Exercise 3A

1 a $f(x) = x^3 - x + 5$

$f(-2) = -8 + 2 + 5 = -1 < 0$

$f(-1) = -1 + 1 + 5 = 5 > 0$

There is a change of sign between −2 and −1 so there is at least one root in the interval $-2 < x < -1$.

b $f(x) = x^2 - \sqrt{x} - 10$

$f(3) = 9 - \sqrt{3} - 10 = -2.732... < 0$

$f(4) = 16 - \sqrt{4} - 10 = 4 > 0$

There is a change of sign between 3 and 4 so there is at least one root in the interval $3 < x < 4$.

c $f(x) = x^3 - \dfrac{1}{x} - 2$

$f(-0.5) = (-0.5)^3 + 2 - 2 = -0.125 < 0$

$f(-0.2) = (-0.2)^3 + 5 - 2 = 2.992 > 0$

There is a change of sign between −0.5 and −0.2 so there is at least one root in the interval $-0.5 < x < -0.2$.

d $f(x) = e^x - \ln x - 5$

$f(1.65) = e^{1.65} - \ln 1.65 - 5 = -0.293... < 0$

$f(1.75) = e^{1.75} - \ln 1.75 - 5 = 0.194... > 0$

There is a change of sign between 1.65 and 1.75 so there is at least one root in the interval $1.65 < x < 1.75$.

2 a $f(x) = 3 + x^2 - x^3$

$f(1.8) = 3 + 1.8^2 - 1.8^3 = 0.408 > 0$

$f(1.9) = 3 + 1.9^2 - 1.9^3 = -0.249 < 0$

There is a change of sign so there is a root, α, in the interval [1.8, 1.9].

b Choose interval [1.8635, 1.8645] to test for root.

$f(1.8635) = 3 + 1.8635^2 - 1.8635^3$
$= 0.00138... > 0$

$f(1.8645) = 3 + 1.8645^2 - 1.8645^3$
$= -0.00531... < 0$

There is a change of sign between 1.8635 and 1.8645, so $1.8635 < \alpha < 1.8645$, which gives $\alpha = 1.864$ correct to 3 d.p.

3 a $h(x) = \sqrt[3]{x} - \cos x - 1$

$h(1.4) = \sqrt[3]{1.4} - \cos 1.4 - 1 = -0.0512... < 0$

$h(1.5) = \sqrt[3]{1.5} - \cos 1.5 - 1 = 0.0739... > 0$

There is a change of sign so there is a root, α, in the interval [1.4, 1.5].

b Choose interval [1.4405, 1.4415] to test for root.

$h(1.4405) = \sqrt[3]{1.4405} - \cos 1.4405 - 1$
$= -0.00055... < 0$

$h(1.4415) = \sqrt[3]{1.4415} - \cos 1.4415 - 1$
$= 0.00069... > 0$

There is a change of sign between 1.4405 and 1.4415, so $1.4405 < \alpha < 1.4415$, which gives $\alpha = 1.441$ correct to 3 d.p.

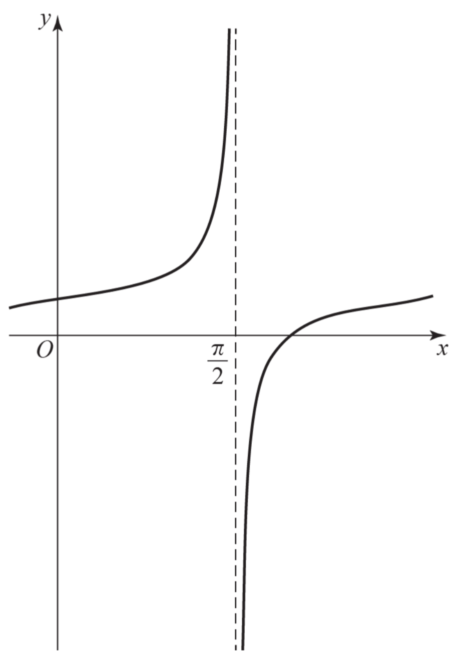
4 a $f(x) = 2 + \tan x$

$f(1.5) = 2 + \tan 1.5 = 16.1... > 0$

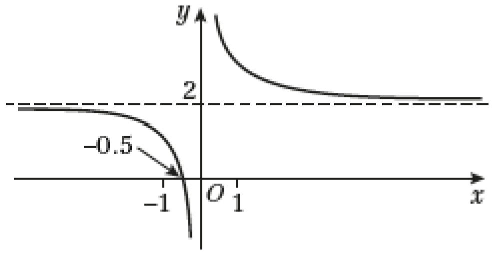
$f(1.6) = 2 + \tan 1.6 = -32.2... < 0$

So there is a change of sign in the interval [1.5, 1.6].

b A sketch shows there is a vertical asymptote in the graph of $y = f(x)$ at $x = \dfrac{\pi}{2} = 1.57...$ So there is no root in the interval [1.5, 1.6].

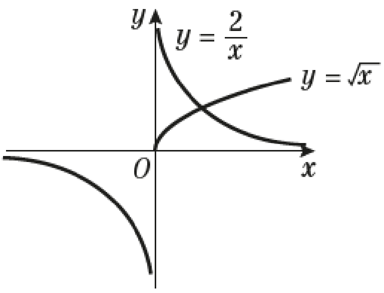


5 A sketch shows a root at −0.5.



Or $f(x) = 0$ when $\frac{1}{x} + 2 = 0 \Rightarrow x = -\frac{1}{2}$
which is in the interval [−1, 1].

6 a



b The curves meet at one point, so there is one value of x that satisfies the equation $\sqrt{x} = \frac{2}{x}$. So $\sqrt{x} = \frac{2}{x}$ has **one** root.

c $f(x) = \sqrt{x} - \frac{2}{x}$

$f(1) = \sqrt{1} - \frac{2}{1} = -1$

$f(2) = \sqrt{2} - \frac{2}{2} = 0.414...$

There is a change of sign, so there is a root, r, between $x = 1$ and $x = 2$.

6 d $\sqrt{x} = \frac{2}{x}$

$x^{\frac{1}{2}} = \frac{2}{x}$

$x^{\frac{1}{2}} \times x = 2$

$x^{\frac{1}{2}+1} = 2$

$x^{\frac{3}{2}} = 2$

$\left(x^{\frac{3}{2}}\right)^2 = 2^2$

$x^3 = 4$

So $p = 3$ and $q = 4$.

e $x^{\frac{3}{2}} = 2$

$\Rightarrow x = 2^{\frac{2}{3}} \left[= (2^2)^{\frac{1}{3}} = 4^{\frac{1}{3}}\right]$

7 a $f(x) = x^4 - 21x - 18$

$f(-0.9) = 0.6561 + 18.9 - 18 = 1.5561 > 0$

$f(-0.8) = 0.4096 + 16.8 - 18 = -0.7904 < 0$

The change of sign between −0.9 and −0.8 implies there is at least one root in the interval [−0.9, −0.8].

b $f'(x) = 4x^3 - 21$

$f'(x) = 0 \Rightarrow 4x^3 = 21$

$x = \sqrt[3]{\frac{21}{4}} = 1.738...$

$f(1.738) = 1.738^4 - 21 \times 1.738 - 18$
$= -45.373...$

Stationary point is (1.74, −45.37) to 2 d.p.

c $f(x) = (x-3)(x^3 + ax^2 + bx + c)$

$f(x) = x^4 + (a-3)x^3 + (b-3a)x^2 + (c-3b)x - 3c$

Comparing coefficients…

$a = 3, b = 9, c = 6$

d