

There is a sign change, therefore a root lies between $x=1.416...$ and $x=2$.



By similar triangles:

$$\frac{2-x_4}{x_4-1.416...}=\frac{9}{0.841...}$$

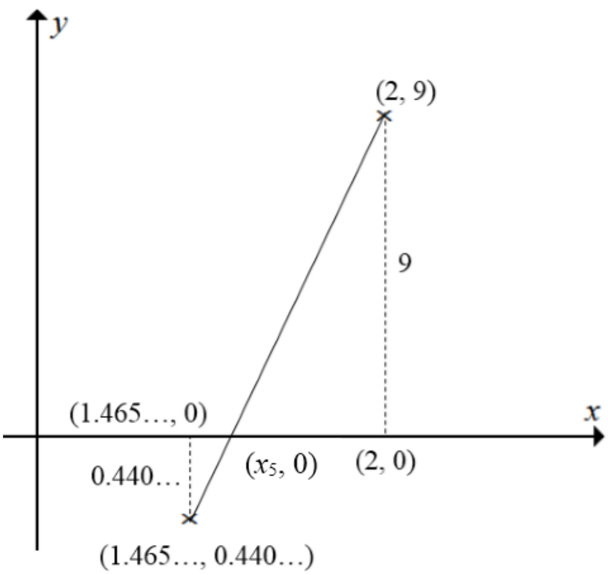
$$0.841...(2-x_4)=9(x_4-1.416...)$$

$$9.841...x_4=14.426...$$

$$x_4=1.465...$$

$$\text{f}(1.465...)=-0.440...$$

There is a sign change, therefore a root lies between $x=1.465...$ and $x=2$.



2 b By similar triangles:

$$\frac{2-x_5}{x_5-1.465...}=\frac{9}{0.440...}$$

$$0.440...(2-x_5)=9(x_5-1.465...)$$

$$9.440...x_5=14.066...$$

$$x_5=1.490...$$

$$\mathrm{f}(1.490...)=-0.220...$$

Two successive approximations give $x = 1.5$, accurate to 1 d.p.

3 a $\frac{3}{x}+3=x\Rightarrow x-\frac{3}{x}-3=0$

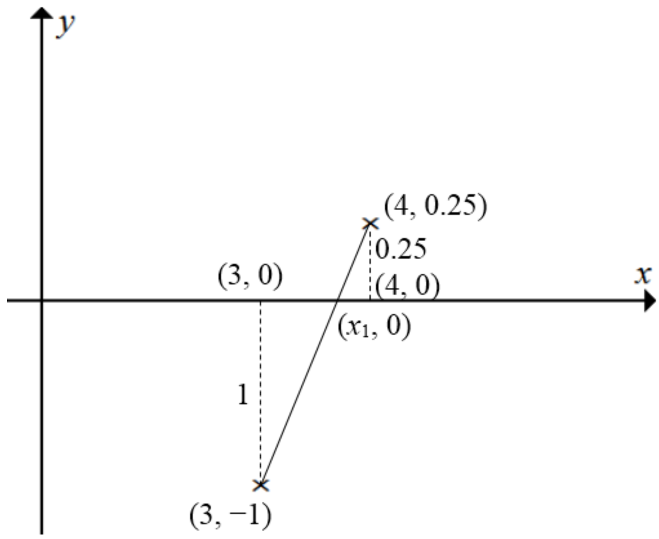
Let $\mathrm{f}(x)=x-\frac{3}{x}-3$

$\mathrm{f}(3)=-1$

$\mathrm{f}(4)=0.25$

There is a sign change, therefore a root lies between $x = 3$ and $x = 4$.

