

Exercise 1B

1 a $z^2 + 121 = 0$

$$z^2 = -121$$

$$z = \pm\sqrt{-121}$$

$$= \pm\sqrt{121}\sqrt{-1}$$

$$= \pm 11i$$

b $z^2 + 40 = 0$

$$z^2 = -40$$

$$z = \pm\sqrt{-40}$$

$$= \pm\sqrt{4}\sqrt{10}\sqrt{-1}$$

$$= \pm\sqrt{4}\sqrt{10}i$$

$$= \pm 2i\sqrt{10}$$

c $2z^2 + 120 = 0$

$$2z^2 = -120$$

$$z^2 = -60$$

$$z = \pm\sqrt{-60}$$

$$= \pm\sqrt{4}\sqrt{15}\sqrt{-1}$$

$$= \pm 2i\sqrt{15}$$

d $3z^2 + 150 = 38 - z^2$

$$4z^2 = -112$$

$$z^2 = -28$$

$$z = \pm\sqrt{-28}$$

$$= \pm\sqrt{4}\sqrt{7}\sqrt{-1}$$

$$= \pm 2i\sqrt{7}$$

e $z^2 + 30 = -3z^2 - 66$

$$4z^2 = -96$$

$$z^2 = -24$$

$$z = \pm\sqrt{-24}$$

$$= \pm\sqrt{4}\sqrt{6}\sqrt{-1}$$

$$= \pm 2i\sqrt{6}$$

1 f

$$6z^2 + 1 = 2z^2$$

$$4z^2 = -1$$

$$z^2 = -\frac{1}{4}$$

$$z = \pm\sqrt{\frac{-1}{4}}$$

$$= \pm\sqrt{\frac{1}{4}}\sqrt{-1}$$

$$= \pm \frac{1}{2}i$$

2 a $(z-3)^2 - 9 = -16$

$$(z-3)^2 = -7$$

$$z-3 = \pm i\sqrt{7}$$

$$z = 3 \pm i\sqrt{7}$$

b $2(z-7)^2 + 30 = 6$

$$2(z-7)^2 = -24$$

$$(z-7)^2 = -12$$

$$z-7 = \pm 2i\sqrt{3}$$

$$z = 7 \pm 2i\sqrt{3}$$

c $16(z+1)^2 + 11 = 2$

$$16(z+1)^2 = -9$$

$$(z+1)^2 = -\frac{9}{16}$$

$$z+1 = \pm \frac{3}{4}i$$

$$z = -1 \pm \frac{3}{4}i$$

3 [Note that **3a**, **3b** & **3c** use the quadratic formula, and **3d**, **3e** & **3f** use completion of the square]

a $a=1, b=2, c=5$

$$\begin{aligned} z &= \frac{-2 \pm \sqrt{(4-20)}}{2} \\ &= \frac{-2 \pm 4i}{2} \\ &= -1 \pm 2i \end{aligned}$$

b $a=1, b=-2, c=10$

$$\begin{aligned} z &= \frac{2 \pm \sqrt{(4-40)}}{2} \\ &= \frac{2 \pm 6i}{2} \\ &= 1 \pm 3i \end{aligned}$$

c $a=1, b=4, c=29$

$$\begin{aligned} z &= \frac{-4 \pm \sqrt{(16-116)}}{2} \\ &= \frac{-4 \pm 10i}{2} \\ &= -2 \pm 5i \end{aligned}$$

d $z^2 + 10z + 26 = 0$

$$\begin{aligned} (z+5)^2 - 25 + 26 &= 0 \\ (z+5)^2 &= -1 \\ z+5 &= \pm i \\ z &= -5 \pm i \end{aligned}$$

e

$$\begin{aligned} z^2 + 5z + 25 &= 0 \\ \left(z + \frac{5}{2}\right)^2 - \frac{25}{4} + 25 &= 0 \\ \left(z + \frac{5}{2}\right)^2 &= -\frac{75}{4} \\ z + \frac{5}{2} &= \pm \frac{i\sqrt{25 \times 3}}{\sqrt{4}} \\ z &= -\frac{5}{2} \pm \frac{5i\sqrt{3}}{2} \end{aligned}$$

3 f

$$\begin{aligned} z^2 + 3z + 5 &= 0 \\ \left(z + \frac{3}{2}\right)^2 - \frac{9}{4} + 5 &= 0 \\ \left(z + \frac{3}{2}\right)^2 &= -\frac{11}{4} \\ z + \frac{3}{2} &= \pm \frac{i\sqrt{11}}{2} \\ z &= -\frac{3}{2} \pm \frac{i\sqrt{11}}{2} \end{aligned}$$

4 [Note that **4a** uses completion of the square and **4b** & **4c** use the quadratic formula]

a

$$\begin{aligned} 2z^2 + 5z + 4 &= 0 \\ z^2 + \frac{5}{2}z + 2 &= 0 \\ \left(z + \frac{5}{4}\right)^2 - \frac{25}{16} + 2 &= 0 \\ \left(z + \frac{5}{4}\right)^2 &= -\frac{7}{16} \\ z + \frac{5}{4} &= \pm \frac{i\sqrt{7}}{4} \\ z &= -\frac{5}{4} \pm \frac{i\sqrt{7}}{4} \end{aligned}$$

b Use the quadratic formula with $a=7, b=-3$ and $c=3$

$$\begin{aligned} z &= \frac{-(-3) \pm \sqrt{(-3)^2 - (4)(7)(3)}}{2(7)} \\ &= \frac{3 \pm \sqrt{9-84}}{14} \\ &= \frac{3 \pm \sqrt{-75}}{14} \\ &= \frac{3}{14} \pm \frac{5\sqrt{3}}{14}i \end{aligned}$$

- 4 c Use the quadratic formula with $a = 5$, $b = -1$ and $c = 3$

$$\begin{aligned} z &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(3)}}{2(5)} \\ &= \frac{1 \pm \sqrt{1 - 60}}{10} \\ &= \frac{1}{10} \pm \frac{\sqrt{59}}{10}i \end{aligned}$$

- 5 Method 1: Completing the square

$$\begin{aligned} (z - 4)^2 - 16 + 21 &= 0 \\ (z - 4)^2 &= -5 \\ z - 4 &= \pm i\sqrt{5} \\ z &= 4 \pm i\sqrt{5} \end{aligned}$$

So $z_1 = 4 + i\sqrt{5}$ and $z_2 = 4 - i\sqrt{5}$

Method 2: using the quadratic formula

$$\begin{aligned} z &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(21)}}{2(1)} \\ &= \frac{8 \pm \sqrt{-20}}{2} \\ &= \frac{8 \pm i\sqrt{4}\sqrt{5}}{2} \\ &= 4 \pm i\sqrt{5} \end{aligned}$$

- 6 $z^2 + bz + 11 = 0$ has two distinct complex

roots when

$$\begin{aligned} b^2 - 4(1)(11) &< 0 \\ b^2 &< 44 \\ -\sqrt{44} &< b < \sqrt{44} \\ \text{or } -2\sqrt{11} &< b < 2\sqrt{11} \end{aligned}$$