

Exercise 2A

$$1 \text{ a } 3x^2 + 7x - 4 = 0$$

$$a = 3, b = 7, c = -4$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{7}{3}$$

$$b \quad \alpha\beta = \frac{c}{a} = -\frac{4}{3}$$

$$c \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \left(\frac{-\frac{7}{3}}{-\frac{4}{3}} \right)$$

$$= \frac{7}{4}$$

$$d \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(-\frac{7}{3}\right)^2 - 2\left(-\frac{4}{3}\right)$$

$$= \frac{49}{9} + \frac{8}{3}$$

$$= \frac{73}{9}$$

$$2 \text{ a } 7x^2 - 3x + 1 = 0$$

$$a = 7, b = -3, c = 1$$

$$\alpha + \beta = -\frac{b}{a} = \frac{3}{7}$$

$$b \quad \alpha\beta = \frac{c}{a} = \frac{1}{7}$$

$$c \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{\frac{3}{7}}{\frac{1}{7}}$$

$$= 3$$

$$d \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{3}{7}\right)^2 - 2\left(\frac{1}{7}\right)$$

$$= \frac{9}{49} - \frac{2}{7}$$

$$= \frac{9}{49} - \frac{14}{49}$$

$$= -\frac{5}{49}$$

$$3 \text{ a } 6x^2 - 9x + 2 = 0$$

$$a = 6, b = -9, c = 2$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-9}{6} = \frac{3}{2}$$

$$b \quad \alpha^2 \times \beta^2 = (\alpha\beta)^2 = \left(\frac{c}{a}\right)^2 = \left(\frac{2}{6}\right)^2 = \frac{1}{9}$$

$$c \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{3}{2}}{\frac{1}{3}} = \frac{9}{2}$$

$$d \quad (\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$$

$$\text{So } \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{1}{3}\right)\left(\frac{3}{2}\right)$$

$$= \frac{27}{8} - \frac{3}{2}$$

$$= \frac{27}{8} - \frac{12}{8}$$

$$= \frac{15}{8}$$

$$4 \quad ax^2 + bx + c = 0$$

$$\alpha = 2 \text{ and } \beta = -3$$

$$\alpha + \beta = 2 + (-3) = -1 = -\frac{b}{a}$$

$$\alpha\beta = (2)(-3) = -6 = \frac{c}{a}$$

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$-\frac{b}{a} = -1 \Rightarrow \frac{b}{a} = 1$$

$$\frac{c}{a} = -6$$

So equation is $x^2 + x - 6 = 0$,
with $a = 1, b = 1$ and $c = -6$

$$5 \quad ax^2 + bx + c = 0$$

$$\alpha = -\frac{1}{2} \text{ and } \beta = -\frac{1}{3}$$

$$\alpha + \beta = -\frac{1}{2} - \frac{1}{3} = -\frac{5}{6} = -\frac{b}{a}$$

$$\alpha\beta = \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right) = \frac{1}{6} = \frac{c}{a}$$

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$-\frac{b}{a} = -\frac{5}{6}, \text{ so } \frac{b}{a} = \frac{5}{6}$$

$$\frac{c}{a} = \frac{1}{6}$$

Further Pure Maths 1

Solution Bank

5 So equation is $x^2 + \frac{5}{6}x + \frac{1}{6} = 0$

or $6x^2 + 5x + 1 = 0$. So
 $a = 6, b = 5$ and $c = 1$.

6 $ax^2 + bx + c = 0$

$$\alpha = \frac{-1+i}{2} \text{ and } \beta = \frac{-1-i}{2}$$

$$\alpha + \beta = -1 = -\frac{b}{a}$$

$$\alpha\beta = \left(\frac{-1+i}{2}\right)\left(\frac{-1-i}{2}\right) = \frac{1-i^2}{4} = \frac{2}{4} = \frac{1}{2} = \frac{c}{a}$$

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$-\frac{b}{a} = -1 \Rightarrow \frac{b}{a} = 1$$

$$\frac{c}{a} = \frac{1}{2}$$

So equation is $x^2 + x + \frac{1}{2} = 0$

or $2x^2 + 2x + 1 = 0$. So
 $a = 2, b = 2$ and $c = 1$.

7 a The other root, β , is the complex conjugate. So $\beta = -1 + 4i$

b $\alpha = -1 - 4i$ and $\beta = -1 + 4i$

$$\alpha + \beta = -2 = -\frac{b}{a}$$

$$\alpha\beta = (-1 - 4i)(-1 + 4i) = 1 - 16i^2 = 17 = \frac{c}{a}$$

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$-\frac{b}{a} = -2 \Rightarrow \frac{b}{a} = 2$$

$$\frac{c}{a} = 17$$

So equation is $x^2 + 2x + 17 = 0$,
 with $a = 1, b = 2$ and $c = 17$.

8 The sum of the roots is 4,
 so $\alpha + \beta = 4$

Also $-\frac{b}{a} = 4$, but $-\frac{b}{a} = \frac{-(k-3)}{k} = \frac{3-k}{k}$

So $\frac{3-k}{k} = 4$

$$3 - k = 4k$$

$$3 = 5k$$

$$k = \frac{3}{5}$$

9 The roots are α and $-\alpha$.

$$\alpha + -\alpha = 0 = -\frac{b}{a}$$

If $-\frac{b}{a} = 0$, then $b = 0$

$$b = 0 \Rightarrow 16 + n = 0 \Rightarrow n = -16$$

10 The roots are reciprocals, so the roots are α
 and $\frac{1}{\alpha}$.

$$\alpha\left(\frac{1}{\alpha}\right) = 1 = \frac{c}{a}$$

But $\frac{c}{a} = \frac{k}{6}$

So $\frac{k}{6} = 1 \Rightarrow k = 6$

11 The roots are k and $2k$.

$$k + 2k = 3k = -\frac{b}{a}$$

But $-\frac{b}{a} = -\frac{4}{m}$, so $3k = -\frac{4}{m}$

$$(k)(2k) = 2k^2 = \frac{c}{a}$$

But $\frac{c}{a} = \frac{4m}{m} = 4$, so $2k^2 = 4$

If $2k^2 = 4, k^2 = 2$ and $k = \pm\sqrt{2}$.

$$3k = -\frac{4}{m}, \text{ so } m = -\frac{4}{3k}$$

If $k = \sqrt{2}, m = -\frac{4}{3\sqrt{2}} = -\frac{2\sqrt{2}}{3}$

If $k = -\sqrt{2}, m = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}$