b



By similar triangles:

$$\frac{4-x_1}{x_1-3}=\frac{0.25}{1}$$

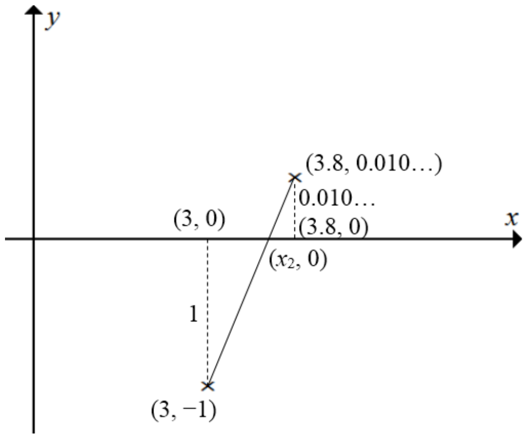
$$4-x_1=0.25(x_1-3)$$

$$1.25x_1=4.75$$

$$x_1=3.8$$

$$\mathrm{f}(3.8)=0.010...$$

There is a sign change, therefore a root lies between $x = 3$ and $x = 3.8$.



By similar triangles:

$$\frac{3.8-x_1}{x_1-3}=\frac{0.010...}{1}$$

$$3.8-x_1=0.010...(x_1-3)$$

$$1.01...x_1=3.831...$$

$$x_1=3.793$$

Two successive approximations give $x = 3.8$, accurate to 1 d.p.

4 **a** $2x\cos x-1=0$

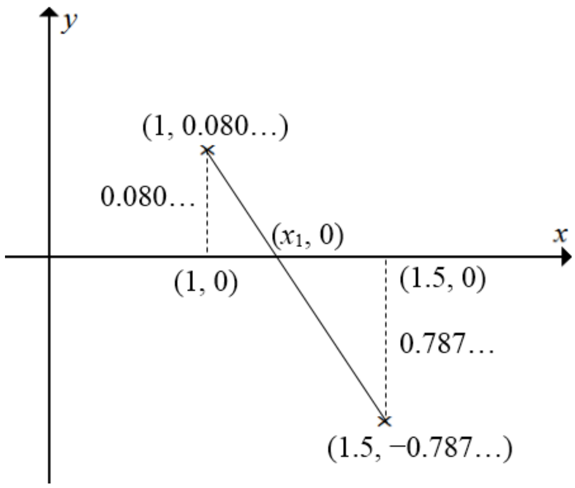
Let $\mathrm{f}(x)=2x\cos x-1$

$\mathrm{f}(1)=0.080...$

$\mathrm{f}(1.5)=-0.787...$

There is a sign change, therefore a root lies between $x = 1$ and $x = 1.5$.

b



By similar triangles:

$$\frac{1.5-x_1}{x_1-1}=\frac{0.787...}{0.080...}$$

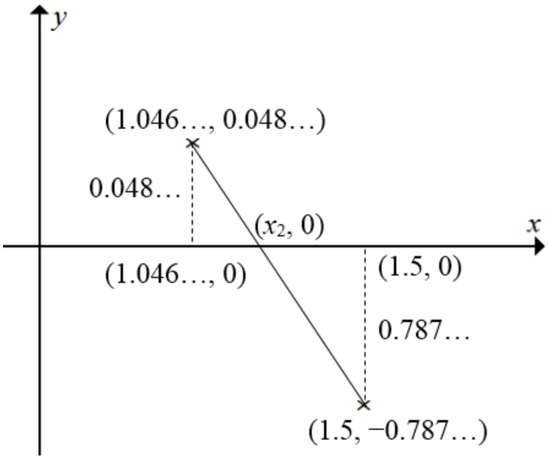
$$0.080...(1.5-x_1)=0.787...(x_1-1)$$

$$0.867...x_1=0.907...$$

$$x_1=1.046...$$

$$\text{f}(1.046)=0.048...$$

There is a sign change, therefore a root lies between $x = 1.046…$ and $x = 1.5$.



By similar triangles:

$$\frac{1.5-x_2}{x_2-1.046...}=\frac{0.787...}{0.048...}$$

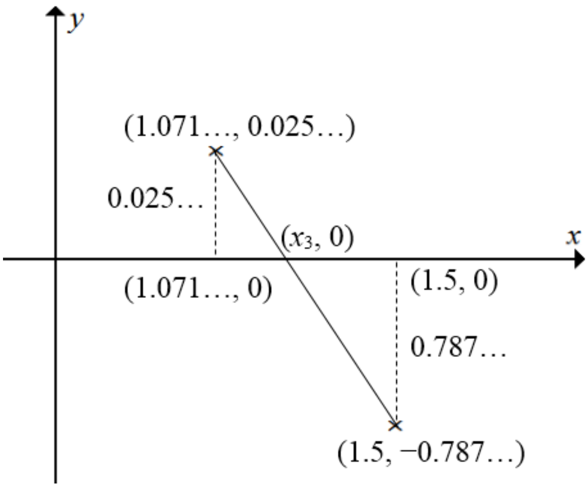
$$0.048...(1.5-x_2)=0.787...(x_2-1.046...)$$

