OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4725**

Further Pure Mathematics 1

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

1 Use formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(r+1) = \tfrac{1}{3}n(n+1)(n+2)\,.$$ [5]

2 The cubic equation $x^3 - 6x^2 + kx + 10 = 0$ has roots $p - q$, p and $p + q$, where q is positive.

(i) By considering the sum of the roots, find p. [2]

(ii) Hence, by considering the product of the roots, find q. [3]

(iii) Find the value of k. [3]

3 The complex number $2 + \mathrm{i}$ is denoted by z, and the complex conjugate of z is denoted by z^*.

(i) Express z^2 in the form $x + \mathrm{i}\,y$, where x and y are real, showing clearly how you obtain your answer. [2]

(ii) Show that $4z - z^2$ simplifies to a real number, and verify that this real number is equal to zz^*. [3]

(iii) Express $\dfrac{z+1}{z-1}$ in the form $x + \mathrm{i}\,y$, where x and y are real, showing clearly how you obtain your answer. [3]

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 3^{2n} - 1\,.$$

(i) Write down the value of u_1. [1]

(ii) Show that $u_{n+1} - u_n = 8 \times 3^{2n}$. [3]

(iii) Hence prove by induction that each term of the sequence is a multiple of 8. [4]

5 **(i)** Show that

$$\frac{1}{2r-1}-\frac{1}{2r+1}=\frac{2}{4r^2-1}.$$ [2]

(ii) Hence find an expression in terms of n for

$$\frac{2}{3}+\frac{2}{15}+\frac{2}{35}+\ldots+\frac{2}{4n^2-1}.$$ [4]

(iii) State the value of

(a) $\displaystyle\sum_{r=1}^{\infty}\frac{2}{4r^2-1}$, [1]

(b) $\displaystyle\sum_{r=n+1}^{\infty}\frac{2}{4r^2-1}$. [1]

6 In an Argand diagram, the variable point P represents the complex number $z = x + \mathrm{i}\,y$, and the fixed point A represents $a = 4 - 3\mathrm{i}$.

(i) Sketch an Argand diagram showing the position of A, and find $|a|$ and $\arg a$. [4]

(ii) Given that $|z-a| = |a|$, sketch the locus of P on your Argand diagram. [3]

(iii) Hence write down the non-zero value of z corresponding to a point on the locus for which

(a) the real part of z is zero, [1]

(b) $\arg z = \arg a$. [2]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by **A**. [3]

(ii) The value of $\det \mathbf{A}$ is 5. Show clearly how this value relates to your diagram in part **(i)**. [3]

A represents a sequence of two elementary geometrical transformations, one of which is a rotation R.

(iii) Determine the angle of R, and describe the other transformation fully. [3]

(iv) State the matrix that represents R, giving the elements in an exact form. [2]

8 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 2 & -1 \\ 2 & 3 & -1 \\ 2 & -1 & 1 \end{pmatrix}$, where a is a constant.

(i) Show that the determinant of **M** is $2a$. [2]

(ii) Given that $a \neq 0$, find the inverse matrix $\mathbf{M}^{-1}$. [4]

(iii) Hence or otherwise solve the simultaneous equations

$$\begin{aligned} x + 2y - z &= 1, \\ 2x + 3y - z &= 2, \\ 2x - y + z &= 0. \end{aligned}$$

[3]

(iv) Find the value of k for which the simultaneous equations

$$\begin{aligned} 2y - z &= k, \\ 2x + 3y - z &= 2, \\ 2x - y + z &= 0, \end{aligned}$$

have solutions. [3]

(v) Do the equations in part **(iv)**, with the value of k found, have a solution for which $x = z$? Justify your answer. [2]