Exercise 1G

1 a $r = \sqrt{(2^2 + 2^2)} = \sqrt{8} = 2\sqrt{2}$

$\tan\alpha = \frac{2}{2} = 1. \quad \alpha = \frac{\pi}{4}$

$\theta = \frac{\pi}{4}$

$2 + 2\mathrm{i} = 2\sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)$

b $r = \sqrt{(0^2 + 3^2)} = \sqrt{9} = 3$

$\tan\alpha = \infty \quad \alpha = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

$3\mathrm{i} = 3\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$

c $r = \sqrt{((-3)^2 + 4^2)} = \sqrt{25} = 5$

$\tan\alpha = \frac{4}{3}. \quad \alpha = 0.927 \text{ rad.}$

$\theta = \pi - \alpha = 2.21$

$-3 + 4\mathrm{i} = 5(\cos 2.21 + \mathrm{i}\sin 2.21)$

d $r = \sqrt{(1^2 + (-\sqrt{3})^2)} = \sqrt{4} = 2$

$\tan\alpha = \frac{\sqrt{3}}{1}. \quad \alpha = \frac{\pi}{3}$

$\theta = -\frac{\pi}{3}$

$1 - \sqrt{3}\mathrm{i} = 2\left(\cos\left(\frac{-\pi}{3}\right) + \mathrm{i}\sin\left(\frac{-\pi}{3}\right)\right).$

e $r = \sqrt{((-2)^2 + (-5)^2)} = \sqrt{29}$

$\tan\alpha = \frac{5}{2}. \quad \alpha = 1.190 \text{ rad}$

$\theta = -(\pi - \alpha) = -1.95$

$-2 - 5\mathrm{i} = \sqrt{29}(\cos(-1.95) + \mathrm{i}\sin(-1.95)).$

f $r = \sqrt{((-20)^2 + 0^2)} = \sqrt{400} = 20$

$\tan\alpha = 0$

$\theta = \pi$

$-20 = 20(\cos\pi + \mathrm{i}\sin\pi)$

g $r = \sqrt{(7^2 + (-24)^2)} = \sqrt{625} = 25$

$\tan\alpha = \frac{24}{7}. \quad \alpha = 1.287 \text{ rad}$

$\theta = -1.29$

$7 - 24\mathrm{i} = 25(\cos(-1.29) + \mathrm{i}\sin(-1.29))$

h $r = \sqrt{((-5)^2 + 5^2)} = \sqrt{50} = 5\sqrt{2}$

$\tan\alpha = \frac{5}{5} = 1. \quad \alpha = \frac{\pi}{4}.$

$\theta = \pi - \alpha = \frac{3\pi}{4}$

$-5 + 5\mathrm{i} = 5\sqrt{2}\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right).$

2 a $\frac{3}{1+\mathrm{i}\sqrt{3}} = \frac{3(1-\mathrm{i}\sqrt{3})}{(1+\mathrm{i}\sqrt{3})(1-\mathrm{i}\sqrt{3})}$

$= \frac{3 - 3\mathrm{i}\sqrt{3}}{1(1-\mathrm{i}\sqrt{3}) + \mathrm{i}\sqrt{3}(1-\mathrm{i}\sqrt{3})}$

$= \frac{3 - 3\mathrm{i}\sqrt{3}}{1 - \mathrm{i}\sqrt{3} + \mathrm{i}\sqrt{3} - 3\mathrm{i}^2}$

$= \frac{3 - 3\mathrm{i}\sqrt{3}}{4}$

$= \frac{3}{4} - \frac{3\sqrt{3}}{4}\mathrm{i}$

$r = \sqrt{\left(\left(\frac{3}{4}\right)^2 + \left(-\frac{3\sqrt{3}}{4}\right)^2\right)} = \sqrt{\left(\frac{9}{16} + \frac{27}{16}\right)}$

$= \sqrt{\left(\frac{36}{16}\right)} = \frac{3}{2}$

$\tan\alpha = \frac{3\sqrt{3}}{4} \div \frac{3}{4} = \sqrt{3} \quad \alpha = \frac{\pi}{3}$

$\theta = -\frac{\pi}{3}$

$\frac{3}{1+\mathrm{i}\sqrt{3}} = \frac{3}{2}\left(\cos\left(\frac{-\pi}{3}\right) + \mathrm{i}\sin\left(\frac{-\pi}{3}\right)\right)$

2 b $\dfrac{1}{2-\mathrm{i}} = \dfrac{2+\mathrm{i}}{(2-\mathrm{i})(2+\mathrm{i})}$

$= \dfrac{2+\mathrm{i}}{2(2+\mathrm{i})-\mathrm{i}(2+\mathrm{i})}$

$= \dfrac{2+\mathrm{i}}{4+2\mathrm{i}-2\mathrm{i}-\mathrm{i}^2}$

$= \dfrac{2+\mathrm{i}}{5} = \dfrac{2}{5}+\dfrac{1}{5}\mathrm{i}$

$r = \sqrt{\left(\left(\dfrac{2}{5}\right)^2+\left(\dfrac{1}{5}\right)^2\right)} = \sqrt{\left(\dfrac{4}{25}+\dfrac{1}{25}\right)}$

$= \sqrt{\dfrac{5}{25}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

$\tan\alpha = \dfrac{1}{5}\div\dfrac{2}{5} = \dfrac{1}{2}.$ $\quad \alpha = 0.4636$ rad.