$$0.835...x_2=0.895...$$

$$x_2=1.071...$$

$$\text{f}(1.071)=0.025...$$

There is a sign change, therefore a root lies between $x = 1.071…$ and $x = 1.5$.



By similar triangles:

$$\frac{1.5 - x_3}{x_3 - 1.071...} = \frac{0.787...}{0.025...}$$

$$0.025...(1.5 - x_3) = 0.787...(x_3 - 1.071...)$$

$$0.812...x_3 = 0.881...$$

$$x_3 = 1.085...$$

Two successive approximations give $x = 1.1$, accurate to 1 d.p.

5 a $x^3 - 2x^2 - 3 = 0$

$$f(x) = x^3 - 2x^2 - 3$$

$$f(2) = -3$$

$$f(3) = 6$$

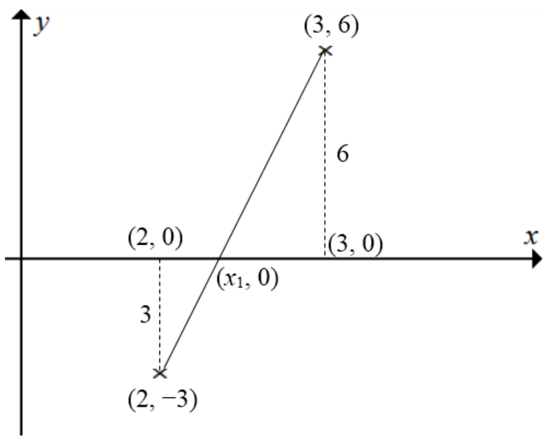
There is a sign change between f (2) and f (3), therefore there is a root of the equation in the interval [2, 3].

$$f'(x) = 3x^2 - 4x = x(3x - 4)$$

For $x > \frac{4}{3}$, $f'(x) > 0$ and the function is increasing.

Therefore the root which lies in the interval [2, 3] must be the largest possible root of the equation.

b



By similar triangles:

$$\frac{3 - x_1}{x_1 - 2} = \frac{6}{3}$$

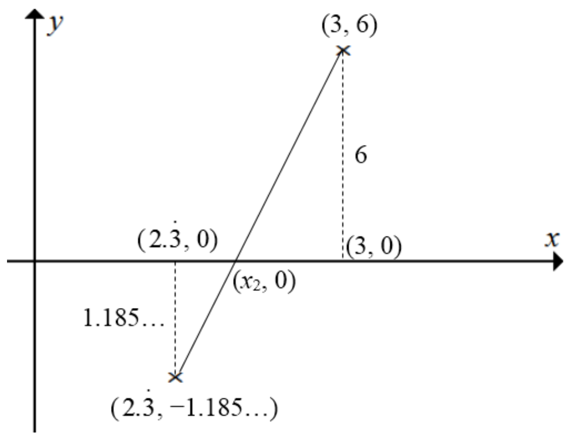
$$3(3 - x_1) = 6(x_1 - 2)$$

$$9x_1 = 21$$

$$x_1 = 2.\dot{3}$$

$$f\left(2.\dot{3}\right) = -1.185...$$

There is a sign change, therefore a root lies between $x = 2.\dot{3}$ and $x = 3$.



