## Exercise 1H

**1 a** $z^2+2z+26=0$

Using the quadratic formula gives

$$z=\frac{-2\pm\sqrt{4-4(1)(26)}}{2}$$
$$=\frac{-2\pm10\text{i}}{2}$$

$\alpha=-1+5\text{i},\ \beta=-1-5\text{i}$ (or vice versa)

**b** $\alpha+\beta=(-1+5\text{i})+(-1-5\text{i})=-2$

**c** $\alpha\beta=(-1+5\text{i})(-1-5\text{i})$
$=-1(-1-5\text{i})+5\text{i}(-1-5\text{i})$
$=1+5\text{i}-5\text{i}-25\text{i}^2$
$=26$

**2 a** $z^2-8z+25=0$

$$z=\frac{8\pm\sqrt{(-8)^2-4(1)(25)}}{2}$$
$$=\frac{8\pm6\text{i}}{2}$$

$\alpha=4+3\text{i},\ \beta=4-3\text{i}$ (or vice versa)

**b** $\alpha+\beta=(4+3\text{i})+(4-3\text{i})=8$

**c** $\alpha\beta=(4+3\text{i})(4-3\text{i})$
$=4(4-3\text{i})+3\text{i}(4-3\text{i})$
$=16-12\text{i}+12\text{i}-9\text{i}^2$
$=25$

**3 a** The other root is the complex conjugate $2-3\text{i}$.

**b** $\left(z-(2+3\text{i})\right)\left(z-(2-3\text{i})\right)$
$=z^2-(2-3\text{i})z-(2+3\text{i})z+(2+3\text{i})(2-3\text{i})$
$=z^2-2z+3\text{i}z-2z-3\text{i}z+4-6\text{i}+6\text{i}-9\text{i}^2$
$=z^2-4z+4+9$
$=z^2-4z+13$

Equation is $z^2-4z+13=0$
where $b=-4,\ c=13$

**4 a** The other root is the complex conjugate $5+\text{i}$.

**b** $\left(z-(5-\text{i})\right)\left(z-(5+\text{i})\right)$
$=z^2-(5+\text{i})z-(5-\text{i})z+(5-\text{i})(5+\text{i})$
$=z^2-5z-\text{i}z-5z+\text{i}z+25+5\text{i}-5\text{i}-\text{i}^2$
$=z^2-10z+25+1$
$=z^2-10z+26$

So, $p=-10$ and $q=26$

**5** The second root is the complex conjugate of $z_1=-5+4i$

i.e. $z_2=-5-4i$

$\left(z-(-5+4\text{i})\right)\left(z-(-5-4\text{i})\right)$
$=z^2-(-5-4\text{i})z-(-5+4\text{i})z+(-5+4\text{i})(-5-4\text{i})$
$=z^2+5z+4\text{i}z+5z-4\text{i}z+25+20\text{i}-20\text{i}-16\text{i}^2$
$=z^2+10z+25+16$
$=z^2+10z+41$

So, $b=10$ and $c=41$

**6** The other root is $1-2\text{i}$.

If the roots are $\alpha$ and $\beta$, the equation is
$(z-\alpha)(z-\beta)=z^2-(\alpha+\beta)z+\alpha\beta=0$

$\alpha+\beta=(1+2\text{i})+(1-2\text{i})=2$
$\alpha\beta=(1+2\text{i})(1-2\text{i})$
$=1(1-2\text{i})+2\text{i}(1-2\text{i})$
$=1-2\text{i}+2\text{i}-4\text{i}^2=5$

Equation is $z^2-2z+5=0$

**7** The other root is 3 + 5i.

If the roots are $\alpha$ and $\beta$, the equation is
$(z-\alpha)(z-\beta)=z^2-(\alpha+\beta)z+\alpha\beta=0$

$$\alpha+\beta=(3-5\text{i})+(3+5\text{i})=6$$
$$\begin{aligned}\alpha\beta&=(3-5\text{i})(3+5\text{i})\\&=3(3+5\text{i})-5\text{i}(3+5\text{i})\\&=9+15\text{i}-15\text{i}-25\text{i}^2=34\end{aligned}$$

Equation is $z^2-6z+34=0$

**8 a** $$\begin{aligned}z&=\frac{5}{3-\text{i}}\\&=\frac{5}{(3-\text{i})}\times\frac{(3+\text{i})}{(3+\text{i})}\\&=\frac{15+5\text{i}}{9+3\text{i}-3\text{i}-\text{i}^2}\\&=\tfrac{15}{10}+\tfrac{5}{10}\text{i}\\&=\tfrac{3}{2}+\tfrac{1}{2}\text{i}\end{aligned}$$

**b** If $z=\frac{3}{2}+\frac{1}{2}\text{i}$ is a root, then $z=\frac{3}{2}-\frac{1}{2}\text{i}$ is also a root.

$$\begin{aligned}0&=\left(z-\left(\tfrac{3}{2}+\tfrac{1}{2}\text{i}\right)\right)\left(z-\left(\tfrac{3}{2}-\tfrac{1}{2}\text{i}\right)\right)\\&=z^2-\left(\tfrac{3}{2}-\tfrac{1}{2}\text{i}\right)z-\left(\tfrac{3}{2}+\tfrac{1}{2}\text{i}\right)z\\&\qquad+\left(\tfrac{3}{2}+\tfrac{1}{2}\text{i}\right)\left(\tfrac{3}{2}-\tfrac{1}{2}\text{i}\right)\\&=z^2-\tfrac{3}{2}z+\tfrac{1}{2}\text{i}z-\tfrac{3}{2}z-\tfrac{1}{2}\text{i}z\\&\qquad+\tfrac{9}{4}-\tfrac{3}{4}\text{i}+\tfrac{3}{4}\text{i}-\tfrac{1}{4}\text{i}^2\\&=z^2-3z+\tfrac{10}{4}\\&=z^2-3z+\tfrac{5}{2}\end{aligned}$$

So $p=-3$ and $q=\frac{5}{2}$

**9** $z=5+q\text{i}$
The second root is the complex conjugate, $z=5-q\text{i}$

$$\begin{aligned}0&=\left(z-(5+q\text{i})\right)\left(z-(5-q\text{i})\right)\\&=z^2-(5-q\text{i})z-(5+q\text{i})z\\&\qquad+(5+q\text{i})(5-q\text{i})\\&=z^2-5z+q\text{i}z-5z-q\text{i}z\\&\qquad+25-5q\text{i}+5q\text{i}-q^2\text{i}^2\\&=z^2-10z+25+q^2\end{aligned}$$

So $z^2-10z+25+q^2\equiv z^2-4pz+34$

Equating $z$ terms:
$4p=10\Rightarrow p=\frac{5}{2}$

Equating constant terms:
$25+q^2=34\Rightarrow q^2=9$
$\Rightarrow q=\pm3$

But $q>0$, and hence $q=3$