$\theta = 0.46$

$\dfrac{1}{2-\mathrm{i}} = \dfrac{\sqrt{5}}{5}(\cos 0.46 + \mathrm{i}\sin 0.46)$

c $\dfrac{1+\mathrm{i}}{1-\mathrm{i}} = \dfrac{(1+\mathrm{i})(1+\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}$

$= \dfrac{1(1+\mathrm{i})+\mathrm{i}(1+\mathrm{i})}{1(1+\mathrm{i})-\mathrm{i}(1+\mathrm{i})} = \dfrac{1+\mathrm{i}+\mathrm{i}+\mathrm{i}^2}{1+\mathrm{i}-\mathrm{i}-\mathrm{i}^2}$

$= \dfrac{2\mathrm{i}}{2} = \mathrm{i}$

$r = \sqrt{\left(0^2+1^2\right)} = 1$

$\tan\alpha = \infty.$ $\quad \alpha = \dfrac{\pi}{2}$

$\theta = \dfrac{\pi}{2}$

$\dfrac{1+\mathrm{i}}{1-\mathrm{i}} = 1\left(\cos\dfrac{\pi}{2}+\mathrm{i}\sin\dfrac{\pi}{2}\right)$

3 a $5\left(\cos\dfrac{\pi}{2}+\mathrm{i}\sin\dfrac{\pi}{2}\right)$

$= 5(0+\mathrm{i})$

$= 5\mathrm{i}$

3 b $\dfrac{1}{2}\left(\cos\dfrac{\pi}{6}+\mathrm{i}\sin\dfrac{\pi}{6}\right)$

$= \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}\mathrm{i}\right)$

$= \dfrac{\sqrt{3}}{4}+\dfrac{1}{4}\mathrm{i}$

c $6\left(\cos\dfrac{5\pi}{6}+\mathrm{i}\sin\dfrac{5\pi}{6}\right)$

$= 6\left(-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}\mathrm{i}\right)$

$= -3\sqrt{3}+3\mathrm{i}$

d $3\left(\cos\left(-\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{2\pi}{3}\right)\right)$

$= 3\left(-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}\mathrm{i}\right)$

$= -\dfrac{3}{2}-\dfrac{3\sqrt{3}}{2}\mathrm{i}$

e $2\sqrt{2}\left(\cos\left(-\dfrac{\pi}{4}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{4}\right)\right)$

$= 2\sqrt{2}\left(\dfrac{1}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}\mathrm{i}\right)$

$= 2-2\mathrm{i}$

f $-4\left(\cos\dfrac{7\pi}{6}+\mathrm{i}\sin\dfrac{7\pi}{6}\right)$

$= -4\left(-\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}\mathrm{i}\right)$

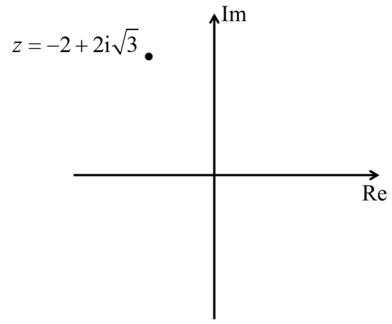
$= 2\sqrt{3}+2\mathrm{i}$

4 a $z = 4\left(\cos\left(\dfrac{2\pi}{3}\right)+\mathrm{i}\sin\left(\dfrac{2\pi}{3}\right)\right)$

$= 4\left(-\dfrac{1}{2}+\mathrm{i}\dfrac{\sqrt{3}}{2}\right)$

$= -2+2\mathrm{i}\sqrt{3}$

4 b



5 $|z| = 7$ and $\arg z = \dfrac{11\pi}{6}$

$$z = 7\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$$
$$= 7\cos\left(\frac{11\pi}{6}\right) + 7i\sin\left(\frac{11\pi}{6}\right)$$
$$= \frac{7\sqrt{3}}{2} - \frac{7}{2}i$$

So $p = \dfrac{7\sqrt{3}}{2}$ and $q = -\dfrac{7}{2}$

6 $|z| = 5$ and $\arg z = -\dfrac{4\pi}{3}$

$$z = 5\left(\cos\left(-\frac{4\pi}{3}\right) + \sin\left(-\frac{4\pi}{3}\right)\right)$$
$$= 5\cos\left(-\frac{4\pi}{3}\right) + 5i\sin\left(-\frac{4\pi}{3}\right)$$
$$= -\frac{5}{2} + \frac{5i\sqrt{3}}{2}$$

So $a = -\dfrac{5}{2}$ and $b = \dfrac{5\sqrt{3}}{2}$.