Exercise 3B

1 $x^2 - 7 = 0$

Let $f(x) = x^2 - 7$

$f(2) = -3$

$f(3) = 2$

Since there is a change of sign between f (2) and f (3), the equation has a root in the interval [2, 3].

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	−3	3	2	2.5	−0.75
2.5	−0.75	3	2	2.75	0.5625
2.5	−0.75	2.75	0.5625	2.625	−0.109…
2.625	−0.109…	2.75	0.5625	2.6875	0.222…
2.625	−0.109…	2.6875	0.222…	2.65625	0.055…
2.625	−0.109…	2.65625	0.055…	2.640625	−0.027…
2.640625	−0.027…	2.65625	0.055…	2.6484375	0.0142…

Therefore $x = 2.6$ (1 d.p.)

2 **a** $x^3 - 7x + 2 = 0$

Let $f(x) = x^3 - 7x + 2$

$f(2) = -4$

$f(3) = 8$

Since there is a change of sign between f (2) and f (3), one root of the equation lies in the interval [2, 3].

b

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	−4	3	8	2.5	0.125
2	−4	2.5	0.125	2.25	−2.35…
2.25	−2.35…	2.5	0.125	2.375	−1.22…
2.375	−1.22…	2.5	0.125	2.4375	−0.580…
2.4375	−0.580…	2.5	0.125	2.46875	−0.234…
2.46875	−0.234…	2.5	0.125	2.484375	−0.056…
2.484375	−0.056…	2.5	0.125	2.4921875	0.033…
2.484375	−0.056…	2.4921875	0.033…	2.48828125	−0.011…
2.48828125	−0.011…	2.4921875	0.033…	2.490234375	0.010…

Therefore $x = 2.49$ (2 d.p.)

3 a $x^3+2x^2-8x+3=0$

Let $f(x)=x^3+2x^2-8x+3$

$f(-5)=-32$

$f(0)=3$

$f(1)=-2$

$f(2)=3$

There is a change of sign between −5 and 0, between 0 and 1 and between 1 and 2. Therefore, one root lies between −5 and 0, one root lies between 0 and 1 and one root lies between 1 and 2.
So, the largest positive root lies in the interval [1, 2].

b

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1	−2	2	3	1.5	−1.125
1.5	−1.125	2	3	1.75	0.484…
1.5	−1.125	1.75	0.484…	1.625	−0.427…
1.625	−0.427…	1.75	0.484…	1.6875	0.0007…
1.625	−0.427…	1.6875	0.0007…	1.65625	−0.220…
1.65625	−0.220…	1.6875	0.0007…	1.671875	−0.111…

Therefore $x = 1.7$ (1 d.p.)

4 a $\frac{x}{2}-\frac{1}{x}=0$

Let $f(x)=\frac{x}{2}-\frac{1}{x}$

$f(1)=-\frac{1}{2}$

$f(2)=\frac{1}{2}$

Since there is a change of sign between f(1) and f(2), the equation has a root in the interval [1, 2].

b

a	f(a)	b	f(b)	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
1	−0.5	2	0.5	1.5	0.083…
1	−0.5	1.5	0.083…	1.25	−0.175
1.25	−0.175	1.5	0.083…	1.375	−0.039…
1.375	−0.039…	1.5	0.083…	1.4375	0.023…

Therefore $x = 1.4$ (2 s.f.)

5 $6x - x^3 = 0$

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
2	4	3	−9	2.5	−0.625
2	4	2.5	−0.625	2.25	2.109…
2.25	2.109…	2.5	−0.625	2.375	0.853…
2.375	0.853…	2.5	−0.625	2.4375	0.142…

Therefore $x = 2.4$ (2 s.f.)