By similar triangles:

$$\frac{3-x_2}{x_2-2.\dot{3}}=\frac{6}{1.185...}$$

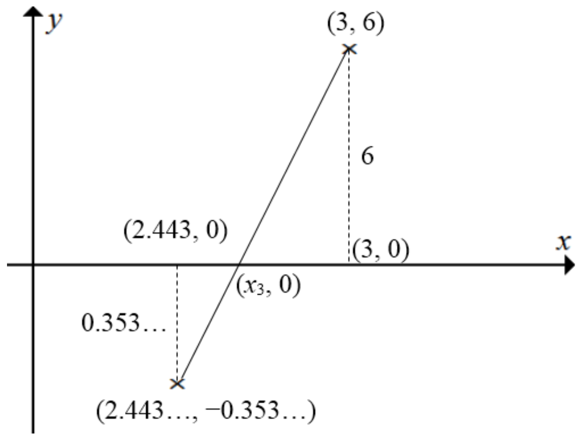
$$1.185...(3-x_2)=6\left(x_2-2.\dot{3}\right)$$

$$7.185...x_2=17.555...$$

$$x_2=2.443...$$

$$f(2.443...)=-0.353...$$

There is a sign change, therefore a root lies between $x=$ 2.443... and $x=3$.



By similar triangles:

$$\frac{3-x_3}{x_3-2.443...}=\frac{6}{0.353...}$$

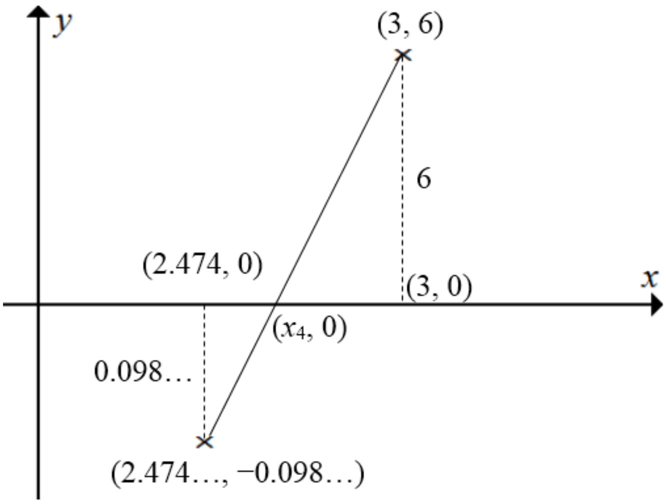
$$0.353...(3-x_3)=6(x_3-2.443...)$$

$$6.353...x_3=15.717$$

$$x_3=2.474...$$

$$f(2.474...)=-0.098...$$

There is a sign change, therefore a root lies between $x = 2.474…$ and $x = 3$.



By similar triangles:

$$\frac{3-x_4}{x_4-2.474...}=\frac{6}{0.098...}$$

$$0.098...(3-x_4)=6(x_4-2.474...)$$

$$6.098...x_4=15.140...$$

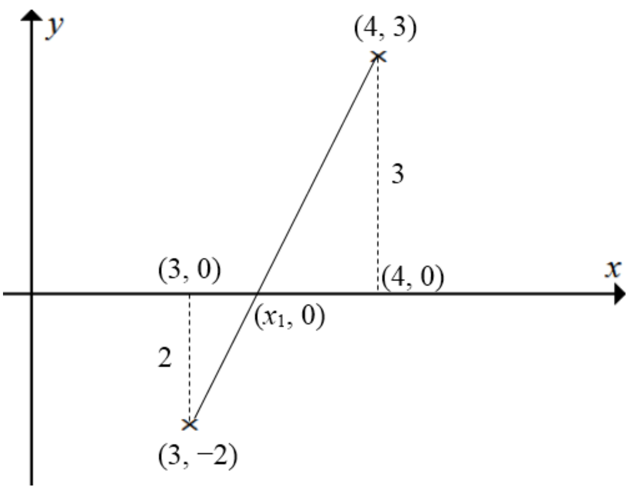
$$x_4=2.482$$

Two successive approximations give $x = 2.5$, accurate to 1 d.p.

6 $f(x)=2^x-3x-1$

$f(3)=-2$

$f(4)=3$



By similar triangles:

$$\frac{4-x_1}{x_1-3}=\frac{3}{2}$$

$$2(4-x_1)=3(x_1-3)$$

$$5x_1=17$$

$$x_1 = \frac{17}{5}$$
$$= 3.4$$